Statistics and probability

8 Probability

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Contents

Prior learning
Chapter 8

Parent guide
Chapter 8

Curriculum guide
Chapter 8

Australian Curriculum statements

Chance

Identify complementary events and use the sum of probabilities to solve problems

Describe events using language of 'at least', exclusive 'or' (A or B but not both), inclusive 'or' (A or B or both) and 'and'.

Represent such events in two-way tables and Venn diagrams and solve related problems

There are endless situations that involve making decisions. Your ability to make an informed decision can be assisted by accurately estimating the probability or likelihood of future events taking place. Decisions involving probability are made by everyone – from the business person examining a new business opportunity, to the scientist testing a new treatment for a disease, to the office worker deciding whether to take an umbrella to work on a particular day.
Politicians are another group of people who have a keen interest in the likelihood of events – especially at election times or when certain issues become 'hot' topics for debate. They often look at the results of opinion polls to help with their decision making.

Mathematical literacy

Maths dictionary

The mathematical words below have special meanings that you will learn in this chapter. It is important that you learn to spell them and gradually learn what they mean in mathematics. You may find the glossary or online mathematical dictionary useful for this purpose.

certain	event	impossible	simulation
chance	expected frequency	likelihood	theoretical probability
complement	experiment	likely outcome	tree diagram
complementary event	experimental probability	outcome	trial
die/dice	favourable outcome	possible	two-step probability
element	fifty-fifty chance	probability	two-way table
equally likely		random	unlikely
even chance		sample point	Venn diagram
		sample space	

8.1 Basic probability

When you consider what could happen in the future, you make judgements about the **chances** or **likelihoods** of particular things occurring.
People often have a 'feeling' that something will happen. Some things appear to be certain while others appear to have little or no chance of occurring. In mathematics, there are terms that can be used to describe the chance of something happening.

Important!

Likelihood

Something that cannot happen is called **impossible**.
Something that might or might not occur is **possible**. The **possible outcomes** (**events**) of a situation are all the things that could happen.
Something that must occur is called a **certain** event.
Something that is just as likely to occur as not occur is called an **even chance** or **fifty-fifty chance**.
When we compare possible events, we say that the event with a greater chance of occurring is the **more likely** event and the other is the **less likely** event.
Possible events that have the same chance of occurring are called **equally likely** events.

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Example 1

A buttered piece of bread falls on the floor. Compare the chances of the possible outcomes.

Solution

List what can happen.	**The possible outcomes are that the toast lands buttered-side down or buttered-side up.**
Compare the chances.	**The chances of landing buttered-side up or buttered side down are really the same.**
Write the answer.	**When a piece of toast falls on the floor, it is equally likely to land buttered-side up or buttered-side down.**

Investigate: Certain and impossible events

Work in groups of 4 or 5 for this investigation.
There are a number of sayings about the certainty or impossibility of events.
Benjamin Franklin (1706–1790), one of the Founding Fathers of the United States, claimed that he wrote the following in a letter to a friend: 'Our Constitution is in actual operation. Everything appears to promise that it will last; but in this world nothing is certain but death and taxes.' This has given rise to the now famous saying 'Nothing is certain but death and taxes'.

- Discuss the certainty of different events with your group.
- Try to identify some events that are certain.
- Make a list of any events you agree are certain.

Another saying that relates to the likelihood of events comes from Charles Dickens's *Pickwick Papers* (1837). Dickens wrote: 'Nothing is impossible, anything can happen, as in Mary said Tom would never call her again, but I told her, "Never say never." '

- What does the adage 'Never say never' suggest about impossibility of events?
- Try to identify some events that are impossible.
- Make a list of any events you agree are impossible.
- Discuss the findings of your group with other groups in the class.
- What do you conclude about the wisdom of these adages?

You may have heard of Murphy's Law, which says that 'Anything that can go wrong, will go wrong'. According to Murphy's Law, buttered toast will always land buttered-side down. This is not really true, but you are more likely to notice this outcome because you then have to clean up the mess. When you notice something more often, you tend to think it occurs more often. If you wanted to check the likelihood of different events, you could perform an experiment to work out which event occurred more often. In some cases, you could use existing information.

Investigate: Buttered toast

A piece of buttered toast is the same on both sides, except that one side has a coating on it. You could use any square shape to represent a piece of toast, such as a square cut from a packing carton.

1 Make some pretend toast by cutting squares from a packing carton.
2 Label one side of your 'toast' as buttered.
3 Toss your 'toast' and record which side it lands.
4 Repeat the experiment many times.
5 Put together the results for the whole class.
6 Work out some conclusions about Murphy's Law.

In mathematics, **probability** is the measure of how likely an event is. **Experimental probability** uses real data to work out chances or likelihoods.

Important!

Probability

Each time we try an experiment it is called a **trial**.
The result of the trial is called an **outcome**.
An **event** is one or more outcomes of an experiment.
The result that we are interested in is called the **favourable outcome** of a trial.
More likely outcomes have higher frequencies than less likely outcomes.

$$\text{Relative frequency} = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

The relative frequency of an event is its **probability**. Probabilities can be expressed as decimals between 0 and 1, as fractions, or as percentages.
You can use the probability of an event to predict the number of times an event is likely to occur in the future. This is called the **expected frequency**.

$$\text{Expected frequency} = \text{Probability} \times \text{Number of trials}$$

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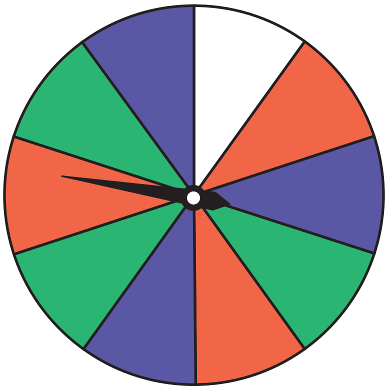
Example 2

The spinner shown here is spun and the colour it landed on was recorded in the table.



Colour	Red	Blue	Green	White
Frequency	28	33	31	8

a What is a trial in this experiment?
b What is the total frequency for this experiment?
c What is the relative frequency of spinning red?
d What is the relative frequency of spinning white?
e If the same spinner was spun 25 times how many times would you expect it to finish on green?

Solution

a Each time you do an experiment is a trial. — **A trial is one spin of the spinner.**

b Add up the frequencies. — **Total frequency** $= 28 + 33 + 31 + 8 = 100$

c The frequency of red is 28. — **Relative frequency of red** $= \frac{28}{100}$

Simplify. — $= \frac{7}{25} = 0.28$ **or** 28%

d The frequency of white is 8. — **Relative frequency of white** $= \frac{8}{100}$

Simplify. — $= \frac{2}{25} = 0.08$ **or** 8%

e Calculate the relative frequency of green. — **Relative frequency of green** $= \frac{31}{100}$

Simplify. — $= 0.31$

Write as a probability. — **Probability of green** $= 0.31$

Write the formula. — **Expected frequency** $=$ **Probability** $\times$ **Number of trials**

Substitute values. — $= 0.31 \times 25$

$= 7.75$

Write the answer. — **You would expect the spinner to land on green about 8 times in 25 spins.**

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Example 3

Thirty people in McLeod Street, Cairns, were asked where they were born. The results were:

Place of birth	Cairns	Elsewhere in Queensland	Interstate	Overseas
Frequency	9	9	4	8

a Which is more likely, being born in Cairns or interstate?
b What is the relative frequency of being born overseas?
c If another 50 people in Shields Street were asked, how many would you expect to have been born in Cairns?
d If you asked the same question in the Flinders Street Mall in Townsville, would you expect similar results? Why or why not?

Solution

a Being born in Cairns has the higher frequency.
Being born in Cairns is more likely.

b Write the formula.
Relative frequency of overseas birth
$$= \frac{\text{Frequency of event}}{\text{Total frequency}}$$

Substitute values.
$$= \frac{8}{30}$$

Simplify.
$$= \frac{4}{15} \approx 0.27 \text{ or } 27\%$$

On your calculator. **4** ÷ **15** = → 0.266666667

c Find the relative frequency.
Relative frequency of Cairns birth
$$= \frac{\text{Frequency of event}}{\text{Total frequency}}$$

Substitute values and simplify.
$$= \frac{9}{30} = \frac{3}{10} = 0.3$$

Write the formula.

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	Expected frequency = Probability × Number of trials
Substitute values.	= 0.3 × 50
Evaluate.	= 15
Write the answer in words.	**You would expect 15 people to be born in Cairns.**
d The conditions are not the same.	**You would not expect similar results because the conditions are different. For example, you would expect fewer born in Cairns.**

You do not need to perform experiments in order to calculate probabilities. You already know from experience that the possible outcomes of certain experiments are equally likely. For example, you know from experience that, when you toss a coin, there is no difference between the outcomes of heads and tails that would make one more likely than the other.

Important!

Theoretical probability

Collected data is not used to calculate **theoretical probability**.
It is based on a list, called the **sample space**, that contains all the possible outcomes. Each outcome in the sample space is listed exactly as many times as it is possible for it to occur.
Each element of the sample space is called an **element** or **sample point**.
The number of ways that an event can occur is written as n(Event).
The **probability** of an event is written as P(Event) and is a measure of its likelihood.
Probability is calculated from the sample space using the formula:

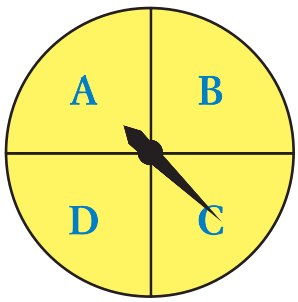
$$P(\text{Event}) = \frac{n(\text{Event})}{n(\text{Sample space})}$$

Example 4

The spinner shown on the right has four letters.

a What is a sample space for this spinner?

b Are the outcomes equally likely?



Solution

a There are four possible numbers that the spinner can land on.	**The sample space is {A, B, C, D}.**
b Each region of the pointer is the same size—a quarter of the circle.	**The outcomes are equally likely.**

Example 5

A cubic die has six faces each with a different number between 1 and 6.

a List the sample space.

b Are the outcomes equally likely?

Solution

a There are six different numbers of dots that are possible. **The sample space is {1, 2, 3, 4, 5, 6}.**

b The die is a cube and the faces are the same size. **The outcomes are equally likely.**

Before using theoretical probability to calculate a probability, you must make sure that the outcomes are equally likely.

Example 6

In each of the following cases, determine if the outcomes are equally likely. If the outcomes are equally likely, write the sample space.

a A matchbox is thrown and it is noted whether it falls flat, on the narrow side or on its end.

b A normal six-sided die is thrown and the uppermost number noted.

Solution

a The sides of a matchbox are different sizes. **The outcomes are not equally likely.**

b The sides of a normal die are all the same. **The outcomes are equally likely.**

List all possible outcomes. **The sample space is {1, 2, 3, 4, 5, 6}.**

The word **random** is used to indicate that something happens by chance – without intention. In probability, random events are equally likely.

Example 7

A bag contains three red, two yellow and five green marbles. A marble is taken out at random.

a Find the probability that the marble is yellow.

b If the marble is replaced and this is done 40 times, on how many occasions would you expect the marble to be yellow?

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Solution

a List the possibilities using letters. — **The sample space is {R, R, R, Y, Y, G, G, G, G, G}.**

Write the formula for this case. — $P(\text{Yellow}) = \dfrac{n(\text{Yellow})}{n(\text{Sample space})}$

Substitute values. — $= \dfrac{2}{10} = 0.2$

b Write the formula. — **Expected frequency = Probability × Number of trials**

Substitute values. — $= 0.2 \times 40$

$= 8$

Write the answer. — **You would expect to get a yellow marble about eight times.**

Exercise 8.1 Basic probability

Understanding

Extra questions
Exercise 8.1

Worked solutions
Exercise 8.1

1 Rate each of these events as 'impossible', 'very unlikely', 'even chance', 'very likely' or 'certain'.
- **a** You draw a black card from a normal deck of playing cards.
- **b** There will be a sunny day next week.
- **c** You will check your Facebook account today.
- **d** When you toss a coin it will show 'heads'.
- **e** Your dog has two tails.
- **f** When a six sided die is rolled, the upper face will show an even number.
- **g** You will vote in an Australian Government election tomorrow.
- **h** You will vote in an Australian Government election some time in the next 20 years.

2 **a** Write down some impossible events. **b** Write down some certain events.

Worked solutions
Exercise 8.1

3 Is the chance of each of the following events more than or less than an even chance?
- **a** selecting a diamond from a normal deck of playing cards
- **b** picking the winner of the next Melbourne Cup
- **c** getting married before you are 40
- **d** eating take-away food next week
- **e** owning your own car by the end of the year
- **f** celebrating your 50th birthday

Worked solutions
Exercise 8.1

4 **a** Draw the scale of likelihood shown below on your notebook.

b Mark the position of each of these types of chances on the scale you have drawn.

highly likely	rarely	often
never	perhaps	fifty-fifty
very uncertain	maybe	sure thing
doubtful	highly unlikely	against the odds
no chance	likely	some chance
improbable	probable	not likely

See Example 1

5 Compare the chances of the outcomes for each of the following possibilities.

a Getting head or getting a tail when tossing a coin.

b Getting a 6 or getting a number other than 6 when rolling a die.

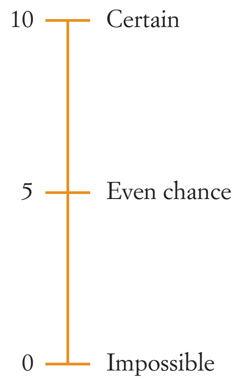
c Winning or losing a car race against the world champion driver.

d Winning or losing a game of scrabble against a 5-year-old.

e Being dealt a pair or being dealt a flush in five cards.

6 Consider the Scale of likelihood on the right where a certain event has a likelihood score of '10', an even chance has a likelihood score of '5' and an impossible event has a likelihood score of '0'.
Use the scale to give a likelihood score between 0 and 10 to each of the following events.

Scale of likelihood



a You will represent Australia in basketball in the next Olympic games.

b It will rain next week.

c You will catch a fish without bait on the hook.

d When you toss a coin it will show 'heads'.

e You will draw a spade from a normal deck of playing cards.

f You will roll a number less than 7 when you roll a six-sided die.

g There will be a full moon tonight.

h It will snow in Darwin tomorrow.

i You will pass your next maths test.

7 Arrange these events in order, from the least likely to the most likely:

a You have a brother or a sister.

b The next person you speak to is female.

c Your favourite sports team will win their next game.

d There will be no car accidents in Australia tomorrow.

e No one at your school has a birthday today.

f It will not rain tomorrow.

g You will eat take away food this week.

h A cyclone will cross the Australian coastline this week.

i The Sun will set tomorrow.

j It will be cold tomorrow.

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8 **a** Copy this table into your workbook. *See* Example 2

Outcome	Tally	Frequency
Tail		
Head		

b Toss a coin 50 times and record the outcome of each toss in the table.

c What is a trial in this experiment?

d What is the total frequency?

e What is the relative frequency of a head?

f What is the relative frequency of a tail?

9 **a** Copy this table into your workbook. *See* Example 2

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		

b Roll a die 40 times and record the outcome of each roll in the table.

c What is a trial in this experiment?

d What is the total frequency?

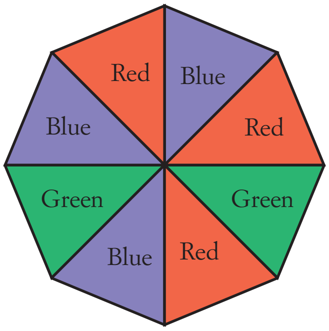
e What is the relative frequency of a 6?

f What is the relative frequency of a 2?

g What is the relative frequency of an even number?

Worked solutions
Exercise 8.1

10 Make a copy of the spinner shown on the right. Cut it out and stick it onto some light cardboard. Place a toothpick or something similar through the centre so that it can be spun so that it lands on one of the flat edges.



a Copy this table into your workbook.

Outcome	Tally	Frequency
Blue		
Green		
Red		

b Spin the spinner 40 times and record the outcome of each spin in the table.

c What is a trial in this experiment?

d What is the total frequency?

e What is the relative frequency of blue?

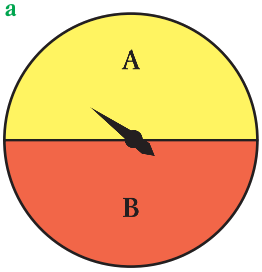
f What is the relative frequency of green?

g What is the relative frequency of red?

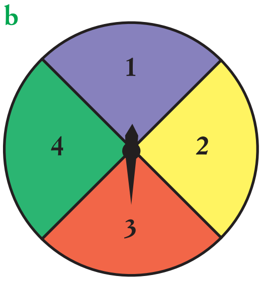
See Example 4

11 Use the spinners below to answer the following questions.

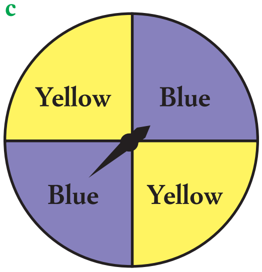
a



b



c



i List the sample space for the each of the spinners.

ii Are each of the events equally likely?

See Example 5

12 A bag contains five green marbles, three blue marbles, two red marbles and two yellow marbles. One marble is drawn at random from the bag.

a List the sample space.

b Are the four different colour outcomes equally likely?

13 For the spinners in question **11**, calculate the probability of the pointer landing on yellow.

14 For the situation in question **12**, calculate the probability of drawing:

a a red marble **b** a green marble

Fluency

15 Use the results of question **8** to answer the following questions.

a According to your results, which outcome (head or tail) is more likely?

b If a coin was tossed 100 times, what would you expect to be the frequency of heads?

c If a coin was tossed 500 times, what would you expect to be the frequency of tails?

16 Use the results of question **9** to answer the following questions.

a According to your results, which outcome (1, 2, 3, 4, 5 or 6) is most likely?

b Compare the probabilities of the outcomes: an even number and an odd number.

c If a die was rolled 1000 times, how many times would you expect it to result in a 5?

d If a die was rolled 200 times, how many times would you expect it to result in an odd number?

17 Use the results of question **10** to answer the following questions.

a According to your results, which outcome (blue, green or red) is least likely?

b Compare the probabilities of the outcomes: red and green.

c If the spinner was spun 100 times, how many times would you expect it to finish on red?

d If the spinner was spun 400 times, how many times would you expect it to finish on green?

Worked solutions
Exercise 8.1
See Example 3

18 Two coins were tossed and the following results obtained.

TH	HT	TT	TH	TT	TT	HT	TH	TH	HH
HT	HT	TH	HH	TT	TH	HT	TH	HT	TT

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a Construct a frequency table.
b Calculate the relative frequencies of each outcome as decimals.
c According to these results, are you more likely to get 'two tails' or 'heads and tails'?

19 A pack of cards was cut and the suit was noted. This was done 25 times and the results were:

See Example 3

Hearts (♥)	Diamonds (♦)	Clubs (♣)	Spades (♠)
7 times	5 times	5 times	8 times

a Calculate the relative frequencies of each outcome as percentages.
b Does it appear that a black card or a red card is more likely?
c Does it appear that a spade or a diamond is more likely?

20 The numbers of oranges in 2 kg bags of oranges were counted. The numbers in each bag were:

See Example 3

14	15	16	14	15	14	15	17	14	15
16	17	14	20	16	17	17	13	15	15
17	17	15	15	14	15	18	17	16	17

a Draw up a frequency table.
b Calculate the relative frequencies of each outcome in as fractions.
c Based on these results, are you more likely to get a bag with 15 or 20 oranges?

21 In each of the following cases, state whether the events are equally likely, and write the sample space accordingly.

See Example 6

a There are five red, three green and four blue marbles in a bag. One marble is chosen from the bag.
b A coin is tossed twice and the number of tails is noted.
c A drawing pin is thrown and it is noted whether it lands point up or point down.
d The score from throwing a dart at a dartboard is noted.
e Two multiple-choice questions have answers A, B and C, with one correct answer for each. Answers are chosen at random and the number of correct answers is noted.

22 There are four red, three green and eight blue marbles in a bag. One marble is taken out.

Worked solutions
Exercise 8.1

See Example 7

a List the sample space.
b What is n(Red marble)?
c Work out the P(Red marble).
d Find P(Green marble).
e Work out P(Blue marble).
f What do all the probabilities add up to?

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Worked solutions
Exercise 8.1

23 A flea-market stall has unlabelled cans of sliced beetroot, pineapple pieces and potato salad for sale. The stallholder says that eight of the cans are pineapple, six are potato salad and two are beetroot. You buy one can.

a Use SB, PP and PS to list the sample space.

b Work out P(SB).

c Work out P(PP).

d Work out P(PS).

24 In a loose leaf folder there are three blank sheets, two graph sheets and five lined sheets of paper. A sheet of paper is taken out at random.

a Write down the sample space.

b Work out n(blank) and P(blank).

c Work out n(graph) and P(graph).

d Work out n(lined) and P(lined).

e What is the most likely kind of paper to be picked out?

See Example 7

25 A normal die is rolled.

a What is the probability of throwing a 6?

b What is $P(2)$?

c What is P(even number)?

d What is the probability of more than 4?

e What is P(prime number)?

f If the die is rolled 150 times, on how many occasions would you expect the result to be a 2?

Problem solving

Worked solutions
Exercise 8.1

26 'Incy wincy spider climbed up the water spout. Down came the rain and washed the spider out!' Some children counted the legs on spiders washed down the gutter after a storm. The numbers of legs they found were as follows:

7	6	7	7	8	7	5	7	7	7
7	6	5	6	8	7	7	8	7	6
8	8	8	7	6					

a Draw up a frequency table.

b Are you more likely to find a spider with six legs or seven legs?

27 Lettuce plants are supposed to grow better if they are planted out small. Unfortunately, the snails eat more of the plants when they are small. The lettuces are usually big enough after 8 days to survive. The table shows the number of lettuces eaten each day when 50 lettuces were planted.

Day	1	2	3	4	5	6	7	8	9	10
Number eaten	6	4	6	3	5	6	5	0	0	0

a How many lettuces were eaten in the first seven days?

b Are lettuces more likely to be eaten by snails in the first three days or the next three days?

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28 Refer to question **20**.
  a From 140 bags of oranges, how many would you expect to have 20 oranges?
  b From 500 bags of oranges, how many would you expect to have 15 oranges?

Reasoning

29 Would it be wise to use the results of question **19** to predict the number of diamonds you should expect from cutting a pack of cards 300 times?

30 Refer to question **27**.
  a What is the relative frequency of lettuces being killed by snails?
  b What is the relative frequency of lettuces surviving snail attacks?
  c How many lettuces would probably be lost if 80 were planted?
  d How many lettuces should be planted to end up with 50?

31 A lucky dip has 4 prizes worth \$2, 6 prizes worth \$1, 10 prizes worth 20c and 180 prizes worth 10c. It costs 20c per dip to play.
  a What is the probability of getting a prize worth more than 20c?
  b What is the probability of getting only 10c?
  c If you played this game 600 times, how much would you expect to win or lose?

32 A fair coin has been tossed five times and there have been five heads in a row. What is the probability that the next time it lands on tails? Explain your answer.

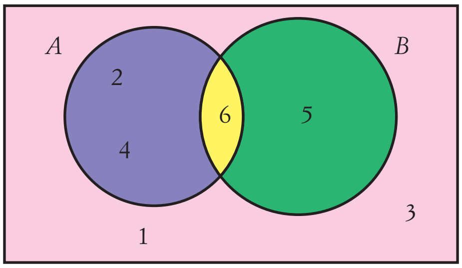
8.2 Two-way tables and complementary events

Sometimes you can use diagrams to help understand situations. When dealing with the outcomes of an experiment, **Venn diagrams** are useful to help understand the relationships between events. A Venn diagram shows all outcomes of the sample space of an experiment in a box. Various events can be shown as circles within the box.

Consider the experiment where a die is rolled and the uppermost number is noted. The sample space for this experiment is {1, 2, 3, 4, 5, 6}. Imagine that you are interested in the following two events:

$$\text{Event } A = \text{an even number} = \{2, 4, 6\}$$
$$\text{Event } B = \text{a number greater than } 4 = \{5, 6\}$$

We can represent the sample space for this experiment and the events A and B in a Venn diagram as shown below.



By looking at the Venn diagram on the previous page you can see the following.

- Event A is shown as the circle shaded blue and yellow and event B is shown as the circle shaded green and yellow.
- The yellow region is the intersection of events A and B and it contains the element that is common to both events: 6.
- The blue region contains elements that are included in event A but not in event B: 2 and 4.
- The green region contains an element that is included in event B but not in event A: 5.
- The pink region contains elements that are not included in event A or event B: 1 and 3.

Example 8

A six-sided die is rolled and the uppermost number noted. What is the probability that the number is:

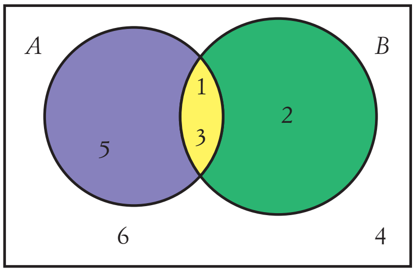
a odd? **b** less than 4?
c odd or less than 4? **d** odd and less than 4?

Solution

Let event A be 'odd number' and event B be 'number less than 4'. List the sample space and the outcomes of each event.

Sample space $= \{1, 2, 3, 4, 5, 6\}$
Event A = **odd number** $= \{1, 3, 5\}$
Event B = **number less than 4** $= \{1, 2, 3\}$

Draw a Venn diagram for this experiment showing events A and B.



a Use the rule for probability.

$$P(\text{Odd}) = \frac{n(\text{Odd})}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{3}{6}$$

Evaluate.

$$= \frac{1}{2}$$

b Use the rule for probability.

$$P(< 4) = \frac{n(< 4)}{n(\text{Sample space})}$$

Substitute required values.

$$= \frac{3}{6}$$

Evaluate.

$$= \frac{1}{2}$$

c Use the Venn diagram to list the outcomes that are either odd or less than 4.

Odd or $< 4 = \{1, 2, 3, 5\}$

Use the rule for probability.

$$P(\text{Odd or} < 4) = \frac{n(\text{Odd or} < 4)}{n(\text{Sample space})}$$

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Substitute required values. $= \frac{4}{6}$

Evaluate. $= \frac{2}{3}$

d Use the Venn diagram to list the outcomes that are both odd and less than 4. **Odd and < 4 = {1, 3}**

Use the rule for probability. $P(\text{Odd and} < 4) = \dfrac{n(\text{Odd and} < 4)}{n(\text{Sample space})}$

Substitute required values. $= \frac{2}{6}$

Evaluate. $= \frac{1}{3}$

When using Venn diagrams, you don't have to list individual elements of the events making up the sample space. Sometimes you only need to list the number of elements in the various events.

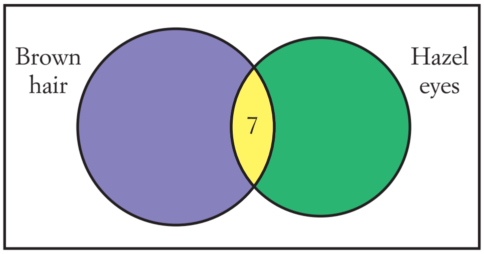
Example 9

Out of a group of 34 students, 22 have brown hair and 19 have hazel coloured eyes. If 7 of the students have brown hair and hazel eyes, find the probability that a student selected at random:

a does not have hazel eyes
b does not have brown hair.

Solution

a Draw a Venn diagram for this experiment showing events 'Brown hair' and 'Hazel eyes'. There are 7 students who have brown hair and hazel eyes. Enter 7 in the intersection of the circles representing the events.



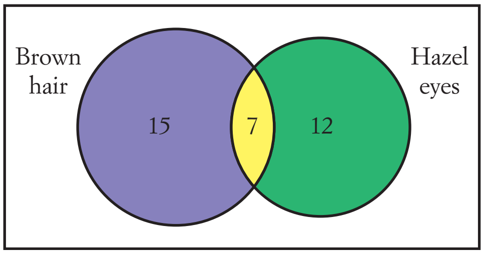
22 students have brown hair. Of these, 7 have hazel eyes.

Number students with brown hair with eye colour that is not hazel = 22 − 7
= 15

19 students have hazel eyes. Of these, 7 have brown hair.

Number students with hazel eyes who do not have brown hair = 19 − 7
= 12

Complete the Venn diagram.



Use the rule for probability. $P(\textbf{Student does not have hazel eyes})$

$= \dfrac{n(\textbf{Eyes not hazel})}{n(\textbf{Sample space})}$

Substitute required values. $= \dfrac{15}{34}$

Evaluate. $\approx 44.1\%$

b Use the rule for probability. $P(\textbf{Student does not have brown hair})$

$= \dfrac{n(\textbf{Hair not brown})}{n(\textbf{Sample space})}$

Substitute required values. $= \dfrac{19}{34}$

Evaluate. $\approx 55.9\%$

Tables can also be useful aids to understanding the outcomes of experiments. Consider the situation where a coin is tossed and a die is rolled.

This experiment consists of two parts and so you can represent the outcomes of this experiment using a **two-way table**. The two-way table is constructed by listing the possible outcomes of one part of the experiment along the top row and the outcomes of the other part down the left had column as shown here.

	1	2	3	4	5	6
H						
T						

The table is completed by filling in the cells as shown.

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Example 10

Use the two-way table for tossing a coin and rolling a six-sided die to find the probability that the outcome is:

a a head and an even number
b a head or an even number
c a tail or a number less than 3

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Solution

a Draw up the table.

	1	2	3	4	5	6
H	H1	H2	H3	H4	H5	H6
T	T1	T2	T3	T4	T5	T6

Use the table to find the number of elements in the sample space. $n(\text{Sample space}) = 12$

List the elements of the outcome. H and even = {H2, H4, H6}

Find the number of elements in the desired outcome. $n(\text{H and even}) = 3$

Use the rule for probability. $P(\text{H and even}) = \dfrac{n(\text{H and even})}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{3}{12}$

Evaluate. $= \dfrac{1}{4}$

b List the elements of the outcome. H or even = {H1, H2, H3, H4, H5, H6, T2, T4, T6}

Use the table to find the number of elements in desired outcome. $n(\text{H or even}) = 9$

Use the rule for probability. $P(\text{H or even}) = \dfrac{n(\text{H or even})}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{9}{12}$

Evaluate. $= \dfrac{3}{4}$

c List the elements of the outcome. T or < 3 = {T1, T2, T3, T4, T5, T6, H1, H2}

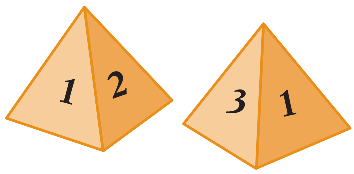
Use the table to find the number of elements in desired outcome. $n(\text{T or} < 3) = 8$

Use the rule for probability. $P(\text{H or even}) = \dfrac{n(\text{T or} < 3)}{n(\text{Sample space})}$

Substitute required values. $= \dfrac{8}{12}$

Evaluate. $= \dfrac{2}{3}$

The faces of two four-sided dice are numbered 1 through 4. The dice are rolled and the numbers on the bottom faces are noted. The two-way table for this experiment is shown here.



	1	2	3	4
1	1, 1	1, 2	1, 3	1, 4
2	2, 1	2, 2	2, 3	2, 4
3	3, 1	3, 2	3, 3	3, 4
4	4, 1	4, 2	4, 3	4, 4

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The probability that the total of the bottom faces is less than 10 is:

$$P(\text{Total} < 10) = \frac{n(\text{Total} < 10)}{n(\text{Sample space})} = \frac{8}{8} = 1$$

The probability that the total of the bottom faces is greater than 10 is:

$$P(\text{Total} > 10) = \frac{n(\text{Total} > 10)}{n(\text{Sample space})} = \frac{0}{8} = 0$$

For this experiment, the event 'total < 10' is a certain event. The event will occur every time. Certain events have a probability of 1.
For this experiment, the event 'total > 10' is an impossible event. The event will never occur. Impossible events have a probability of 0.
When a six-sided die is rolled, the sample space is $\{1, 2, 3, 4, 5, 6\}$.
The probability of each outcome is equally likely and is $\frac{1}{6}$.
Adding the probabilities of all possible outcomes for this experiment gives:

$$P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

These results are true for all experiments.

Important!

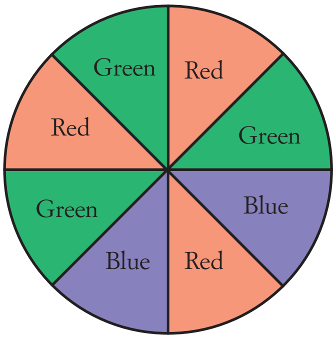
Probabilities of events

The probability of a **certain** event is 1.
The probability of an **impossible** event is 0.
The probabilities of all other events are between 0 and 1.
The sum of the probabilities of all outcomes for an experiment is 1.

Example 11

For this spinner, what is the probability of spinning:

a green
b a colour other than green
c green or a colour other than green



Solution

a List the sample space. — **The sample space is {G, G, G, R, R, R, B, B}.**

Use the probability formula for this case. — $\boldsymbol{P(\text{G}) = \frac{n(\text{G})}{n(\text{Sample space})}}$

Substitute values. — $\boldsymbol{= \frac{3}{8} = 0.375}$

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b 'A colour other than green' can be written as 'not green'. List the favourable outcomes.

Not green $= \{\mathbf{R, R, R, B, B}\}$

Use the probability formula.

$$\boldsymbol{P}(\textbf{Not G}) = \frac{\boldsymbol{n}(\textbf{Not G})}{\boldsymbol{n}(\textbf{Sample space})} = \frac{\mathbf{5}}{\mathbf{8}} = \mathbf{0.625}$$

c List the favourable outcomes.

Green or not green $= \{\mathbf{G, G, G, R, R, R, B, B}\}$

The event 'green or not green' includes all possible outcomes. The event 'green or not green' is certain.

$\boldsymbol{P}$(**Green or not green**) $= \mathbf{1}$

In the previous example you saw that the events 'green' and 'a colour other than green' together made up the entire sample space for the experiment. Events such as these are called **complementary events**.
Because an event and its complement together made up the entire sample space for the experiment, the sums of the probabilities of an event and its complement must equal 1.

Important!

Complementary events

For any event A, the **complement** of the event is shown as A' or $\overline{A}$.
The sum of the probabilities of an event and its complement is 1.
The probability of the complement of event A is the same as the probability that event A will not occur.

$$P(A) + P(A') = 1$$

So $$P(A') = 1 - P(A)$$

or $$P(A) = 1 - P(A')$$

Example 12

Find the complement of each of the following events.

a rolling a die and getting a 5

b it will rain tomorrow

Solution

a List the sample space.

The sample space is {1, 2, 3, 4, 5, 6}.

The complement of getting a 5 is not getting a 5.

The complement of 'getting a 5' is 'getting a 1, 2, 3, 4 or 6'.

b The event is that it will rain tomorrow.

The complement of 'raining tomorrow' is 'not raining tomorrow'.

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Example 13

A spinner only has whole number outcomes. The probability of getting an odd number is 0.38. Calculate the probability of getting an even number.

Solution

Getting an odd number and getting an even number are complementary events.	
Use the rule for the probabilities of an event and its complement.	$P(A) + P(A') = 1$
Substitute of the event and its complement.	$P(\text{Odd}) + P(\text{Even}) = 1$
Substitute for P(odd).	$0.38 + P(\text{Even}) = 1$
Subtract 0.38 from each side of the equation.	$P(\text{Even}) = 1 - 0.38 = 0.62$
State the result.	**The probability of getting an even number is 0.62.**

Exercise 8.2 Two-way tables and complementary events

Fluency

Extra questions
Exercise 8.2

See Example 8

1 A six-sided die is rolled and the uppermost number noted. If A = {an even number} and B = {a number > 2}:

a represent the outcomes of the experiment and events A and B using a Venn diagram.

What is the probability that the number is:

b even?

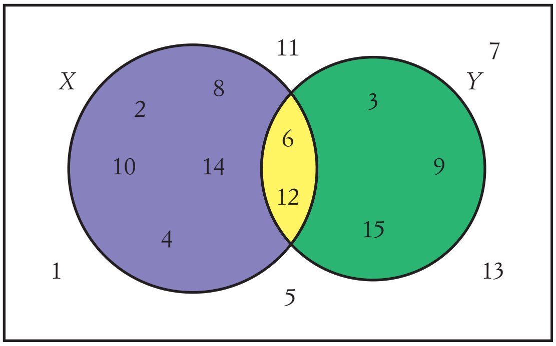
c greater than 2?

d even or greater than 2?

e even and greater than 2?

See Example 8

2 Numbered discs are randomly drawn from a container. X = {an even number} and Y = {a multiple of 3}. The outcomes of the experiment are shown in the Venn diagram below.



What is the probability that a selected number is:

a even?

b a multiple of 3?

c even or a multiple of 3?

d even and a multiple of 3?

e not even?

f not a multiple of 3?

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3 Out of a group of 40 students, 29 listen to an MP3 player on the way to school and 28 have a Facebook account. 17 of the students listen to an MP3 player on the way to school and have a Facebook account. See Example 9

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a student selected at random:

b does not listen to an MP3 player on the way to school

c does not have a Facebook account

4 A second-hand car yard has 233 vehicles. Of these, 63 are red, 44 are four wheel drives and 16 are red four wheel drives. See Example 9

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a vehicle selected at random:

b is red

c is a four wheel drive

d is not red

e is not a four wheel drive

f is neither red nor a four wheel drive.

5 Two six-sided dice are rolled and the total of the uppermost faces noted. Worked solutions Exercise 8.2
See Example 10

a Construct a two-way table to represent this experiment.

Use the two-way table to find the probability that the total is:

b even

c prime

d greater than 6

e less than 5

f even or greater than 6

g even and less than 6

6 A pizza restaurant sells pizzas in four different sizes: family, large, medium and small. The fillings available are meat lovers, Hawaiian, supreme, four cheeses and vegetarian. Over the years, the chef knows that all fillings and sizes are equally likely to be ordered. See Example 10

a Construct a two-way table to represent the possible outcomes when a pizza is ordered.

Use the two-way table to find the probability that the next pizza ordered is:

b a large size with any filling

c a supreme of any size

d a family size with any filling

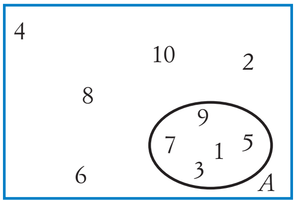
e a vegetarian of any size

f a medium meat lovers

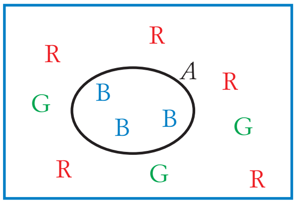
g a medium or a meat lovers

7 Each Venn diagram below shows the sample space for an experiment and an event A. In each case list elements of the complement of A.

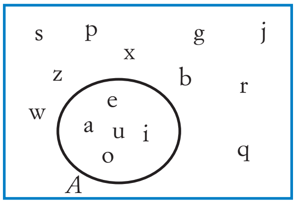
a



b



c



8 Find the complement of each of the following events.

a Getting a head when you toss a coin.

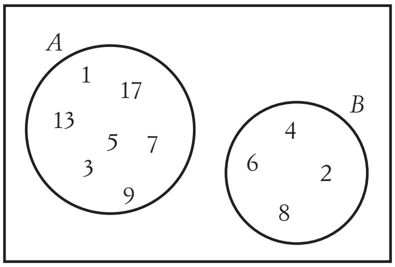
b Selecting a letter of the alphabet and getting a vowel.

c Selecting a month of the year and getting a month that ends in 'er'.
d Winning a game of tennis.
e Getting a number greater than 7 on a spinner with segments numbered 1 through 10.
f Getting an even number when a six-sided die is rolled.

Worked solutions
Exercise 8.2

9 A six-sided die is rolled. List the complement of each of the following events.
a getting a 6
b getting an odd number
c getting a prime number
d getting a number less than 3

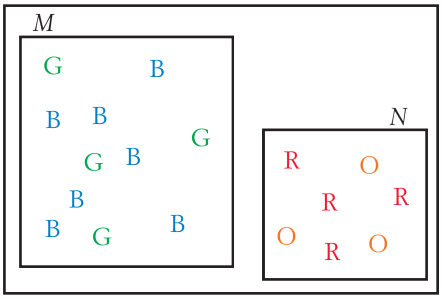
10 A bag contains 11 numbered cards. The Venn diagram on the right shows two events A and B that result when a card is drawn and the number on it noted.



Calculate:
a $P(A)$
b $P(B)$
c $1 - P(A)$
d $1 - P(B)$
e $P(A) + P(B)$

Complete the following:
f $P(A') = \ldots$
g $P(B') = \ldots$

11 A jar contains 18 marbles: each one coloured green, blue, red or orange. The Venn diagram on the right shows two events M and N that result when a marble is drawn and its colour noted.



Calculate:
a $P(M)$
b $P(N)$
c $1 - P(M)$
d $1 - P(N)$
e $P(M) + P(N)$

Complete the following:
f $P(M') = \ldots$
g $P(N') = \ldots$

12 A six-sided die is rolled and the number on the upper most face is noted. What is the complement of the event: a number greater than 4?
Select from A, B, C or D below.
A a number less than 4
B 1 and 2
C 5 and 6
D a number less than or equal to 4

Worked solutions
Exercise 8.2

13 A bag contains 2 green, 4 red, 6 blue and 8 yellow marbles.
A marble is selected at random. What is the probability of:
a getting a red
b getting a marble that isn't red
c getting a yellow marble
d getting a marble that isn't yellow

Worked solutions
Exercise 8.2

14 Two coins are tossed and the uppermost faces are noted.
a Draw a two way table to show all the possible outcomes.
Use the table to find the probability that the outcome is:
b two heads
c not two heads
d a head and a tail
e other than a head and a tail

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15 Two six-sided dice are rolled and the numbers on the uppermost faces are noted.

a Draw a two way table to show all the possible outcomes.

Use the table to find the probability that the total of the faces is:

b 12   **c** not 12

d an even number   **e** an odd number

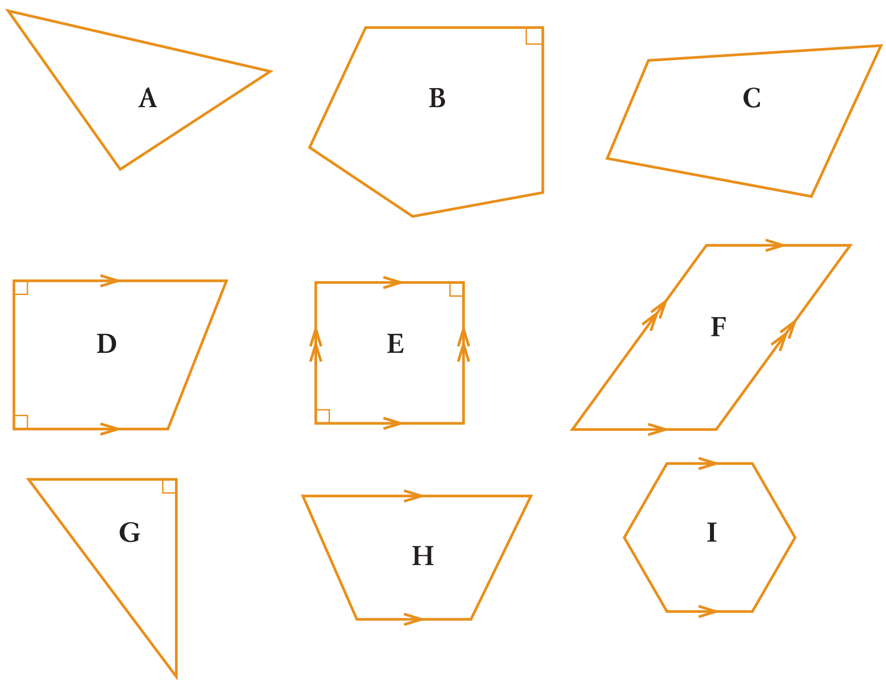
f a prime number   **g** not a prime number

h greater than 8   **i** less than or equal to 8

Problem solving

Worked solutions
Exercise 8.2

16 Some plane shapes are shown below. In an experiment, a shape is selected at ramdom.



If:

$P = \{\text{a shape with four sides}\}$

$Q = \{\text{a shape with a right angle}\}$

$R = \{\text{a shape with a pair of parallel sides}\}$

a Represent the experiment using a Venn diagram.

Use the Venn diagram to calculate the probability that a selected shape:

b has four sides and a right angle

c has four sides or a right angle

d has four sides or parallel sides

e has four sides, no right angle and no parallel sides

f does not have parallel sides

g does not have four sides but does have a right angle

h has four sides, a right angle and parallel sides

17 200 tennis players were asked which of these strokes they considered their weakest stroke(s): the serve, the drop shot, the lob.
Of the 200 players questioned:

- 30 players said none of these were their weakest stroke.
- 20 players said all three of these were their weakest stroke.
- 30 players said their serve and drop shot were their weakest strokes.
- 20 players said only their serve and lob were their weakest strokes.
- 25 players said their drop shot but not their lob was their weakest stroke.
- 42 players said only their lob was their weakest stroke.
- 85 players said their serve was their weakest stroke.

a Represent the experiment using a Venn diagram.

One of the 200 players is randomly selected. Use the Venn diagram to calculate the probability that a selected player thinks:

b only their serve and lob are their weakest strokes
c their serve is not their weakest stroke
d their weakest stroke is their serve
e only their lob is their weakest stroke

18 The letters of the alphabet are written on separate cards and placed in a bag. A card is selected from the bag at random.
a How many possible outcomes are there?
b Are the outcomes equally likely?

What is the probability that the selected card is:

c a, c, d, e, f, g, h, or j **d** not a, c, d, e, f, g, h, or j
e a vowel **f** not a vowel

Worked solutions
Exercise 8.2

19 Marbles numbered 1 through 20 are placed in a bag and one selected from the bag at random.
a How many possible outcomes are there?
b Are the outcomes equally likely?

What is the probability that the selected marble is:

c an even number **d** not even
e a prime number **f** not prime
g a number less than 30 **h** a number greater than 30

Reasoning

20 Twenty-five men and 25 women were asked whether they would prefer to listen to music, play sport or watch TV in their spare time.
Of those questioned:

- 4 men said they preferred to listen to music.
- 13 men said they preferred to play sport.
- a total of 15 people said they preferred to listen to music.
- a total of 19 people said they preferred to play sport.

a Represent the experiment using a two-way table.

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If a randomly selected group of 100 men and 100 women were asked the same question:

b how many people would you expect to say they preferred to listen to music?

c how many women would you expect to say they preferred to watch TV?

d how many men would you expect to say they preferred to play sport?

21 Marbles numbered 1 through 100 are placed in a bag. A marble is taken from the bag at random. What is the probability that the selected marble is:

a a multiple of 3 **b** not a multiple of 3

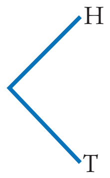
8.3 Two-step probabilities

A probability experiment that involves two steps is known as a **two-step chance experiment**. An example of a simple two-step experiment is tossing a coin twice. One way of representing the outcomes of a two-step experiment is by using a two-way table. You saw how a two-way table constructed in the previous section.

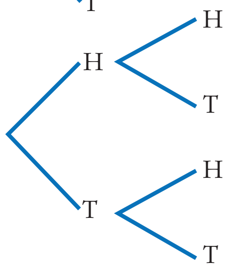
Another method of representing the outcomes of a two-step experiment is called a **tree diagram**.

We will construct a tree diagram for tossing a coin twice.

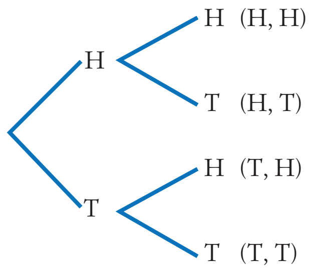
Begin with by marking a point on the page. The outcomes of the first step (H and T) are then listed under each other to the right of the point and a 'branch' is drawn from the point to each outcome.



List the outcomes of the second step to the right of each of the outcomes of the first step and draw branches between each of the outcomes.



Finally, at the end of each branch write the two outcomes in order.



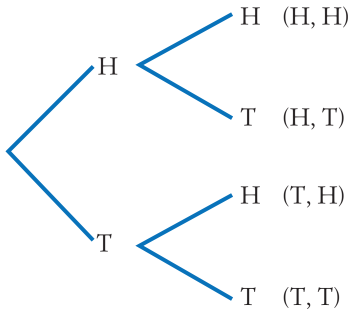
Example 14

A coin is tossed two times. Use a tree diagram to calculate the probability that the outcome of the experiment is:

a two heads **b** two tails
c a head and a tail **d** a head followed by a tail

Solution

a Draw the tree diagram to show the sample space for this experiment.



Use the probability formula. $P(2H) = \dfrac{n(2H)}{n(\text{Sample space})}$

There is only one outcome that is two heads. $= \dfrac{1}{4}$

b Use the probability formula. $P(2T) = \dfrac{n(2T)}{n(\text{Sample space})}$

There is only one outcome that is two tails. $= \dfrac{1}{4}$

c Use the probability formula. $P(\text{H and T}) = \dfrac{n(\text{H and T})}{n(\text{Sample space})}$

There are two favourable outcomes: H, T and T, H. $= \dfrac{2}{4}$

Simplify. $= \dfrac{1}{2}$

d Use the probability formula. $P(\text{H, T}) = \dfrac{n(\text{H, T})}{n(\text{Sample space})}$

There is one favourable outcome: H, T. $= \dfrac{1}{4}$

Example 15

A drawer contains three pairs of socks: one blue, one red and one green. A pair of socks is taken out of the drawer, the colour noted and returned to the drawer. This step is then repeated. Use a tree diagram to calculate the probability that the selected pairs of socks are:

a the same colour **b** different colours
c red and green **d** red followed by green

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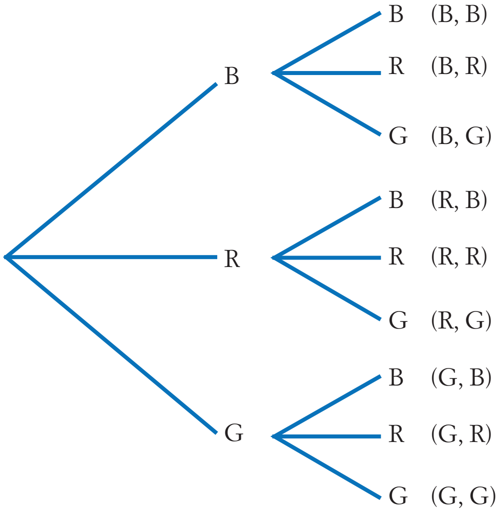
Solution

a Draw the tree diagram to show the sample space for this experiment.



Use the probability formula. $P(\text{Same colour}) = \dfrac{n(\text{Same colour})}{n(\text{Sample space})}$

There are three favourable outcomes: B, B; R, R and G, G. $= \dfrac{3}{9}$

Simplify. $= \dfrac{1}{3}$

b The events 'same colour' and 'different colours' are complementary. $P(\text{Same colour}) + P(\text{Different colours}) = 1$

Substitute the known value. $\dfrac{1}{3} + P(\text{Different colours}) = 1$

Subtract $\frac{1}{3}$ from both sides of the equation. $P(\text{Different colours}) = 1 - \dfrac{1}{3}$

Simplify. $= \dfrac{2}{3}$

c Use the probability formula. $P(\text{R and G}) = \dfrac{n(\text{R and G})}{n(\text{Sample space})}$

There are two favourable outcomes: R, G and G, R. $= \dfrac{2}{9}$

d Use the probability formula. $P(\text{R, G}) = \dfrac{n(\text{R, G})}{n(\text{Sample space})}$

There is only one favourable outcome. $= \dfrac{1}{9}$

In the previous example, the outcomes of each step were the same because the first pair of socks was replaced before the second pair was withdrawn. The situation where the outcomes of each step of an experiment are the same is said to involve **replacement**. In some experiments there is no replacement.

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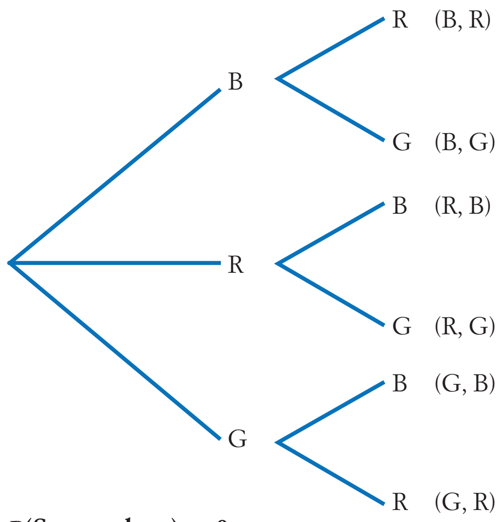
Example 16

A jar contains a red, blue and green disc. A disc is taken out of the jar, the colour noted and not returned. Another disc is then removed and the colour noted. Use a tree diagram to calculate the probability that the selected discs are:

a the same colour **b** different colours
c red and green **d** red followed by green

Solution

a Draw the tree diagram to show the sample space for this experiment.



There are no favourable outcomes. $P(\text{Same colour}) = 0$

b Every outcome consists of different colours. $P(\text{Different colours}) = 1$

c Use the probability formula. $P(\text{R and G}) = \dfrac{n(\text{R and G})}{n(\text{Sample space})}$

There are two favourable outcomes: R, G and G, R. $= \dfrac{2}{6}$

Simplify. $= \dfrac{1}{3}$

d Use the probability formula. $P(\text{R, G}) = \dfrac{n(\text{R, G})}{n(\text{Sample space})}$

There is only one favourable outcome. $= \dfrac{1}{6}$

It is also possible to show the outcomes of a two-step experiment without replacement using a two-way table. Consider the situation in the previous example. The two-way table is set up as shown here.

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	B	R	G
B			
R			
G			

There is no replacement for this experiment, so if a blue disc is selected in the first step, a blue disc can't be selected in the second step. Using this reasoning, the table can be completed as follows:

	B	R	G
B		B, R	B, G
R	R, B		R, G
G	G, B	G, R	

Example 17

A bag contains one red, two blue and two green marbles. A marble is taken out of the bag, the colour noted and not returned. Another marble is then removed and the colour noted. Use a two-way table to calculate the probability that the selected marbles are:

a the same colour **b** different colours
c blue and green **d** blue followed by green

Solution

a Draw the table and complete the cells. Remember to block out the cells that are not possible due to no replacement.

	R	B	B	G	G
R		R, B	R, B	R, G	R, G
B	B, R		B, B	B, G	B, G
B	B, R	B, B		B, G	B, G
G	G, R	G, B	G, B		G, G
G	G, R	G, B	G, B	G, G	

Use the probability formula. $P(\text{Same colour}) = \dfrac{n(\text{Same colour})}{n(\text{Sample space})}$

There are 4 favourable outcomes. $= \dfrac{4}{20}$

Simplify. $= \dfrac{1}{5}$

b The events 'same colour' and 'different colours' are complementary. $P(\text{Same colour}) + P(\text{Different colours}) = 1$

Substitute the known value. $\dfrac{1}{5} + P(\text{Different colours}) = 1$

Subtract $\frac{1}{5}$ from both sides of the equation. $P(\text{Different colours}) = 1 - \dfrac{1}{5}$

Simplify. $= \dfrac{4}{5}$

c Use the probability formula. $P(\text{B and G}) = \dfrac{n(\text{B and G})}{n(\text{Sample space})}$

There are 8 favourable outcomes: 4 lots of B, G and 4 lots of G, B. $= \frac{8}{20}$

Simplify. $= \frac{2}{5}$

d Use the probability formula. $P(\text{B, G}) = \frac{n(\text{B, G})}{n(\text{Sample space})}$

There are four favourable outcomes. $= \frac{4}{20}$

Simplify. $= \frac{1}{5}$

Exercise 8.3 Two-step probabilities

Fluency

See Examples 14, 15

Extra questions
Exercise 8.3

1 A four-sided die is rolled and the bottom number noted. A coin is then tossed.

Draw a tree diagram to represent this experiment.

See Examples 14, 15

2 Use the tree diagram drawn for question **1** to calculate the probability that the outcome will contain:

a a 3 and a head
b a 3 or head
c an even number and a head
d a head or an even number

See Examples 14, 15

3 A bag contains four cards numbered 1 through 4. A two digit number is formed by drawing a card, writing down its number, returning it to the bag and then repeating the first step. Draw a tree diagram to represent this experiment.

Worked solutions
Exercise 8.3

See Examples 14, 15

4 Use the tree diagram drawn for question **3** to calculate the probability of obtaining:

a a number ending in 4
b an even number
c a number greater than 20
d a number less than or equal to 20
e a number with the digits 1 and 4
f the number 14

See Examples 14, 15

5 Astrid wants to buy a new car. She doesn't care whether it's a 4-door sedan, a 2-door sedan or a hatch back. The only colours she likes are black, yellow or green and she likes these colours equally. Draw a tree diagram to represent the various cars that Astrid is likely to buy.

See Examples 14, 15

6 Use the tree diagram drawn for question **5** to calculate the probability that Astrid bought:

a a 4-door sedan of any colour
b a green car
c a yellow 2-door sedan
d a car that is black or yellow

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7 A drawer contains five pairs of socks: one red, two grey and two blue. A pair of socks is taken out of the drawer, the colour noted and returned to the drawer. This step is then repeated. Draw a two-way table to represent this experiment.

8 Use the two-way table drawn for question **7** to calculate the probability that the selected pairs of socks are:

Worked solutions
Exercise 8.3

a the same colour
b different colours
c blue and grey
d blue and then grey
e both red
f both grey

9 A card is drawn from a normal deck of 52 playing cards, the suit noted and then returned to the deck. This step is then repeated. Draw a two-way table to represent this experiment.

10 Use the two-way table drawn for question **9** to calculate the probability that the selected cards are:

a both ♥
b a ♥ and a ♦
c both black cards
d the same suit
e a ♠ followed by a ♦
f the same colour
g a red card and a black card
h a red card followed by a black card

11 A bag contains four cards numbered 1 through 4. A two digit number is formed by first drawing a card and writing down its number. This card is not returned to the bag. Another card is then drawn from the bag and its number written down. Draw a tree diagram to represent this experiment.

See Example 16

12 Use the tree diagram drawn for question **11** to calculate the probability of obtaining:

See Example 16

a a number ending in 4
b an even number
c a number greater than 20
d a number less than or equal to 20
e a number with the digits 1 and 4
f the number 14

Problem solving

13 A card is drawn from a normal deck of 52 playing cards, the suit noted and then returned to the deck. Another card is then drawn from the deck and its rank (A, 2, 3, 4, 5, . . ., J, Q. K) is noted. Draw a two-way table to represent this experiment and calculate the probability that the selected cards are:

a a ♥ and then a K
b a red card and a 4
c a ♠ or a ♦ and a 10
d a black card and a J or Q

14 A drawer contains three pairs of socks: two red and one blue. A pair of socks is taken out of the drawer, the colour noted and then put aside. Another pair of socks is then taken out of the draw and its colour noted. Draw a tree diagram to represent this experiment and calculate the probability that the selected pairs of socks are:

See Example 16

a the same colour
b different colours
c red followed by blue
d blue followed by red
e both red
f both blue

15 A drawer contains five pairs of socks: one red, two grey and two blue. A pair of socks is taken out of the drawer, the colour noted and then put aside. Another pair of socks is then taken out of the draw and its colour noted. Draw a two-way table to represent this experiment and calculate the probability that the selected pairs of socks are:

Worked solutions
Exercise 8.3

See Example 17

a the same colour
b different colours
c blue and grey
d blue and then grey
e both red
f both grey

See Example 17

16 A bag contains six cards numbered 1 through 6. A two digit number is formed by first drawing a card and writing down its number. This card is not returned to the bag. Another card is then drawn from the bag and its number written down. Draw a two-way table to represent this experiment and calculate the probability of obtaining:

a an odd number
b a number ending in 3
c a number greater than 30
d a number less than or equal to 30
e a number with the digits 2 and 5
f the number 25

17 A family has two children. What is the probability of:

a no boys
b two girls
c the younger being a girl
d a boy and a girl

Reasoning

18 In the game rock-paper-scissors, two players simultaneously display the gesture for a rock, scissors or paper using their hands. The objective is to select a gesture which defeats that of the opponent. The outcome of each round is settled as follows:

- rock beats scissors
- scissors beats paper
- paper beats rock

If both players choose the same gesture, the game is tied.

If you are playing an opponent and each of you randomly chooses the rock, scissors or paper gesture each time you play find the probability that:

a you win the round
b you lose the round
c the round is tied

19 How many different 2-digit numbers can be formed from the digits 1, 3, 5, 7 and 9 if:

a no digit can be repeated
b repetitions are allowed

20 A coin is tossed three times. Find the probability of getting at least two tails.

21 A coin is tossed four times. Find the probability that at least one head occurs.

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Chapter 8 summary

- An **event** is something that could happen. Something that cannot happen is called **impossible**.
- An event that might or might not occur is **possible**. The **possible outcomes** (**events**) of a situation are all the things that could happen. An event that must occur is called a **certain** event.
- **More likely** events have a greater chance of occurring than **less likely** events. **Equally likely** events have the same chance of occurring.
- Something that is just as likely to occur as not occur is called an **even chance** or **fifty-fifty chance**.
- **Experimental probability** uses real data to work out chances. Each time we try an experiment it is called a **trial**. The result that we are interested in is called the **favourable outcome** of a trial. More likely outcomes have higher frequencies than less likely outcomes.
- The **probability** of an outcome can be measured as a decimal between 0 and 1, a fraction or a percentage using the **relative frequency**.

$$\text{Relative frequency} = \frac{\text{Frequency of event}}{\text{Total Frequency}} = \frac{\text{Number of favourable outcomes}}{\text{Number of trials}}$$

- **Expected frequency** = Probability × Number of trials
- **Theoretical probability** is based on the **equally likely outcomes**. The **sample space** lists all possible outcomes, each known as an **element** or **sample point**.
- The number of ways that an event can occur is written as n(**event**).
- The **probability** of an event is written as P(**Event**) and is calculated from the sample space using the formula:

$$P(\text{Event}) = \frac{n(\text{Event})}{n(\text{Sample space})}$$

- The probability of a certain event is 1 and the probability of an impossible event is 0. The probabilities of all other events lie between 0 and 1.
- A **random** event happens by chance.
- A **tree diagram** or a **two-way table** can be used to represent the outcomes of an experiment. Events can also be represented using **Venn diagrams**.
- The sum of the probabilities of all outcomes for an experiment is 1.
- An event (E) and its **complement** (E' or $\overline{E}$) together made up the entire sample space for the experiment. The probability of the complement of event E is the same as the probability that event E will not occur.

$$P(E) + P(E') = 1$$

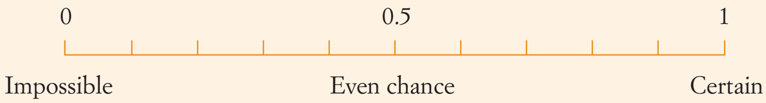
$$\text{So} \quad P(E') = 1 - P(E)$$

$$\text{or} \quad P(E) = 1 - P(E')$$

- A probability experiment that involves two stages or steps is known as a **two-step chance experiment**.
- The situation where the outcomes of each step of a two-step chance experiment are the same is said to involve **replacement**.

Understanding

1 Copy this probability scale.



Place the following events on the probability scale using the letters a, b, c, etc.

a You will find a leprechaun today.
b You will be have a birthday at some time next year.
c Someone in your class will go shopping this week.
d Someone in your class will watch TV for more than an hour today.
e No one in your class will not use a mobile phone this week.

2 Is the chance of each of the following events more than or less than an even chance?

a selecting a club from a normal deck of playing cards
b a student in your class will drive a car home from school today
c a student at your school will complete year 12
d getting a number greater than 2 when a six-sided die is rolled
e students in your class will attend classes at school on Christmas Eve

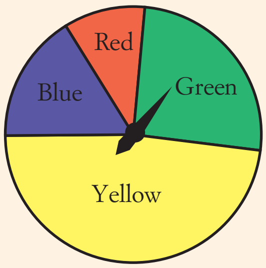
See Example 1

3 Compare the chance of tossing a head with the chance of drawing a king from a standard pack of playing cards.

See Example 2

4 Use the spinner shown on the right to answer these questions.

a Are the outcomes equally likely?
b Which colour is most likely to be spun?
c Which colour is least likely to be spun?
d List the outcomes in order, from most likely to least likely.



See Example 3

5 The table below shows the numbers of different breeds of cattle on a particular property.

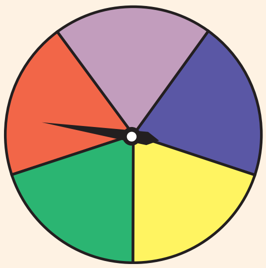
Breed	Angus	Hereford	Brahmin	Simmental	Charolais
Number	120	350	200	180	130

One of the cattle from was stuck in a waterhole. What breed is the steer most likely to be?

See Example 3

6 This spinner is spun and the colour where the pointer lands is noted in the table below.

Outcome	Tally	Frequency
Blue	𝍸 𝍸 𝍸 𝍸 ||||	
Green	𝍸 𝍸 𝍸 𝍸	
Red	𝍸 𝍸 𝍸 |||	
Yellow	𝍸 𝍸 𝍸 ||	
Pink	𝍸 𝍸 𝍸 𝍸 |	



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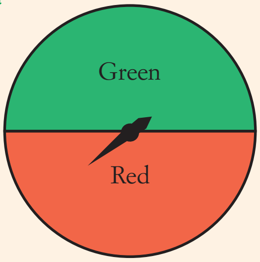
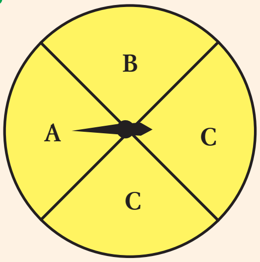
a Complete the frequency column of the table.
b What is the total frequency?
c What is the relative frequency of blue?
d What is the relative frequency of red?
e What is the relative frequency of pink?

7 Use the spinners below to answer the following questions. *See* Examples 4-6

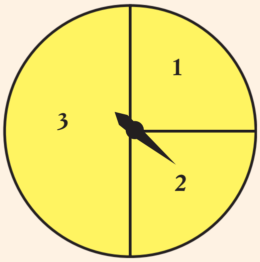
a



b

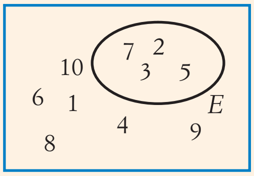


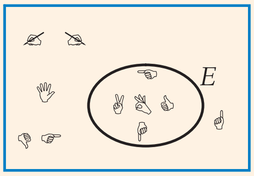
c

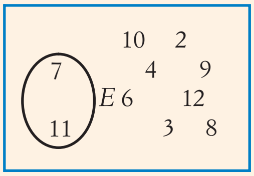


i List the sample space for the each of the spinners.
ii Are each of the events equally likely?

8 Each Venn diagram below shows the sample space for an experiment and an event *E*. In each case list elements of the complement of *E*.







Fluency

9 Write down some equally likely events.

10 Calculate the relative frequencies of each breed in question **5**.

11 A six-sided die was rolled and the number of dots on the upper face noted. The results were: *See* Example 3

1	2	3	4	5	6
8 times	6 times	5 times	10 times	6 times	5 times

a Calculate the total frequency for this experiment.
b Calculate the relative frequencies of each outcome as percentages.
c Which number appears most likely?
d Which number appears least likely?
e Does it appear that an odd or even number is more likely?

12 A bag contains six red and nine green marbles. *See* Examples 4-6
a List the sample space.
b Find the probability of drawing a red marble.

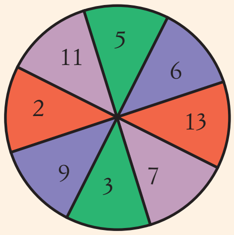
See Examples 4-6

13 There are 9 green, four red and one pink and 6 blue marbles in a bag. One marble is taken out.

a List the sample space.
b What is n(blue marble)?
c Work out the P(blue marble).
d Find P(green marble).
e Work out P(red marble).
f Work out P(pink marble).
g What do all the probabilities add up to?

See Examples 4-6

14 For the spinner shown on the right:



a list all the outcomes
b find all probabilities of each of the outcomes
c what is P(even number)
d what is P(odd number)
e find $P(\text{number} < 10)$

See Example 7

15 A six-sided die is rolled.

a What is the probability of rolling a 5?
b What is the probability of rolling an even number?

See Example 8

16 Discs numbered 1 through 20 are randomly drawn from a container.

$$A = \{\text{an odd number}\} \text{ and } B = \{\text{multiple of } 3\}.$$

a Represent the outcomes of this experiment using a Venn diagram.

Use the Venn diagram to calculate the probability that a randomly selected number is:

b odd
c a multiple of 3
d an odd multiple of 3
e odd or a multiple of 3
f not odd
g not a multiple of 3

See Example 9

17 A group of 63 students goes on a picnic. Of those, 27 went swimming, 19 played volleyball and 33 went kayaking. 9 students went swimming and played volleyball, 6 played volleyball and went kayaking, 12 went swimming and went kayaking and 4 did all three activities.

a Represent this situation using a Venn diagram.

Use the Venn diagram to find the probability that a student selected at random:

b participated in none of the three events
c participated in exactly one event
d only went kayaking

See Example 10

18 Two six-sided dice are rolled and the total of the uppermost faces noted.

a Construct a two-way table to represent this experiment.

Use the two-way table to find the probability that the total is:

b odd
c not prime
d less than 7
e greater than 8
f even or greater than 8
g even and greater than 8

See Examples 11, 12

19 A six-sided die is rolled. What is the probability of:

a rolling a 3?
b not rolling a 3?

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20 A bag contains 8 red marbles, 2 black marbles and 5 white marbles in. One marble is randomly taken out of the bag. *See* Examples 11, 12

a How many different outcomes are possible?
b What is the probability that a black marble is selected?
c What is the probability that the selected marble is not black?

21 A bag contains four cards numbered 2, 5, 6 and 9. A two digit number is formed by drawing a card, writing down its number, returning it to the bag and then repeating the first step. *See* Examples 14, 15

a Draw a tree diagram to represent this experiment.

Use this tree diagram to calculate the probability of obtaining:

b a number ending in 6
c an even number
d a number greater than 55
e a number less than or equal to 55
f a multiple of 5
g not a multiple of 5

22 A box contains five chocolates sweets – one milk chocolate, two dark chocolates and two white chocolates. A chocolate is taken out of the box and eaten. Another chocolate is then taken and eaten. *See* Example 17

a Draw a two-way table to represent this experiment.

Use this two-way table to calculate the probability that the selected chocolates were:

b the same type
c different types
d a dark and a white one
e a dark one followed by a white one
f both white
g both dark

Problem solving

23 The amount of petrol (in litres) sold by a service station as shown below.

63	25	50	21	18	26	39	29	45	31
35	53	19	50	44	63	26	20	53	40
58	48	49	21	50					

Use this information to decide if the next petrol sale is likely to be more or less than 30 L.

24 If 100 head of cattle from the property in question **5** are sold, how many would you expect to be Brahmins?

25 A normal coin was tossed 150 times and landed tails 72 times.

a What is the relative frequency of obtaining:

i a head? **ii** a tail?

b Compare the results of part **a** with the theoretical probability of tossing a head or a tail.

26 It is estimated that 4% of the Australian population have red hair. If there are 260 people in a cinema, how many would you expect to be redheads?

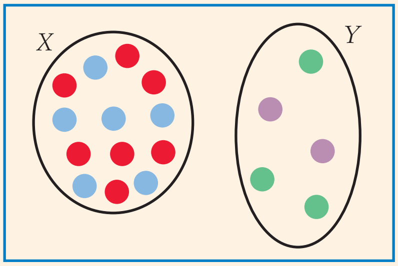
27 Of a total of 100 people at a business conference 26 pay their bills using BPAY, 33 use internet banking and 31 use telephone banking. 8 said they use BPAY and internet banking, 12 said they use internet banking and telephone banking and 11 said they use telephone banking and BPAY. 5 said they use all three methods to pay their bills. What is the probability that a randomly selected conference goer: *See* Example 9

a only uses internet banking to pay bills?

b does not use any of the three methods (BPAY, internet banking or telephone banking) to pay bills?

See Example 12

28 A jar contains 18 discs: each one coloured either red, blue, green or purple. The Venn diagram on the right shows two events X and Y that result when a disc is drawn and its colour noted.



Calculate:

a $P(X)$
b $P(Y)$
c $1 - P(X)$
d $1 - P(Y)$
e $P(X) + P(Y)$

Complete the following:

f $P(X') = \ldots$
g $P(Y') = \ldots$

29 A family has three children. What is the probability of:

a no girls
b exactly two boys
c the youngest being a boy
d a boy and a two girls

Reasoning

30 A six-sided die is rolled 1000 times. Would you be confident to predict the number of times a 4 will be rolled based on the results of question **11**?

31 The faces of a twelve-sided die are numbered 1 through 12 and the faces on a six-sided die are numbered 1 through 6. The two dice are rolled and the numbers on the upper faces are noted. This is repeated 500 times.

a How many times would you expect to get two sixes?
b How many times would you expect to get a total of 10?
c How many times would you expect to get a total get a total greater than 10?

See Example 15

32 A coin is tossed three times and the outcomes noted.

a Draw a tree diagram to represent this experiment.
b List the sample space.
c Find P(exactly two tails).
d What is P(at least two tails)?
e Find P(a head and two tails).

The experiment is repeated 100 times.

f How many times would you expect to get at least one tail?
g How many times would you expect to get a tail and two heads?

33 A coin is tossed four times. Find the probability that the outcome has exactly two tails.

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Chapter 8: Probability

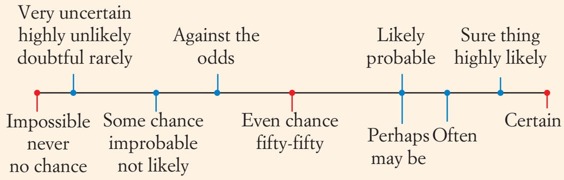
Exercise 8.1

1 **a** Even chance **b** Very likely
c Answers will vary **d** Even chance
e Very unlikely **f** Even chance
g Impossible **h** Very likely

2 **a** Answers can be anything that cannot happen.
b Answers can be anything that must happen.

3 **a** less **b** less **c** more
d more **e** less **f** more

4 Positions are not intended to be exact.



5 **a** They are equally likely.
b Getting a 6 is less likely than not getting a 6.
c You are more likely to lose than to win.
d You are more likely to win than to lose.
e You are more likely to get a pair than a flush.

6 **a** 0 **b** Answers will vary
c 1 or 2 **d** 5
e 2 or 3 **f** 10
g 1 or 2 **h** 0
i Answers will vary

7 Answers will vary depending on time of year etc. Your teacher will check these answers.

8–10 Answers will vary. Your teacher will check.

11 **a** **i** {A, B} **ii** Each event is equally likely.
b **i** {1, 2, 3, 4} **ii** Each event is equally likely.
c **i** {Yellow, blue} **ii** Each event is equally likely.

12 **a** {G, G, G, G, G, B, B, B, R, R, Y, Y}
b The different colour outcomes are not equally likely.

13 **a** $P(\text{Y}) = \frac{1}{2}$ **b** $P(\text{Y}) = \frac{1}{4}$ **c** $P(\text{Y}) = \frac{1}{2}$

14 **a** $P(\text{R}) = \frac{1}{6}$ **b** $P(\text{G}) = \frac{5}{12}$

15–17 Answers will vary. Your teacher will check.

18 **a, b**

Outcome	Frequency	Relative frequency
HH	2	0.1
HT	6	0.3
TH	7	0.35
TT	5	0.25
Total	20	

c You are more likely to get ‘heads and tails’ than ‘two tails’.

19 **a** See table below.
b Black or red are equally likely.
c A spade appears more likely, but only a small number of trials was done.

20 **a, b**

Number	Frequency	Relative frequency
13	1	$0.0\dot{3}$
14	6	0.2
15	9	0.3
16	4	$0.1\dot{3}$
17	8	$0.\dot{2}\dot{6}$
18	1	$0.0\dot{3}$
19	0	0
20	1	$0.0\dot{3}$
Total	30	

c A bag with 15 oranges is more likely.

21 **a** Yes; {R, R, R, R, R, G, G, G, B, B, B, B}
b Yes; {HH, HT, TH, TT}
c No
d No
e Yes; {AA, AB, AC, BA, BB, BC, CA, CB, CC}

22 **a** {R, R, R, R, G, G, G, B, B, B, B, B, B, B, B}
b 4
c $\frac{4}{15}$ or about 0.27 or 27%
d $\frac{1}{5}$ or 0.2 or 20%
e $\frac{8}{15}$ or about 0.53 or 53%
f 1

Exercise 8.1

19

	Hearts (♥)	Diamonds (♦)	Clubs (♣)	Spades (♠)	Total
	7 times	5 times	5 times	8 times	25
Relative frequency	0.28	0.2	0.2	0.32	

23 a {SB, SB, PP, PP, PP, PP, PP, PP, PP, PP, PS, PS, PS, PS, PS, PS}

b $\frac{1}{8}$ or 0.125 or 12.5%

c $\frac{1}{2}$ or 0.5 or 50%

d $\frac{3}{8}$ or 0.375 or 37.5%

24 a {B, B, B, G, G, L, L, L, L, L}

b 3 and $\frac{3}{10}$ or 0.3 or 30%

c 2 and $\frac{1}{5}$ or 0.2 or 20%

d 5 and $\frac{1}{2}$ or 0.5 or 50%

e Lined paper

25 a $\frac{1}{6}$ or about 0.17 or 17%

b $\frac{1}{6}$ or about 0.17 or 17%

c $\frac{1}{2}$ or 0.5 or 50%

d $\frac{1}{3}$ or about 0.33 or 33%

e $\frac{1}{2}$ or 0.5 or 50%

f 25 times

26 a

Legs	Frequency
5	2
6	5
7	12
8	6
Total	**25**

b Seven legs

27 a 35 **b** The first three days.

28 a 4 or 5 **b** 150

29 No, as there are not enough trials

30 a 0.7 **b** 0.3

c 56 **d** 167 (166.7)

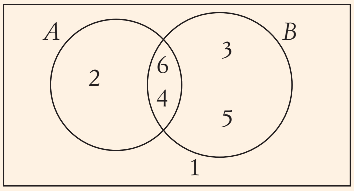
31 a $\frac{1}{20}$ or 0.05 or 5% **b** $\frac{9}{10}$ or 0.9 or 90%

c Lose about \$18

32 The probability is $\frac{1}{2}$ or 0.5 or 50%. The past history does not change the probability.

Exercise 8.2

1 a

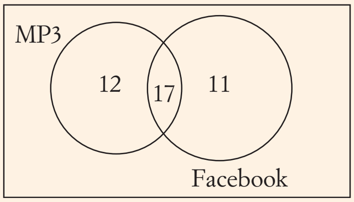


b $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{5}{6}$ **e** $\frac{1}{3}$

2 a $\frac{7}{15}$ **b** $\frac{1}{3}$ **c** $\frac{2}{3}$

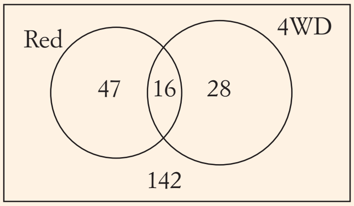
d $\frac{2}{15}$ **e** $\frac{8}{15}$ **f** $\frac{2}{3}$

3 a



b $\frac{11}{40}$ **c** $\frac{3}{10}$

4 a



b $\frac{63}{233} \approx 27.0\%$ **c** $\frac{44}{233} \approx 18.9\%$ **d** $\frac{170}{233} \approx 73.0\%$

e $\frac{189}{233} \approx 81.1\%$ **f** $\frac{142}{233} \approx 60.9\%$

5 a

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{2}$ **c** $\frac{5}{12}$ **d** $\frac{7}{12}$

e $\frac{1}{6}$ **f** $\frac{5}{6}$ **g** $\frac{1}{9}$

6 a See table below.

Exercise 8.2

6

	Meat lovers (ML)	Hawaiian (H)	Supreme (Sup)	4Cheeses (4C)	Vegetarian (V)
F	F, ML	F, H	F, Sup	F, 4C	F, V
L	L, ML	L, H	L, Sup	L, 4C	L, V
M	M, ML	M, H	M, Sup	M, 4C	M, V
S	S, ML	S, H	S, Sup	S, 4C	S, V

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b $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{1}{4}$
e $\frac{1}{5}$ **f** $\frac{1}{20}$ **g** $\frac{2}{5}$

7 a {2, 4, 6, 8, 10}
b {R, R, R, R, R, G, G, G}
c {b, g, j, p, q, r, s, w, x, z}

8 a a tail
b a consonant
c {January, February, March, April, May, June, July, August}
d losing
e {1, 2, 3, 4, 5, 6, 7}
f an odd number

9 a {1, 2, 3, 4, 5} **b** {2, 4, 6}
c {1, 4, 6} **d** {3, 4, 5, 6}

10 a $\frac{7}{11}$ **b** $\frac{4}{11}$ **c** $\frac{4}{11}$
d $\frac{7}{11}$ **e** 1 **f** $P(A') = P(B)$
g $P(B') = P(A)$

11 a $\frac{11}{18}$ **b** $\frac{7}{18}$ **c** $\frac{7}{18}$
d $\frac{11}{18}$ **e** 1 **f** $P(M') = P(N)$
g $P(N') = P(M)$

12 D

13 a 0.2 **b** 0.8 **c** 0.4 **d** 0.6

14 a

	H	T
H	HH	HT
T	TH	TT

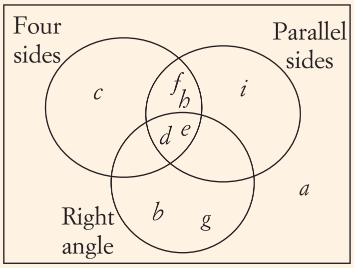
b $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$

15 a

	1	2	3	4	5	6
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

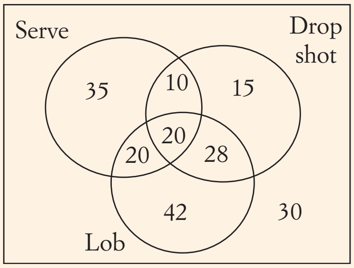
b $\frac{1}{36}$ **c** $\frac{35}{36}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$
f $\frac{5}{12}$ **g** $\frac{7}{12}$ **h** $\frac{5}{18}$ **i** $\frac{13}{18}$

16 a



b $\frac{2}{9}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$ **e** $\frac{1}{9}$
f $\frac{4}{9}$ **g** $\frac{2}{9}$ **h** $\frac{2}{9}$

17 a



b $\frac{1}{10}$ **c** $\frac{23}{40}$ **d** $\frac{17}{40}$ **e** $\frac{21}{100}$

18 a 26 **b** Outcomes are equally likely.
c $\frac{4}{13}$ **d** $\frac{9}{13}$ **e** $\frac{5}{26}$ **f** $\frac{21}{26}$

19 a 20 **b** Outcomes are equally likely.
c $\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{2}{5}$ **f** $\frac{3}{5}$
g 1 **h** 0

20 a

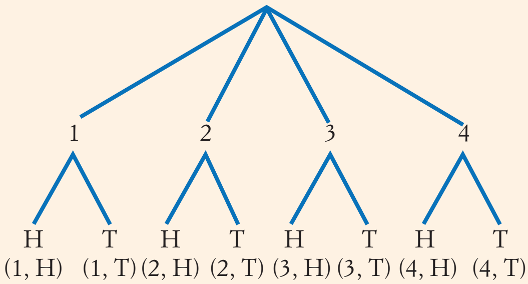
	Music	Sport	TV	Totals
Men	4	13	8	25
Women	11	6	8	25
Totals	15	19	16	50

b 60 **c** 32 **d** 52

21 a $\frac{33}{100}$ **b** $\frac{67}{100}$

Exercise 8.3

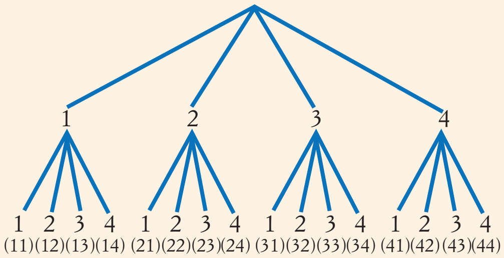
1



Answers

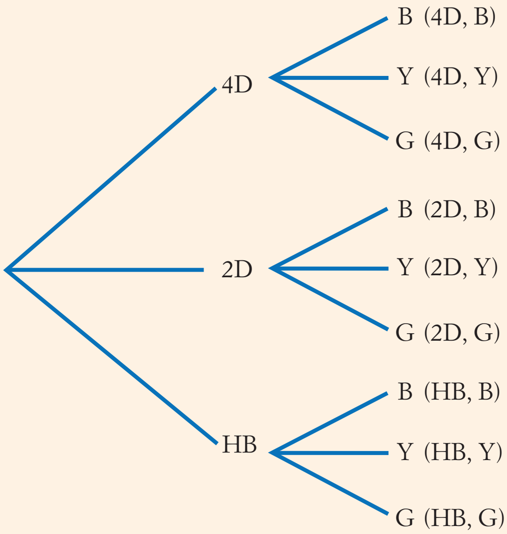
2 **a** $\frac{1}{8}$ **b** $\frac{5}{8}$ **c** $\frac{1}{4}$ **d** $\frac{3}{4}$

3



4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
d $\frac{1}{4}$ **e** $\frac{1}{8}$ **f** $\frac{1}{16}$

5



6 **a** $\frac{1}{3}$ **b** $\frac{1}{3}$ **c** $\frac{1}{9}$ **d** $\frac{2}{3}$

7

	R	**G**	**G**	**B**	**B**
R	RR	RG	RG	RB	RB
G	GR	GG	GG	GB	GB
G	GR	GG	GG	GB	GB
B	BR	BG	BG	BB	BB
B	BR	BG	BG	BB	BB

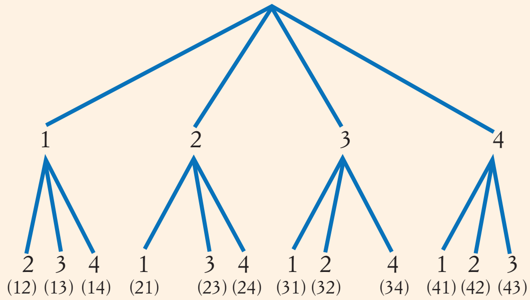
8 **a** $\frac{9}{25}$ **b** $\frac{16}{25}$ **c** $\frac{8}{25}$
d $\frac{4}{25}$ **e** $\frac{1}{25}$ **f** $\frac{4}{25}$

9

	♥	♦	♣	♠
♥	♥♥	♥♦	♥♣	♥♠
♦	♦♥	♦♦	♦♣	♦♠
♣	♣♥	♣♦	♣♣	♣♠
♠	♠♥	♠♦	♠♣	♠♠

10 **a** $\frac{1}{16}$ **b** $\frac{1}{8}$ **c** $\frac{1}{4}$ **d** $\frac{1}{4}$
e $\frac{1}{16}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{4}$

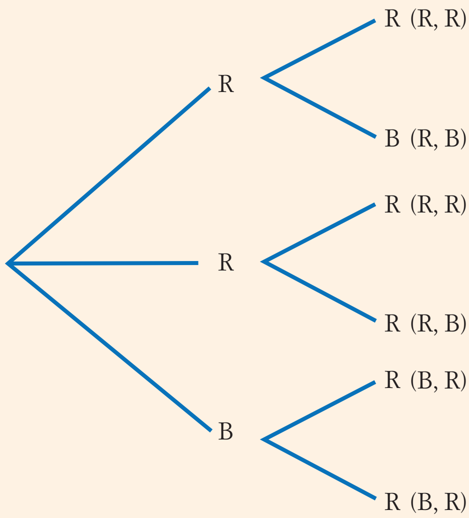
11



12 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$
d $\frac{1}{4}$ **e** $\frac{1}{6}$ **f** $\frac{1}{12}$

13 See table below.
a $\frac{1}{52}$ **b** $\frac{1}{26}$ **c** $\frac{1}{26}$ **d** $\frac{1}{13}$

14



Exercise 8.3

13

	A	2	3	4	5	6	7	8	9	10	J	Q	K
♥	♥A	♥2	♥3	♥4	♥5	♥6	♥7	♥8	♥9	♥10	♥J	♥Q	♥K
♦	♦A	♦2	♦3	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q	♦K
♣	♣A	♣2	♣3	♣4	♣5	♣6	♣7	♣8	♣9	♣10	♣J	♣Q	♣K
♠	♠A	♠2	♠3	♠4	♠5	♠6	♠7	♠8	♠9	♠10	♠J	♠Q	♠K

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a $\frac{1}{3}$ b $\frac{2}{3}$ c $\frac{1}{3}$
d $\frac{1}{3}$ e $\frac{1}{3}$ f 0

15

	R	**G**	**G**	**B**	**B**
R		RG	RG	RB	RB
G	GR		GG	GB	GB
G	GR	GG		GB	GB
B	BR	BG	BG		BB
B	BR	BG	BG	BB	

a $\frac{1}{5}$ b $\frac{4}{5}$ c $\frac{2}{5}$
d $\frac{1}{5}$ e 0 f $\frac{1}{10}$

16

	1	**2**	**3**	**4**	**5**	**6**
1		12	13	14	15	16
2	21		23	24	25	26
3	31	32		34	35	36
4	41	42	43		45	46
5	51	52	53	54		56
6	61	62	63	64	65	

a $\frac{1}{2}$ b $\frac{1}{6}$ c $\frac{2}{3}$
d $\frac{1}{3}$ e $\frac{1}{15}$ f $\frac{1}{30}$

17 a $\frac{1}{4}$ b $\frac{1}{4}$ c $\frac{1}{2}$ d $\frac{1}{2}$

18 a $\frac{1}{3}$ b $\frac{1}{3}$ c $\frac{1}{3}$

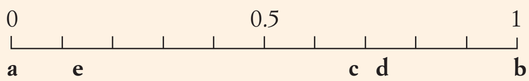
19 a 20 b 25

20 $\frac{1}{2}$

21 $\frac{15}{16}$

Chapter review

1



The positions of c, d and e will vary.

2 a less than an even chance
b less than an even chance
c more than an even chance
d more than an even chance
e less than an even chance

3 There is a greater chance of tossing heads than drawing a king from a normal pack.

4 a No b Yellow
c Red d Yellow, green, blue, red.

5 Hereford

6 a

Outcome	Tally	Frequency
Blue	卌 卌 卌 卌 IIII	24
Green	卌 卌 卌 卌	20
Red	卌 卌 卌 III	18
Yellow	卌 卌 卌 II	17
Pink	卌 卌 卌 卌 I	21

b 100 c 0.24 d 0.18 e 0.21

7 a i {Green, pink}
ii Each event is equally likely.
b i {A, B, C, D}
ii Each event is equally likely.
c i {1, 2, 3}
ii The events are not equally likely.

8 a {1 4 6 8 9 10}
b {✍, ✍, 👎, ☞, ☝, ✋}
c {2, 3, 4, 6, 8, 9, 10, 12}

9 A coin falling heads or tails; cutting a black or red card; etc.

10 See table below.

11 a 40
b 1: 20%, 2: 15%, 3: 12.5%, 4: 25%, 5: 15%, 6: 12.5%
c 4
d 3 or 6
e An even number

Chapter review

10

Breed	**Angus**	**Hereford**	**Brahmin**	**Simmental**	**Charolais**
Relative frequency	0.122, 12.2%	0.357, 35.7%	0.204, 20.4%	0.184, 18.4%	0.133, 13.3%

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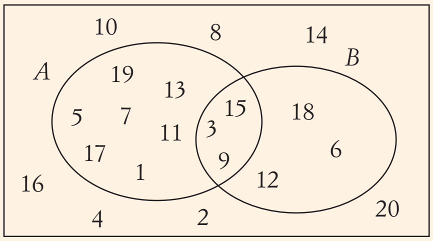
12 **a** {R R R R R R G G G G G G G G G}
b 0.4

13 **a** {G G G G G G G G G R R R R P **B B B B B B**}
b 6 **c** 0.3 **d** 0.45
e 0.2 **f** 0.05 **g** 1

14 **a** {2 3 5 6 7 9 11 13}
b probabilities of each outcome = 0.125
c 0.25
d 0.75
e 0.25

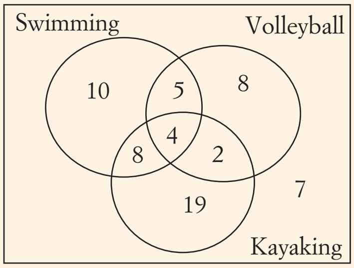
15 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$

16 **a**



b $\frac{1}{2}$ **c** $\frac{3}{10}$ **d** $\frac{3}{20}$
e $\frac{13}{20}$ **f** $\frac{1}{2}$ **g** $\frac{7}{10}$

17 **a**



b $\frac{7}{63} = \frac{1}{9}$ **c** $\frac{37}{63}$ **d** $\frac{19}{63}$

18 **a**

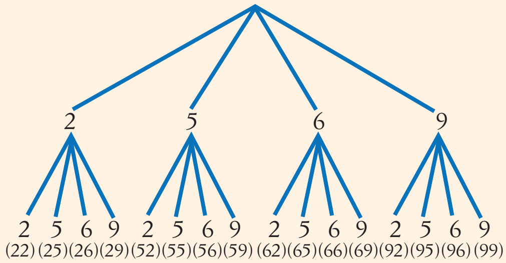
	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{2}$ **c** $\frac{7}{12}$ **d** $\frac{5}{12}$
e $\frac{5}{18}$ **f** $\frac{2}{3}$ **g** $\frac{1}{9}$

19 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$

20 **a** 3 **b** $\frac{2}{15}$ **c** $\frac{13}{15}$

21 **a**



b $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** $\frac{5}{8}$
e $\frac{3}{8}$ **f** $\frac{1}{4}$ **g** $\frac{3}{4}$

22 **a**

	M	**D**	**D**	**W**	**W**
M		M, D	M, D	M, W	M, W
D	D, M		D, D	D, W	D, W
D	D, M	D, D		D, W	D, W
W	W, M	W, D	W, D		W, W
W	W, M	W, D	W, D	W, W	

b $\frac{1}{5}$ **c** $\frac{4}{5}$ **d** $\frac{2}{5}$
e $\frac{1}{5}$ **f** $\frac{1}{10}$ **g** $\frac{1}{10}$

23 Over 30 litres is likely because the frequency is higher.

24 About 20

25 **a** **i** 0.52 **ii** 0.48
b $P(\text{H}) = P(\text{T}) = 0.5$

26 10 or 11

27 **a** $\frac{9}{50}$ **b** $\frac{9}{25}$

28
a $\frac{13}{18}$ **b** $\frac{5}{18}$ **c** $\frac{5}{18}$
d $\frac{13}{18}$ **e** 1 **f** $P(X') = P(Y)$
g $P(Y') = P(X)$

29 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{3}{8}$

30 No. There were too few trials on which to base a prediction.

31 **a** About 7 times
b 41 or 42 times
c About 229 times

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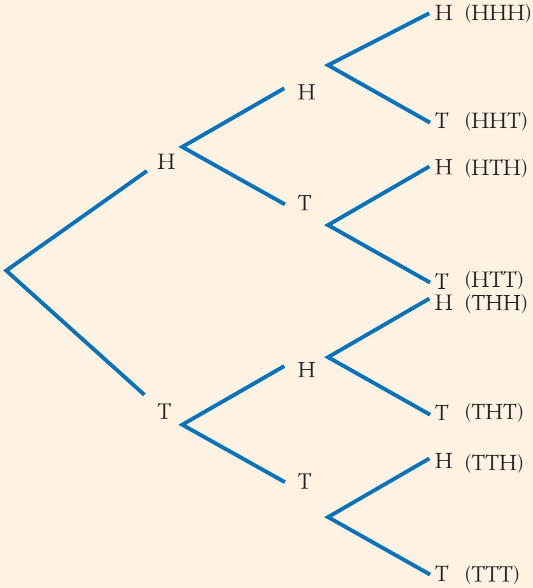
32 a



b {HHH HHT HTH HTT THH THT TTH TTT}

c $\frac{3}{8}$

d $\frac{1}{2}$

e $\frac{3}{8}$

f 87 or 88 times

g 37 or 38 times

33 $\frac{3}{8}$