
**Year 10 Mathematics Extension Investigation**

**Circle Geometry Properties!!! Take Home Part**

## Introduction

In the following tasks, you will be asked to use the Geometry application of your calculator to develop conjectures about circles and their properties. You will then be asked to use your geometrical knowledge to prove your conjectures. The layout for each proof is shown below.

**Diagram**: Include a labelled diagram.


**Given**: State the given information.

**To Prove**: State that which is to be proved.

**Extension to the diagram**:
It may be useful to draw and/or extend lines in the diagram.

**Proof**: Statements need to be supported with reasoning.

Before commencing the tasks, you are advised to look at the information on page 2 of this booklet.

On page 2 you will also find information regarding the Geometry application on the ClassPad CAS calculator. You should check the CAS settings for the Geometry application on your calculator and make any necessary adjustments.

Page 2 may be a useful point of reference when looking for tools within the Geometry application. You will be using a selection of these tools; they are located in the drop down menus on the toolbar.

You will also use the Measurement Box.

## CAS settings for the Geometry application

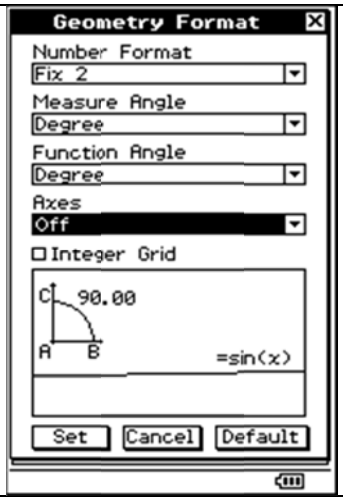
Tap

Tap **Geometry Format**

Set **Measure Angle** to **Degree**

Set **Axes** to **Off**

Tap **Set**

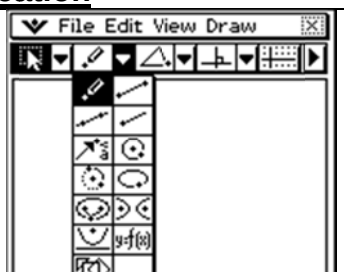

## Tools available in the Geometry application

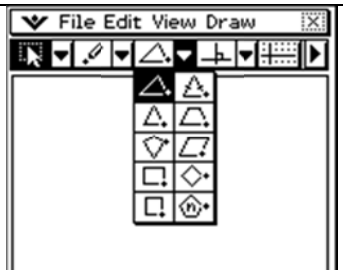
By tapping on the down arrow, ▼, to the right of each of the drop down menus,

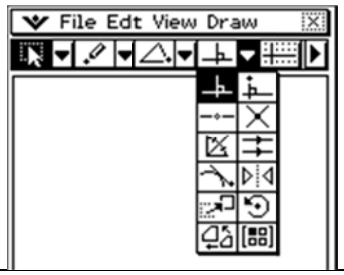
▼ ▼ ▼

a number of tools become available.

When a tool is referred to, the accompanying screen picture shows where the tool may be found.



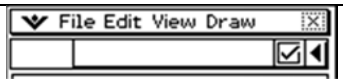




## The Measurement Box

Tapping the ▶ button to the right of the toolbar displays the measurement box.

Tap ◀ to return to the normal toolbar.


normal toolbar




Measurement box

The **Measurement Box** can be used to perform operations such as:

- to view the measurements of a figure
- to specify a measurement of a figure
- to fix a measurement of a figure
- to name a point, line segment, angle, …
- to determine whether a line is tangential to a circle
- to fix a line so that it is tangential to a circle

**NOTE:**

- To **deselect an object** (or objects), tap [ ] and then tap anywhere in space within the Geometry window.

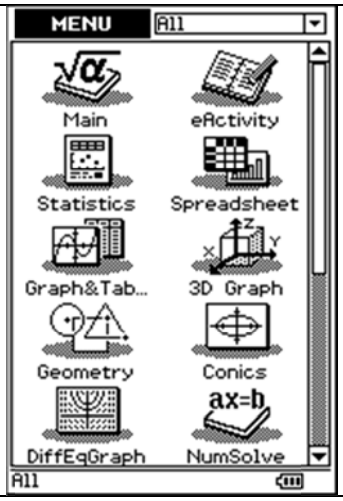
- To undo tap **Edit**, tap **Undo / Redo**.

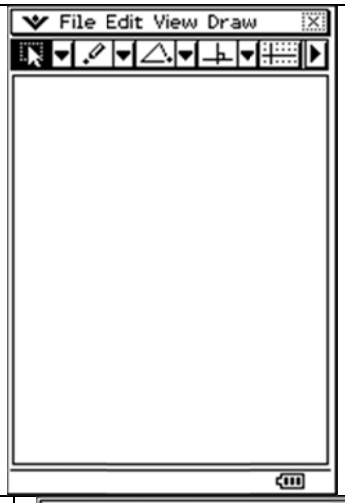
## TASK ONE: The Angle in a Semi-circle

Tap Menu.

Tap Geometry.

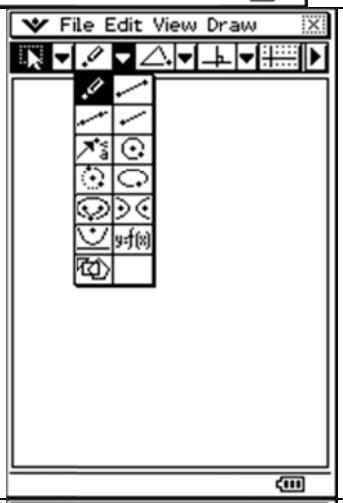
Tap **File**, tap **New**, tap **OK**.



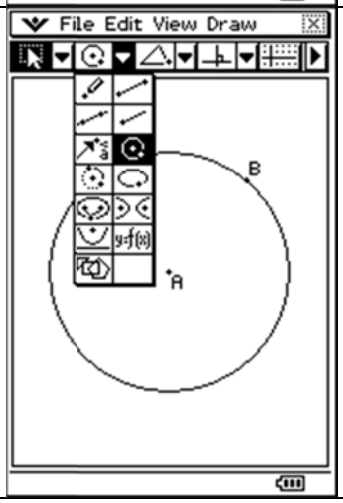

Draw a circle by tapping [ ] and then tapping in two different places in the Geometry window.

Tap **View**, tap **Zoom to Fit**.




Draw a line passing through centre A and point B by tapping [ ], then tap A, tap B.

Tap [ ].

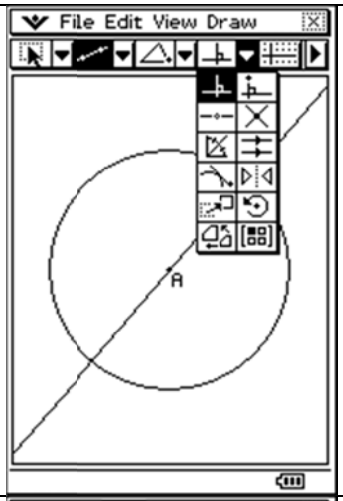


Select the circle and the line by tapping once on each of them.

Choose [⊠] to locate point D, the point of intersection of the circle and the line.

Draw line segment BD by tapping [↗], then tap B, tap D.

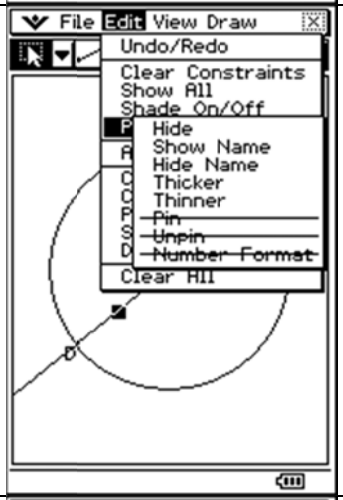
Tap [↖].




To hide the line passing through D, A and B, tap on any part of this line that lies outside the circle, tap **Edit**, tap **Properties**, tap **Hide**.

Tap [↗] to draw two line segments, BE and DE, such that E is a point on the circle.

Tap [↖].



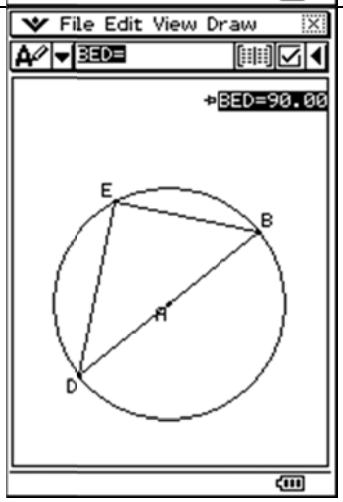

Tap [▶]. Measure the size of ∠BED by tapping once on BE and once on DE. The angle size of ∠BED will be displayed in the measurement box.

Tap on the angle size of ∠BED displayed in the measurement box or drag it into the Geometry window.

Name this angle BED by tapping [A] and using the [abc] tab on the keyboard to type BED, press **=**, press **EXE**.

Hide the keyboard.

Tap in space.



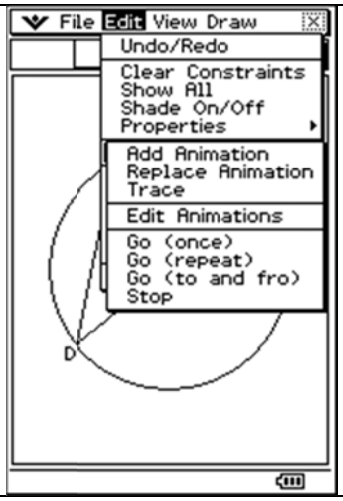

Observe the size of ∠BED as point E moves around the circle by using one of the two methods below:

<u>Method One</u>
Tap E, tap the circle.

Tap **Edit**, tap **Animate**, tap **Add Animation**.

Tap **Edit**, tap **Animate**, tap **Go (once)**.

<u>Method</u> Two
Tap E, tap E a second time and drag it around the circle.



| | |
|---|---|
| Tap **File**, tap **Save** and name the file.<br><br>Note: You may wish to “lock” the circle or BD into place by taping on A or BD and then selecting tick in the measurement box. | |

1. What do you notice about the size of ∠BED as point E moves around the circle?

2. Why do you think ∠BED is referred to as the angle in the semi-circle?

3. Prove the Angle in a Semi-Circle Theorem. (Use the layout on page 1.)

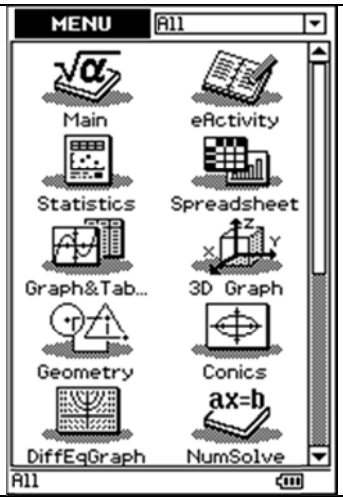
   Hint: This proof can be downloaded at http://www.mathxtc.com/
   (Use with caution!!!)

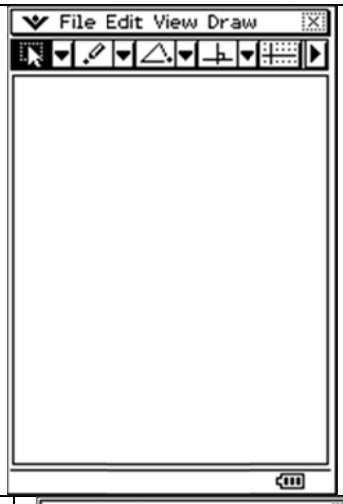
## TASK TWO: The Central Angle

Tap Menu.

Tap Geometry.

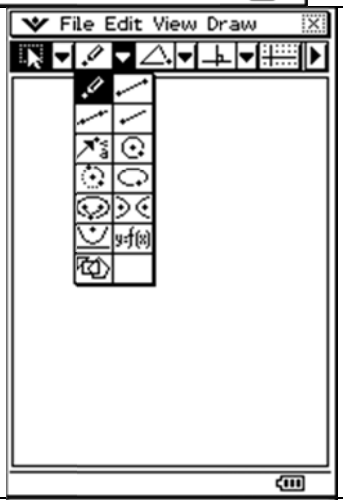
Tap **File**, tap **New**, tap **OK**.




Draw a circle by tapping [ ] and then tapping in two different places in the Geometry window.

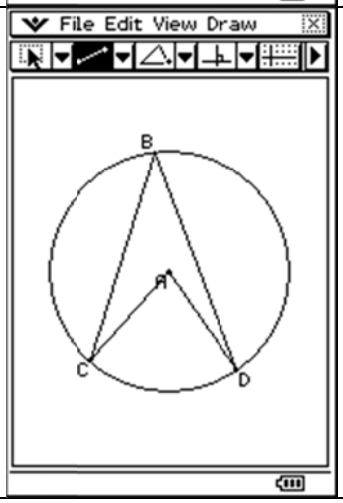
Tap **View**, tap **Zoom to Fit**.

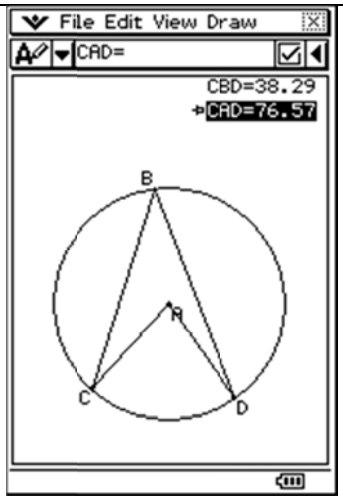
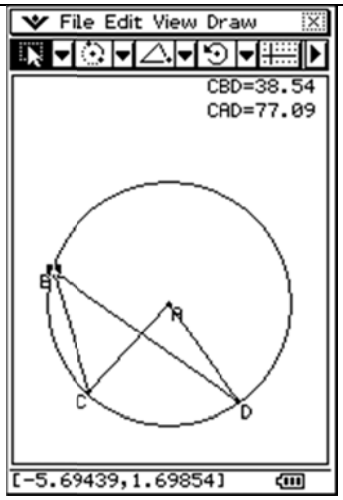



Tap [ ]. Draw radii AC and AD; draw line segments BC and BD such that ∠CBD is subtended at the circumference by arc CD in the same segment as ∠CAD.

Tap [ ].

Tap [ ].



| | |
|---|---|
| Display the size of ∠CBD by tapping BC and BD.<br><br>Tap on the size of ∠CBD and drag it into the Geometry window.<br><br>Name this angle CBD by tapping [A] and using the [abc] tab on the keyboard to type CBD, press **=**. Press **EXE**.<br><br>Tap in space.<br><br>Display the size of ∠CAD by tapping AC and AD.<br><br>Tap on the size of ∠CAD and drag it into the Geometry window.<br><br>Name this angle CAD by tapping [A] and using the [abc] tab on the keyboard to type CAD, press **=**. Press **EXE**.<br><br>Tap in space.<br><br>Hide the keyboard. |  |
| Observe the size of angles ∠CAD and ∠CBD when points B, C and D respectively move on the circle by:<br><br>Tap B. Tap B a second time and drag it around the circumference such that ∠CAD and ∠CBD both remain in the same segment.<br><br>Tap in space.<br><br>Tap C. Tap C a second time and drag it around the circumference such that ∠CAD and ∠CBD both remain in the same segment.<br><br>Tap in space.<br><br>Tap D. Tap D a second time and drag it around the circumference such that ∠CAD and ∠CBD both remain in the same segment.<br><br>Tap **File**, tap **Save** and name the file. |  |

1. On which arc are angles CBD and CAD both subtended?

2. What do you notice about the size of the angle subtended at the centre of the circle, ∠CAD, and the size of the angle subtended at the circumference of the circle, ∠CBD?

3. Prove the Central Angle Theorem. (Use the layout on page 1.)

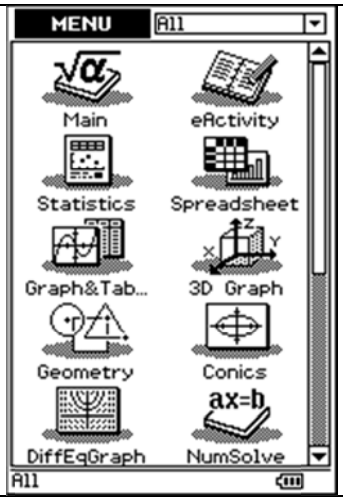
   Hint: This proof can be downloaded at http://www.mathxtc.com/
   (Use with caution!!!)

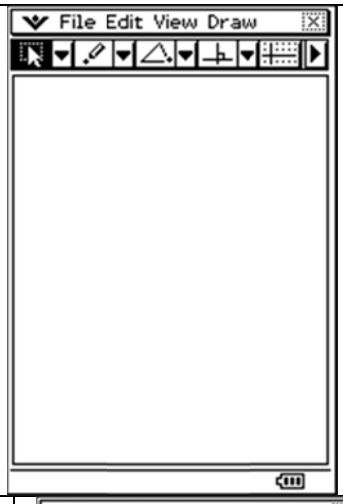
## TASK THREE: Angles in the Same Segment

Tap Menu.

Tap Geometry.

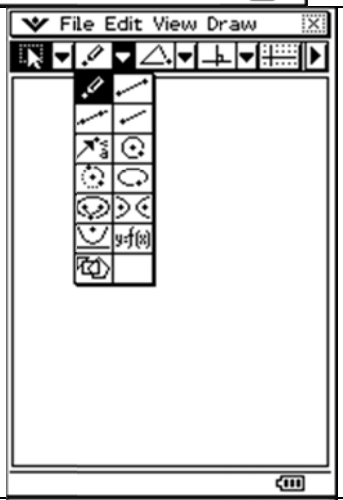
Tap **File**, tap **New**, tap **OK**.




Draw a circle by tapping [⊙] and then tapping in two different places in the Geometry window.

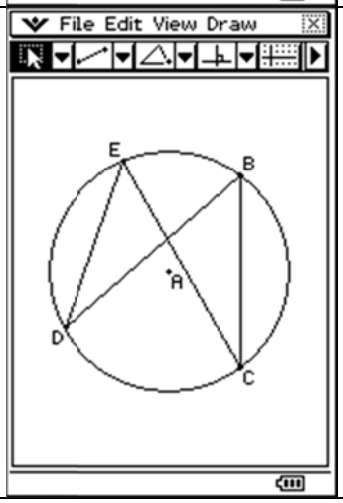
Tap **View**, tap **Zoom to Fit**.

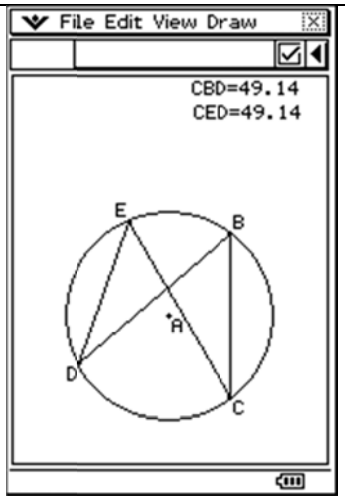
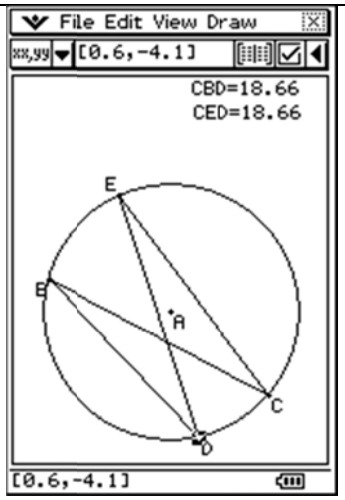



Tap [↗]. Draw ∠CBD and ∠CED, both of which are subtended in the same segment by arc CD.

Tap [↖].

Tap [▶].



| | |
|---|---|
| Display the size of ∠CBD by tapping BC and BD.<br><br>Tap on the size of ∠CBD and drag it into the Geometry window.<br><br>Name this angle CBD by tapping [A] and using the [abc] tab on the keyboard to type CBD, press **=**. Press **EXE**.<br><br>Tap in space.<br><br>Display the size of ∠CED by tapping CE and ED.<br><br>Tap on the size of ∠CED and drag it into the Geometry window.<br><br>Name this angle CED by tapping [A] and using the [abc] tab on the keyboard to type CED, press **=**. Press **EXE**.<br><br>Tap in space.<br><br>Hide the keyboard. |  |
| Observe the size of angles ∠CBD and ∠CED when points B, C and D respectively move on the circle by:<br><br>Tap B. Tap B a second time and drag it around the circumference such that ∠CBD and ∠CED both remain in the same segment.<br><br>Tap in space.<br><br>Tap C. Tap C a second time and drag it around the circumference such that ∠CBD and ∠CED both remain in the same segment.<br><br>Tap in space.<br><br>Tap D. Tap D a second time and drag it around the circumference such that ∠CBD and ∠CED both remain in the same segment.<br><br>Tap **File**, tap **Save** and name the file. |  |

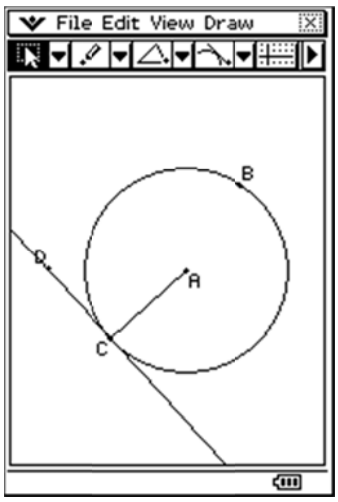
1. What do you notice about the size of the angles, ∠CBD and ∠CED, which are both subtended at the circumference by arc CD?

2. Prove the Angles in the Same Segment Theorem.

   Hint: This proof can be downloaded at http://www.mathxtc.com/
   (Use with caution!!!)

## TASK FOUR: Tangents and Radii

1. Use the Geometry application to draw a circle centre A and radius AB with DC tangential to the circle at C. Draw the radius AC.




Save the file.

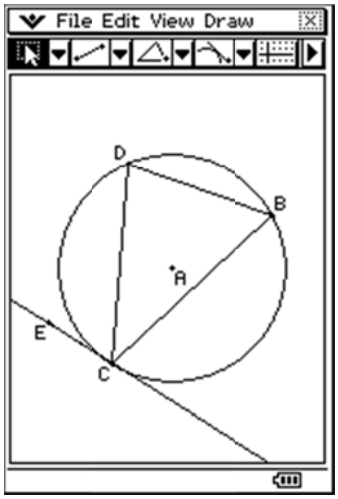
2. What do you notice about the size of $\angle ACD$ as the point C moves on the circle?

3. Prove the Tangent-Radii Theorem.

Hint: This proof can be downloaded at http://www.mathxtc.com/
(Use with caution!!!)

## TASK FIVE: Angles in the Alternate Segment

1. Use the Geometry application to draw a circle centre A and radius AB with EC tangential to the circle at C. B and D are two points on the circle such that ∠CBD is in the alternate segment to ∠DCE.




Note: it will be necessary to draw line segment EC prior to measuring the size of ∠DCE.

Save your file.

2. What do you notice about size of ∠DCE and ∠CBD as points B, C and D respectively are moved on the circle? (Maintain the location of ∠CBD in the alternate segment to ∠DCE.)

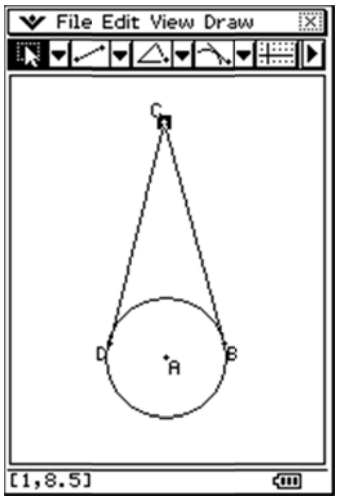
3. Prove the Angles in the Alternate Segment Theorem.

Hint: This proof can be downloaded at http://www.mathxtc.com/
(Use with caution!!!)

## TASK SIX: Lengths of Tangents From an External Point

1. Use the Geometry application to draw a circle centre A and radius AB. Position point C anywhere outside the circle and point D on the circle. Draw line segments CB and CD.

   Tap on CB, tap on the circle and then tap ▶ to display the measurement box. If **No** is displayed, tap the tick, ☑. CB is now a tangent to the circle at B. In a similar manner, make CD a tangent to the circle at D.




   Save your file.

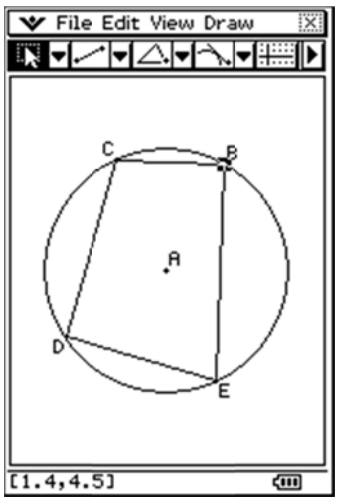
2. What do you notice about the length of tangents CB and CD as you change the location of point C?

3. Prove the Lengths of Tangents Theorem.

   Hint: This proof can be downloaded at http://www.mathxtc.com/
   (Use with caution!!!)

## TASK SEVEN: Cyclic Quadrilaterals

1. Use the Geometry application to draw a circle centre A and radius AB. Draw (cyclic) quadrilateral BCDE such that vertices B, C, D and E are all points on the circle.




   Save the file.

2. Change the shape of this cyclic quadrilateral by moving the vertices B, C, D and E respectively around the circle. What do you notice about the sizes of the opposite angles of this cyclic quadrilateral?

3. Prove the Cyclic Quadrilateral Theorem.

   Hint: This proof can be downloaded at http://www.mathxtc.com/
   (Use with caution!!!)