

INTRODUCTORY CALCULUS - REVISION ASSIGNMENT SERIES

Revision Assignments 4, 5 and 6 will be tested on **Wednesday 5th November 2008**.
Questions will be randomly selected from each of the three assignments. No notes will be allowed.
Solutions will be posted onto www.mathxhc.com

REVISION ASSIGNMENT SIX: 2007 SEMESTER TWO EXAM

ISSUED: MONDAY 20TH OCTOBER 2008

DUE: WEDNESDAY 5TH NOVEMBER 2008

NAME: _____

1. [2, 3 & 4 = 9 marks]

Write an equation for each of the following linear functions

(a)

x	-3	-1	1
y	-20	-10	0

(b) The line perpendicular to $-2x - 3y = 12$ that cuts the y axis at $y = -3$.

(c) The tangent to the curve $y = \frac{-6}{x} + 2x$ at $x = -1$.

2. [2, 3 & 3 = 8 marks]

If $g(x) = 2 - 3x - 2x^2$

Fully simplify your answer for:

(a) $g(0.5)$

(b) $g(a + 1)$

(c) p, if $g(p) = -3$

3. [3, 4, 3 & 3 = 13 marks]

(a) Simplify the following expressing your answers with positive indices:

(i) $\frac{16a^2b^{-3}}{(2ab)^3} \times ab^5$

(ii) $\frac{6^{1-a}}{9^{a-1}} \times 18^{a-1}$

(b) Solve the following, showing use of index rules:

(i) $(2x - 3)^{-2} = 0.25$

(ii) $\log_x 125 = 1.5$

4. [3, 5, 4 & 2 = 14 marks]

A storage container in the shape of a rectangular prism has a volume of 15m^3 . It has a height of h , and the base length, l , is 50% longer than its width.

- (a) Show how the formula $h = \frac{45}{2l^2}$ is determined from the volume.
- (b) The cost of materials for all sides is \$30 per square metre.
Show that the cost of making the container could be determined by the formula:
- $$\text{Cost} = 40l^2 + \frac{2250}{l}$$
- (c) What dimensions will minimize the cost of making the container?
Give each dimension to the nearest cm.
- (d) To the nearest dollar what is the minimum cost of making 100 such containers.

5. [3, 3, 3 & 3 = 12 marks]

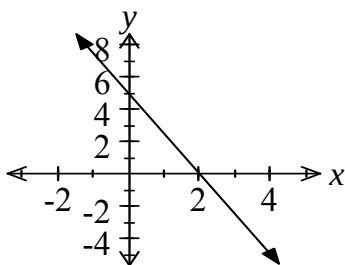
Solve the following equations exactly, where possible, otherwise to two decimal places. Working is expected.

- (a) $2 \log 4 + \log x = \log 4 + \log (x - 2)$
- (b) $\log (3x + 1) = 3$
- (c) $\log_3 \left(\frac{3x + 1}{x} \right) = -2$
- (d) $\log_5 x^2 - \log_5 \left(\frac{1}{x} \right) = 6$

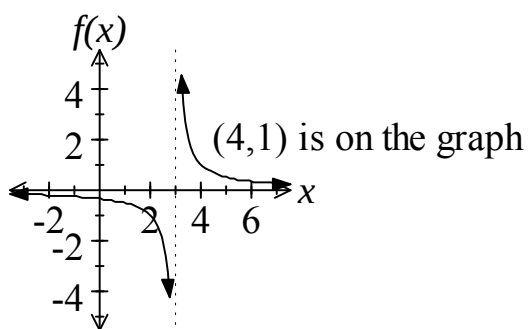
6. [2, 2 & 3 = 7 marks]

Determine the equation for the graphs below:

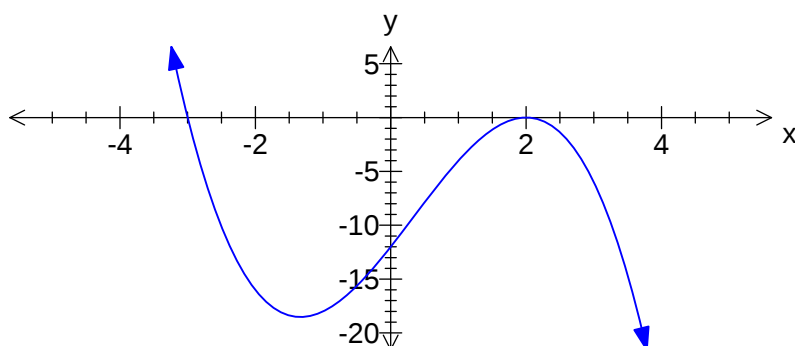
(a)



(b)



(c)



7. [3, 3, 4 & 5= 15 marks]

- (a) Differentiate, with respect to x

$$f(x) = (x^2 + 1)(x + 1)$$

- (b) Differentiate, with respect to x

$$g(x) = \frac{1}{\sqrt{4x + 2}}$$

- (c) Differentiate, with respect to x

$$h(x) = (3x + 2)^3 5\pi x$$

- (d) (i) Differentiate, with respect to x

$$k(x) = \frac{1 - 3x}{3x - 1}$$

- (ii) Explain the significance of your answer in (i).

8. [2, 3 & 2 = 7 marks]

The concentration, C mg/kL, of a chemical in the Roh River at time t days is given by:

$$C = e^{-2t} (5t + 1) \qquad 0 \leq t \leq 5$$

- (a) State an expression for the instantaneous rate of change of C with respect to t.
(b) Find the value of t when the instantaneous rate of change of C with respect to t is 0.
(c) Find the maximum concentration of the chemical, to two decimal places.

9. [7 marks]

Find the two points at which the tangent to the curve $y = \frac{x^3}{3} + \frac{5x^2}{4}$ is parallel to the line $-3x + 2y = 6$

10. [3, 3 & 2 = 8 marks]

A marathon runner is attempting to run the Grand Canyon's desert event over a distance of 2600 km. On his first day he runs 55km, but fatigue and the desert heat cause him to run less each day. Each successive day he runs 1.5% less than the previous day.

- (a) To the nearest km, how far does he run on the third and tenth day?
- (b) After 30 days of running, how far short of completing the event is he?
- (c) On which day does he complete the event?

11. [4, 3 & 2 = 9 marks]

A particle moves in a straight line with acceleration given by the expression $a = 6t \text{ cm s}^{-2}$. The particle's initial displacement is 1cm and its initial velocity is -3 cm s^{-1} . The time t is in seconds and $t \geq 0$.

- (a) Find expressions for the particle's velocity and displacement.
- (b) Describe the motion and position of the particle when $t = 2$ seconds.
- (c) Find the time(s) when the particle is at the origin.

12. [2, 3, 3, 3 & 3 = 14 marks]

Find:

- (a) $\int (3x^2 - 2x + 0.2) dx$
- (b) $\int (1 - 6x)^3 dx$
- (c) $\int \left(\frac{-2}{(1-2x)^5} \right) dx$
- (d) $\int_0^2 e^{-3x-2} dx$ give an exact answer
- (e) $\int \frac{12x}{(3x^2-1)} dx$

13. [5 marks]

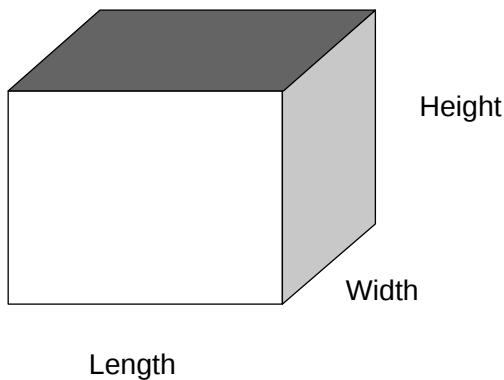
A manufacturer's marginal cost function is:

$$\frac{dc}{dn} = 3n + 0.6n^2$$

If production is presently $n = 200$ units per month, how much more would it cost to increase production to 250 units per month?

14. [2, 3 & 3 = 8 marks]

A glass fish container, without a lid, is in the shape of a rectangular prism with the width equal to the height. The total surface area of glass required is 5400 cm^2 .



- (a) Show that the length of the container can be calculated from the formula.

$$l = \frac{5400 - 2h^2}{3h}$$

- (b) Now show that the volume of the container can be:

$$V = 1800h - \frac{2h^3}{3}$$

- (c) Find the dimensions that will maximize the volume?

15. [2, 2 & 4 =8 marks]

A biologist is monitoring a fire ant infestation and notices that the area grows such that $\frac{dA}{dn} = 0.7A$ where **A** is the area in hectares and **n** is the number of weeks since the infestation was initially found. The initial infestation was estimated to be 1000 hectares.

- (a) Find an expression for the area infested after **n** weeks.
Use this formula to find the size of the infested area after 5 weeks.
- (b) Use logarithms to find the time taken for the infestation to reach 5000 hectares.
- (c) The local Environmental Council conducts a spraying operation to reduce the area on infestation. It predicts that the decay of the infestation after the spraying to be modeled by the equation $\frac{dA}{dn} = -0.5A$.
If the area of infestation after 10 weeks was 9025 hectares find to the nearest week when the spraying program commenced.

16. [8 marks]

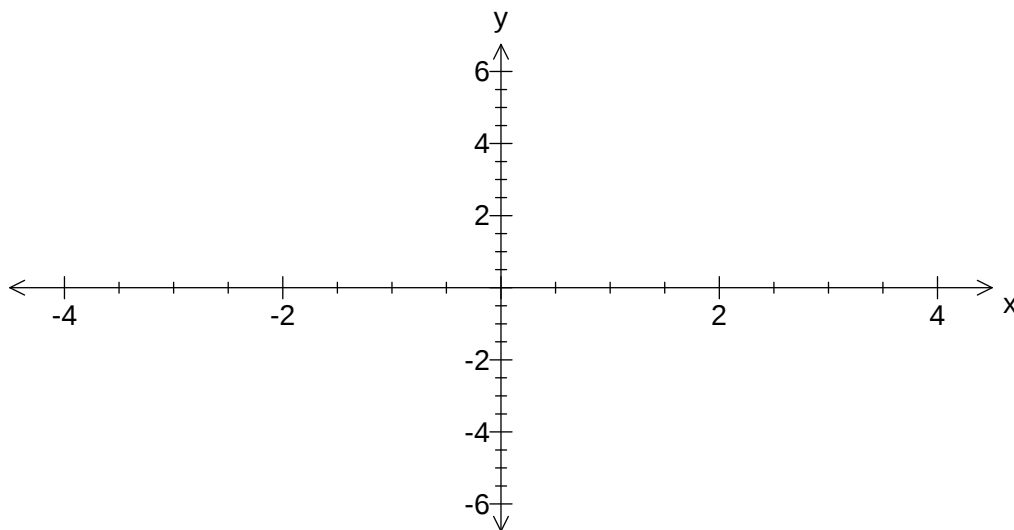
Consider the function $3y = 4x^3 - 3x + 6$.

Using calculus methods give the coordinates of all significant points. Your answer should determine all intercepts, turning points and points of inflexion and their nature.

17. [2, 3, 2 & 5 = 12 marks]

Given $f(x) = -(x^2 - 1)(x - 3)$ and $g(x) = -x^2 + 2x + 3$ For $-3 \leq x \leq 4$

- (a) Find all points of intersection.
(b) Provide a neat sketch of both graphs on the axes provided.



- (c) For your graph give the values of x for which $g(x) > f(x)$?
(d) Use calculus methods to calculate the area between the two curves.

18. [8 marks]

A curve has gradient function $f'(x) = kx + \frac{m}{x^2}$ where k and m are constants.

Find the function $f(x)$ given that the graph passes through $(2, -1)$ and has a turning point at $(-2, 0)$.

19. [3 & 5 = 8 marks]

- (a) Given $a + \frac{a}{3} + \frac{a}{9} + \dots = 15$ Find a .
(b) Given that:

$$1.5, 2x, 7x + 3$$

are the first three terms of a geometric progression and that x is a positive value, then determine the fifth term.

