INTRODUCTORY CALCULUS - REVISION ASSIGNMENT SERIES

Revision Assignments 4, 5 and 6 will be tested on **Wednesday 5th November 2008**.
Questions will be randomly selected from each of the three assignments. No notes will be allowed.
Solutions will be posted onto www.mathxtc.com

REVISION ASSIGNMENT FIVE

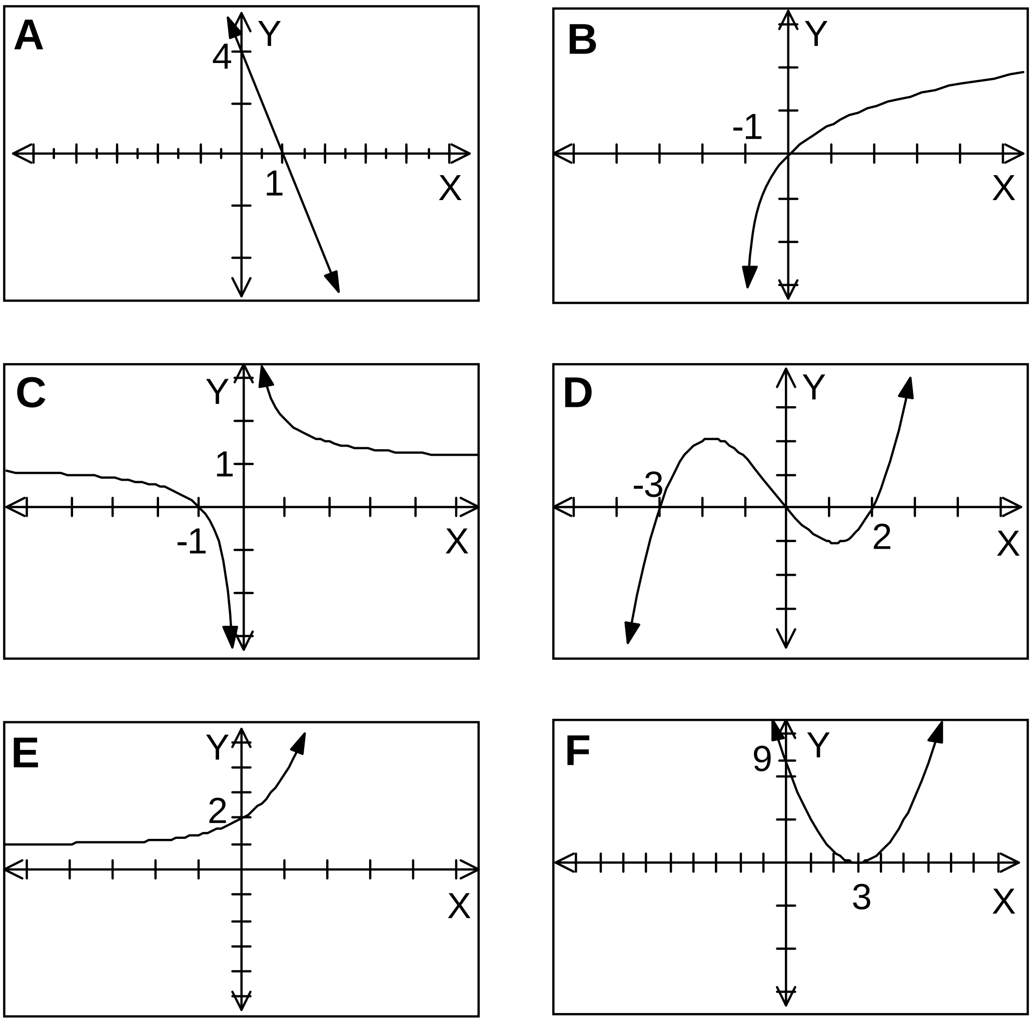
ISSUED: FRIDAY 12TH SEPTEMBER 2008 **DUE: MONDAY 20TH OCTOBER 2008**

NAME:

PART A

1. (2, 2, 2, 2, 2, 2 marks)

The graphs of 6 functions are drawn below. Match the letter of each graph with one of the functions listed below.



$y = 2x$ $\qquad y = x(x-2)(x+3)$ $\qquad y = \ln(x+1)$

$y = \dfrac{1}{x} + 1$ $\qquad y = (x-3)^2$ $\qquad y = \dfrac{1}{4}x + 4$

$y = x^3 - x^2 - 6x$ $\qquad y = -4x + 4$ $\qquad y = e^x + 1$

2. (5, 2, 3, 3 marks)

A function f is defined as

$$f(x) = x^3 + 3x^2 - 4$$

(a) Find the coordinates of all stationary points on the curve. Determine the nature of these points.

(b) Determine the coordinates of any points of inflection.

(c) The function f can also be expressed in the form

$$f(x) = (x + 2)^2(x - 1)$$

Determine the X and Y intercepts of the graph.

(d) Use the information from parts (a) and (b) to draw a sketch of the function.

3. (3, 4, 2, 2 marks)

A travel agent charters an aircraft and offers cheap flights to Hong Kong. The advertised price for the trip is $600 per person if 120 people or less sign up. For every person above 120 the price for each person will be reduced by $2. [For example if 123 people sign up then each person on the flight pays $594].

(a) If n is the number of persons above 120 signing up show that the total revenue, R (in dollars), received by the agent is given by

$$R(n) = 72\,000 + 360n - 2n^2$$

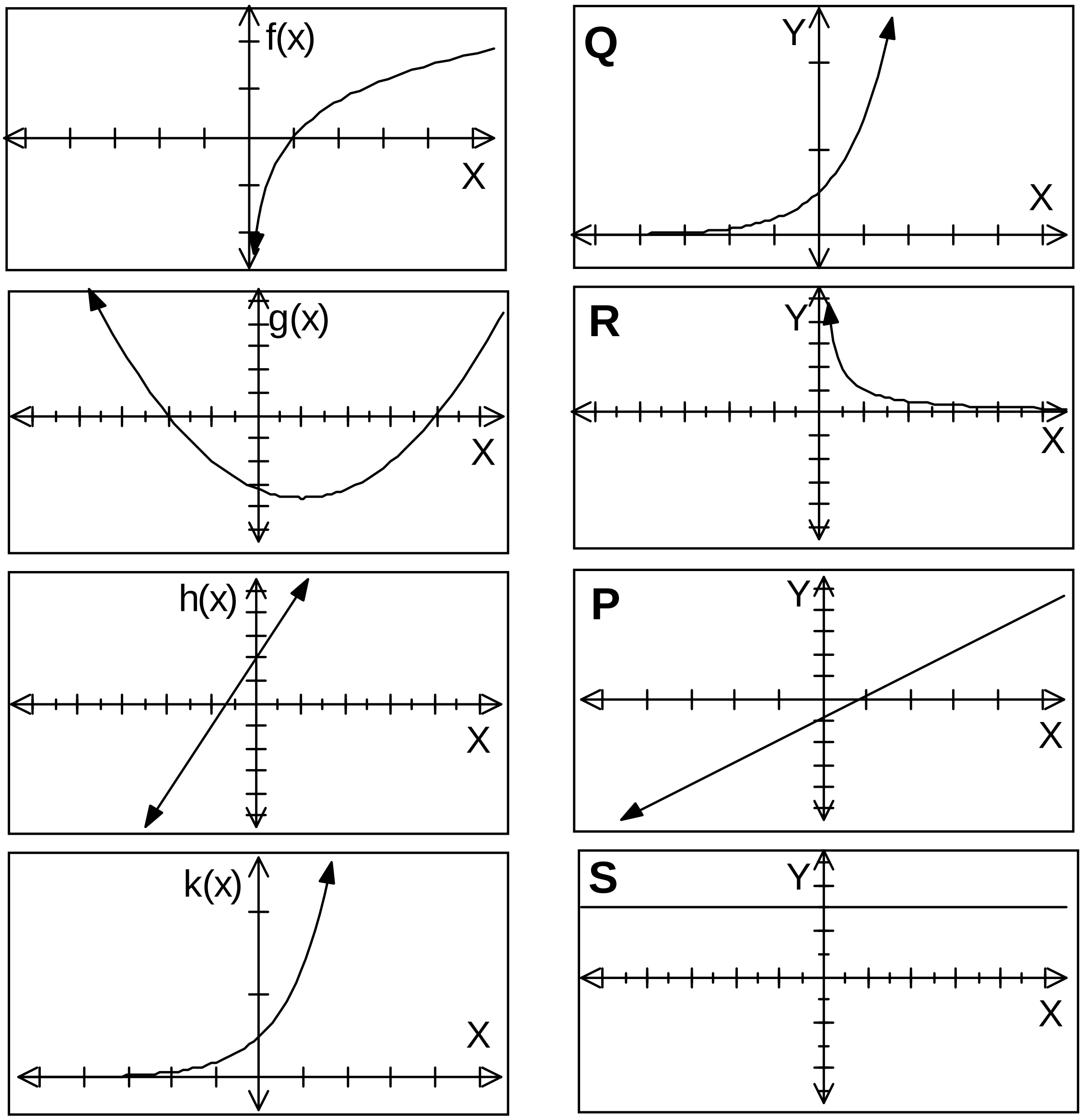
(b) How many passengers will provide the agent with the maximum possible revenue?

(c) What is the maximum revenue the agent can obtain?

(d) What restrictions, if any, need to be placed on the domain of n?

4. (8 marks)

The graphs of functions f, g, h and k are drawn below as well as the graphs of the derivatives of these functions in random order.



Copy and complete the following table which matches each graph with its derivative.

function	f	g	h	k
derivative				

5. (a) (5 marks)

Determine the equation of the tangent to the graph of $y = x^2 - \frac{1}{x^2}$ at the point where x = - 1.

(b) (1, 1, 3 marks)

If $y = e^{2t}$ and $x = 4t^2$

Find (i) $\frac{dy}{dt}$

(ii) $\frac{dx}{dt}$

(iii) hence, use the chain rule to find $\frac{dy}{dx}$

6. (4, 4 marks)

A large gas turbine which is rotating at 3 500 revolutions per minute is switched off and begins to slow down. The following table shows the speed of the turbine at one minute intervals after it is switched off.

minutes after turbine is switched off	1	2	3	4	5
turbine speed (r.p.m.)	2 800	2 240	1 792	1 434	1 147

(a) At what speed is the turbine rotating 12 minutes after it is switched off?

(b) What time elapses after the turbine is switched off before it is rotating at 100 r.p.m.?

7. (4, 3 marks)

Functions f and g are defined as follows.

$$f(x) = \frac{4x}{2x - 1} \qquad g(x) = \ln e^{(-4x + 7)}$$

(a) Determine $\frac{df}{dx}$ and $\frac{dg}{dx}$

(b) Find the value(s) of x for which the graphs of the two functions have the same gradient.

8. (2, 4, 6 marks)

A welfare organisation seeking donations for one of its overseas projects decides to run a media campaign. Past experience shows that contributions are highest at the start of the campaign and tend to diminish as the campaign continues.

The rate at which the donations, D, are being received by the organisation after the campaign has been running for t days is given by

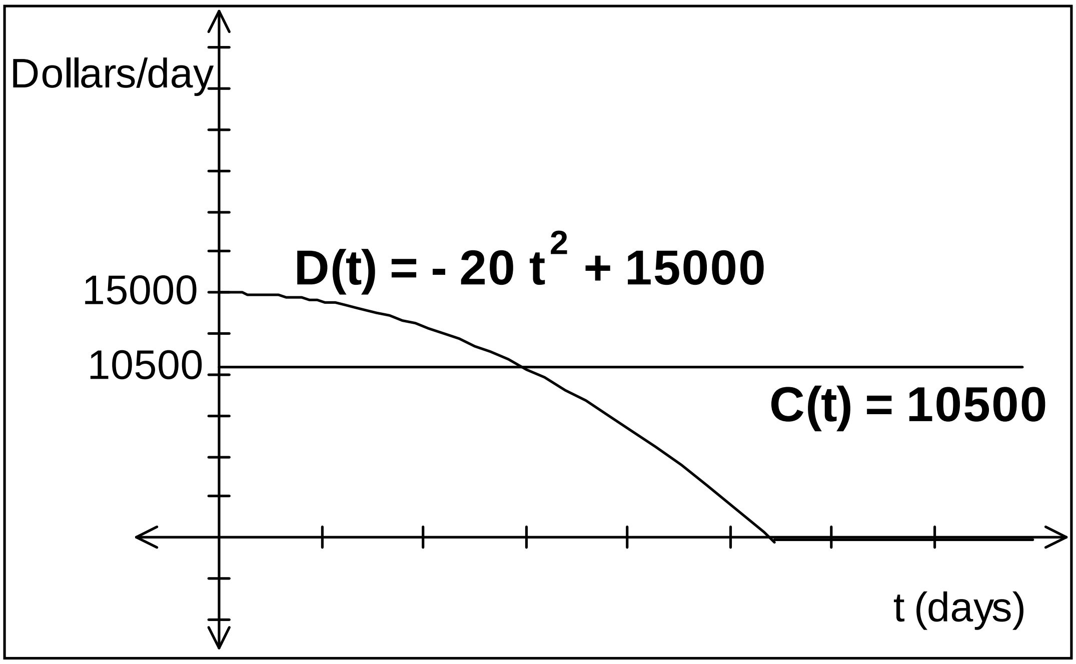
$$D(t) = -20t^2 + 15\,000$$

where D is measured in dollars.

(a) At what rate are the contributions being received at the end of the 5^{th} day?

(b) The organisation spends \$10 500 per day on advertising costs. If the campaign is to cease when the rate of donations is the same as the cost of advertising for how many days should the campaign run?

The graphs of the donations as well as the advertising costs, C, after t days are shown below.



(c) Determine the range of values for t where a profit is made and by evaluating the area between the two graphs find the total profit made by the organisation.

9. (3, 2, 3, 3 marks)

A broken cooling pipe at a nuclear power station results in contaminated water slowly being released into the sea.

The area, A, of seabed which is contaminated t days after the pipe breaks is shown in the following table.

days after accident (t)	5	10	15	20
contaminated area (A) measured in km^2	0.29	1.72	8.49	53.60

The contaminated area can be approximated using the equation

$$A(t) = e^{kt^2} - 1 \qquad \text{where k is a constant.}$$

(a) Choose one set of values from the given table to calculate the value of k correct to 3 decimal places.

Use your 3 decimal place approximation of k in answering parts (b), (c) and (d).

(b) What area of seabed is contaminated 16 days after the pipe breaks?

(c) How many days after the pipe breaks will the area contaminated have reached 500 km^2?

(d) At what rate is the area of contamination spreading 30 days after the pipe breaks?

10. (1, 3, 3, 3, 2, 2 marks)

On a section of the Perth to Armadale railway engineers are experimenting in order to find the most comfortable and economical train speed between two stations P and Q. Train A and train B travel at different velocities between the two stations.

The velocity of train A for the journey, in ms^{-1}, is given by

$$V_A = \frac{-t^2}{120} + t$$

(a) At what velocity is train A travelling 20 seconds after leaving station P?

(b) What is the maximum velocity of train A?

(c) What distance has train A travelled k seconds after leaving station P?

Train B accelerates constantly for 20 seconds until it reaches a speed of 24 ms^{-1}. It then maintains this velocity for 80 seconds before decelerating uniformly to rest in 20 seconds.

(d) Draw a velocity time graph to indicate the change in the velocity of train B throughout its journey.

(e) Determine the velocity equation of train B for the period for which it is accelerating.

(f) What is the distance between the two stations?

11. (a) (2, 2, 3 marks)

Perform the following integrations.

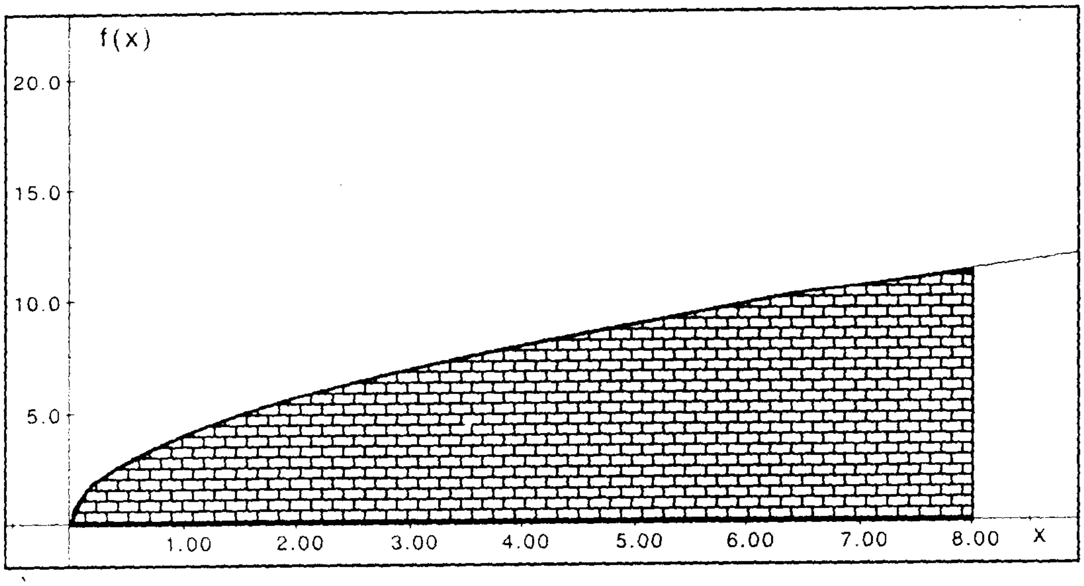
(i) $\int \sqrt{5x + 4}\, dx$

(ii) $\int \left(\dfrac{6}{x} - e^{6x} \right) dx$

(iii) $\int \left(\dfrac{1}{x} + x \right)^2 dx$

(b) (3, 4 marks)

A courtyard which is to be paved with tiles has two straight edges and one curved edge. The curved edge can be approximated by the equation $f(x) = 4\sqrt{x}$ as shown in the diagram below. [All measurements are given in metres]



(i) Approximate the area which is to be tiled using a series of rectangles of width 2 metres and heights given by f(2), f(4), f(6) and f(8).

(ii) By how much does your calculation in part (i) exceed the true area of the courtyard?

12. (2, 4, 3, 4 marks)

The number of seagulls (S) nesting on Rottnest Island has been increasing since the start of 1988 at a rate proportional to the number present

Ie. $\frac{dS}{dt} = kS$

Where k is a constant and t is the number of years since the start of 1988.

(a) Show that $S = Ae^{kt}$ satisfies the differential above. (A is a constant)

(b) At the start of 1988 the number of seagulls was estimated at 16 000. By the beginning of 1993 the population had grown to 19 500.

Determine (i) the value of A

(ii) the value of k correct to two decimal places.

(c) If the population continues to grow at the same rate predict the number of seagulls on the island at the start of year 2 000. Give your answer to the nearest 100 seagulls.

(d) During which year will the seagull population have doubled the population at the start of 1988?

13. (a) (2, 3, 4, 3 marks)

The first three terms of a geometric sequence are as follows

ln 16, ln 4, ln 2

(i) What is the common ratio?

(ii) What is the 8th term of the sequence?

(iii) What is the sum of the first 5 terms of the sequence?

(iv) Determine the sum of an infinite number of terms of the sequence.

(b) (2, 1, 2 marks)

If $y = -e^{-x}$

Find (i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

Hence write down an expression for $\dfrac{d^{70}y}{dx^{70}}$

14. (2, 5 marks)

Functions h and f are defined as follows:

$$h(x) = f'(x) \quad \text{where} \quad f(x) = \ln(4x + 2)$$

(i) Find h(x) in terms of x.
(ii) Determine the area bounded by the graph of function h and the x axis between x = 1 and x = 5.

15. (1, 1, 2, 2, 2, 2 marks)

The Hookemup Fishing Company produces fibreglass fishing lures.
The total cost in dollars of producing n lures is given by

$$C(n) = 20\sqrt{n} + 2n + 900$$

(a) What is the cost of producing 800 lures?

(b) Find the average cost per lure when 800 lures are sold.

(c) Determine the marginal cost, $C'(n)$.

(d) Use the marginal cost to calculate by how much the total production costs will increase due to the production of the 201^{st} lure.

(e) If the company sells the lures for $8 each, write an equation for the profit earned by the company for the sale of n lures.

(f) Hence determine the profit to the company on the sale of 800 lures.

16. (4, 4, 5 marks)

The acceleration, a ms^{-2}, of an object after t seconds is given by

$$A(t) = 200e^{-2t}$$

(a) For how many seconds must the object accelerate before the acceleration reaches 40 ms^{-2}? Give your answer correct to 2 decimal places.

(b) If the object is initially at rest determine an expression for the velocity of the object after t seconds.

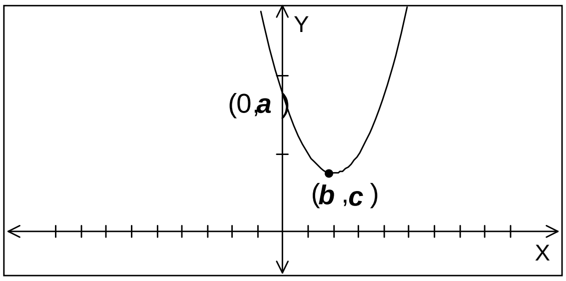
(c) How far does the object travel in the first 2 seconds of motion?

PART B

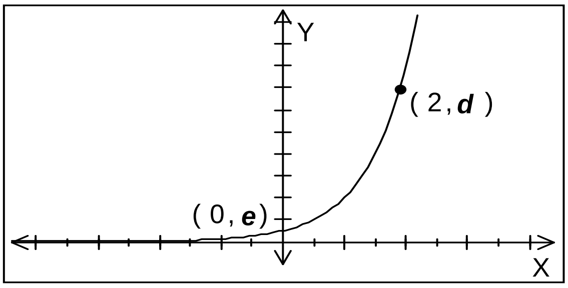
1. (10 marks)

Determine the value of the variables **a**, **b**, **c**, **d**, **e**, **f**, **g**, **h**, **i** and **j** in the following diagrams. The variables appear either in the graph or in the respective function.

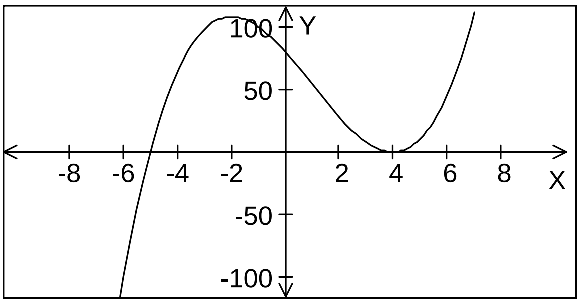
(a) $y = (x - 2)^2 + 3$



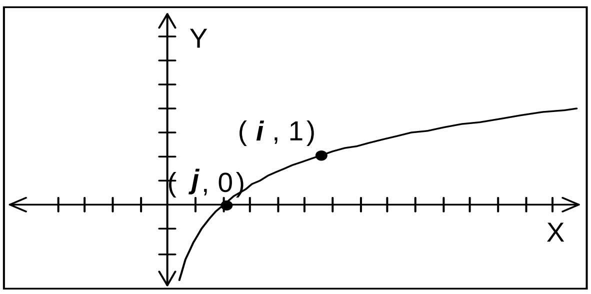
(b) $y = 2^{2x}$



(c) $y = (x + \boldsymbol{f})(x + \boldsymbol{g})(x + \boldsymbol{h})$



(d) $y = \ln x$



2. (12 marks)

(a) The fuel consumption (C) of a tractor, when travelling between 10 km/h and 80 km/h, varies according to the equation

$$C(x) = \frac{1}{50}\left(\frac{200}{x} + x\right) \text{L/km}$$

where x is the speed of the tractor in km/h.

(i) What is the fuel consumption of the tractor (in L/km) when it is travelling at 20 km/h?

(ii) How many litres of fuel does the tractor use in travelling 50 km at an average speed of 20 km/h?

(iii) At what speed should the tractor travel so that fuel consumption is a minimum?

(b) Two functions P and Q are such that P(2) = 3 and Q(2) = 5.

If we let $P'(x) = \frac{dP}{dx}$ and $Q'(x) = \frac{dQ}{dx}$

then $P'(2) = -1$ and $Q'(2) = 4$

Find (i) $R'(2)$ where $R(x) = P(x)Q(x)$

(ii) $T'(x)$ where $T(x) = \frac{P(x)}{Q(x)}$

3. (10 marks)

A windup toy car is released and travels along the ground with an acceleration a cm/s ^{2}, at time t seconds after release where a = - 2t + 5 until the car comes to rest.

(a) What is the acceleration and velocity of the car at the instant it is released?

(b) Determine the average velocity of the car for the first 3 seconds of its motion.

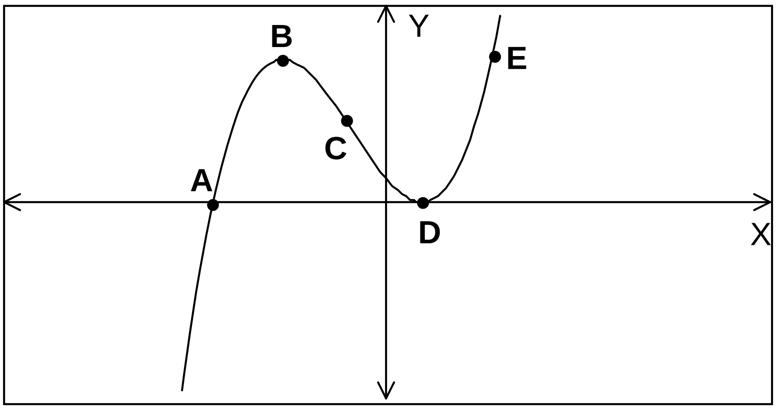
(c) For how many seconds is the car in motion?

(d) Write down an expression for the distance, S, travelled by the car t seconds after it was released.

(e) What distance does the car travel before coming to rest?

4. (13 marks)

The graph of a function $y = (x - 1)^2(x + 5)$ is shown below.



(a) Determine (i) the x and y intercepts of the graph.

(ii) the coordinates of the point B (a local maximum).

(iii) the coordinates of the point C (a point of inflection).

(c) Copy and complete the table below indicating whether y' and y'' are positive or negative within the indicated intervals on the graph.

Interval	**AB**	**BC**	**CD**	**DE**
y'				
y''				

5. (13 marks)

(a) By calculating $\frac{dy}{dx}$ explain why the function $y = (2x + 3)^3$ is never decreasing.

(b) Determine the area of the region bounded by the curve $y = \sqrt{2x + 1}$, the X axis and the lines $x = 0$ and $x = 3$.

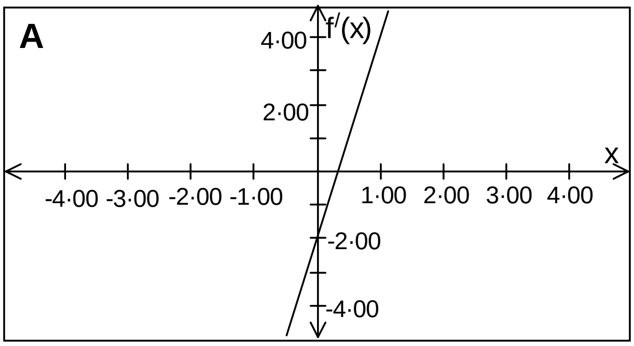
(c) Find the equation of the tangent to the curve $g(x) = \ln x + \frac{1}{x}$ at the point where $x = 1$.

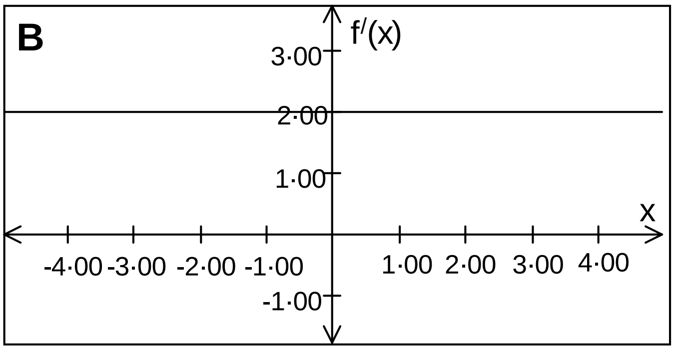
6. (8 marks)

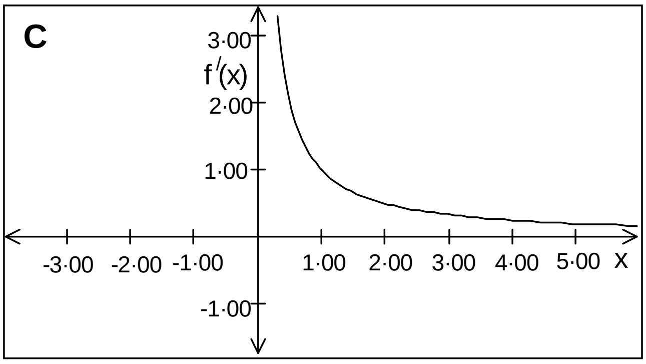
(a) Match each of the following functions with the graph of its derivative.

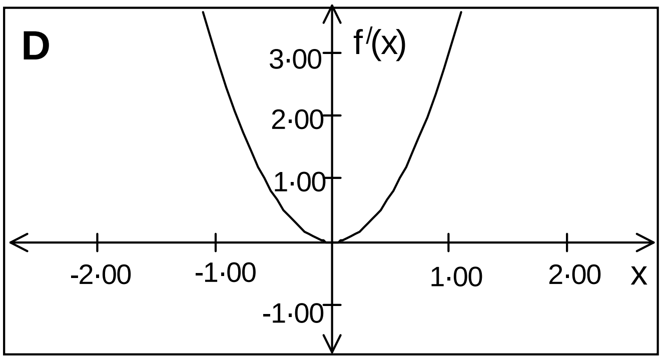
(i) $f(x) = 2x - 3$ (ii) $f(x) = \ln x$ (iii) $f(x) = 3x^2 - 2x + 1$

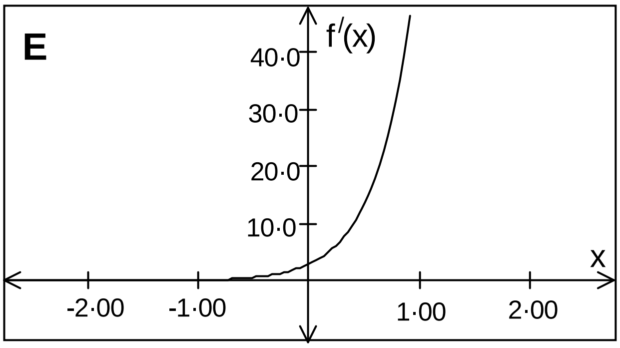
(iv) $f(x) = e^{3x}$ (v) $f(x) = x^3 + 5$











(b) Find an expression for $\frac{dp}{dx}$ in terms of x given

$$P = 4q \qquad \text{and} \qquad q = 2r + 3 \qquad \text{and} \qquad r = 5x^2$$

7. (8 marks)

A crystal is grown in a solution in a laboratory. The rate of growth of the crystal is given by

$$\frac{dV}{dt} = -\frac{1}{20}(t + 2)^2 + 50 \text{ mm}^3\text{/hour}$$

where V is the volume of the crystal at time t.

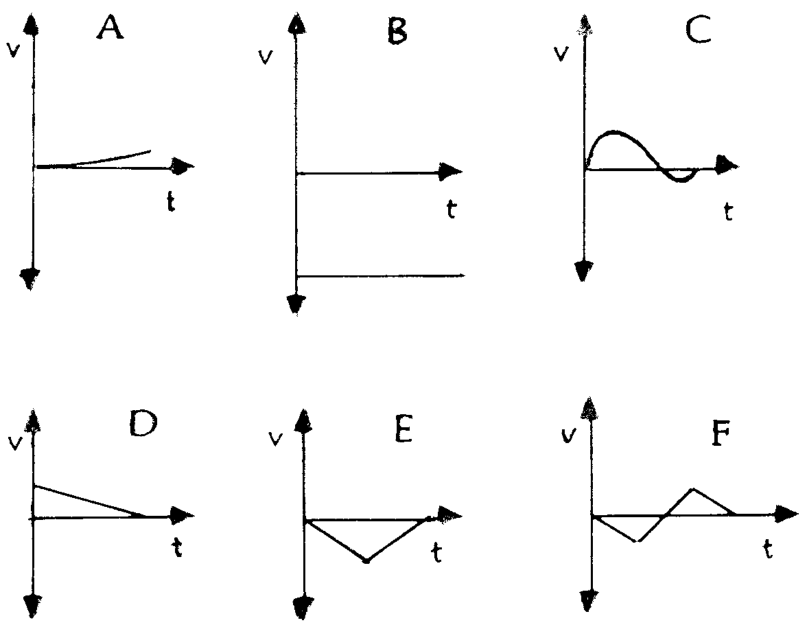
(a) Determine an expression for the volume V, after t hours, if the volume of the crystal at time t = 0 is 10 mm^3.

(b) Hence, find by how much the volume of the crystal increases between t = 10 and t = 20 hours.

8. (14 marks)

(a) The graphs show the velocities of 6 objects as they move along the x axis for equal intervals of time. Assuming the scales on all graphs are the same determine which object(s)

(i) has zero acceleration throughout its journey
(ii) ends up at its starting point
(iii) ends up furthest from its starting point
(iv) has the greatest (positive) average velocity
(v) experiences the greatest instantaneous acceleration.



(b) In February 1994 the Canning River was affected by an algal bloom. On the 10^{th} of February an estimated 54 000 m^2 of river was full of algae and the bloom was spreading at t6he rate of 6% per day.

ie $$\frac{dA}{dt} = 0.06A$$

where A is the area of river affected (measured in m^2) t days after February 10^{th}.

(i) Show that the equation above can be obtained by differentiating $A = A_0 e^{0.06t}$ where A_0 is a constant.

(ii) What is the value of A_0?

(iii) What area of river was affected on February 15^{th}?

(iv) On what date did the area affected first exceed 100 000 m^2?

9. (11 marks)

A bus company operates a shuttle service between the airport and the city. The company charges $12 for the trip and carries 400 passengers/day.

The company estimates that they will lose 20 customers per day for each increase of $1 in the fare.

(a) How many passengers will the company expect if they charge $15 for the trip?

(b) Show that the total fare (F) received by the company when charging $d for the trip is given by

$$F(d) = d(640 - 20d) \qquad \text{where } d \geq 12$$

(c) What fare (to the nearest dollar) should the company charge in order to maximise the amount of fares collected?

(d) Find the maximum amount the company can charge for a trip if it hopes to collect at least $3 000 per day in fares. Give your answer to the nearest dollar.

10. (9 marks)

(a) For what values(s) of x will the tangents to the curves $f(x) = e^{x}$ and $g(x) = e^{2x}$ be parallel?

(b) A function g is defined as

$$G(x) = \ln(2x + 5)$$

Determine (i) g(2)

(ii) g(2 + h) where h is a constant

(iii) the value of $\frac{dg}{dx}$ when x = 2.

11. (12 marks)

The rate of oil flow from a well has been decreasing steadily at the rate of 3% per month. The well is presently producing 770 000 barrels of oil per month.

(a) Determine the flow of oil from the well measured in barrels/month

(i) in one months time.

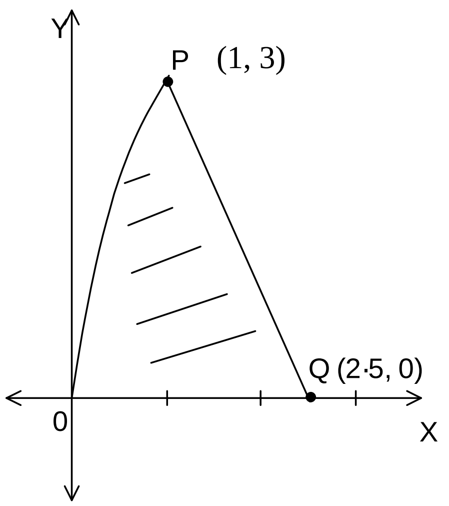
(ii) In six months time.

(b) Find the total amount of oil the well is expected to produce during the next 12 months.

(c) The oil well will be shut down once the oil flow falls below 400 000 barrels/month. For how many more months will the well be expected to be in operation?

12. (15 marks)

(a) The diagram shows the pattern for a new windsurfer sail where all measurements are given in metres.



(i) Find the equation of the line PQ.

(ii) If the curve OP is determined by the equation

$$y = x^3 - 3x^2 + 4x$$

calculate the area of the sail, in square metres, correct to 2 decimal places. [Hint: Divide the sail area into two regions.]

(b) Find, correct to 2 decimal places, the sum of an infinite number of terms of the series

$$\frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \ldots.$$

13. (15 marks)

(a) A company issues shares to increase its working capital. The rate at which the number (N) of shares sold per day has been increasing is given by

$$\frac{dN}{dt} = 60e^{\frac{1}{4}}$$

where t is the number of days after the shares were first released.

(i) If 20 000 shares were sold initially (i.e. when t = 0), determine an expression for N(t), the number of sales on day t.

(ii) How many shares were sold on the 8^{th} day after they were released for sale?

(iii) How many days after release would the expected number of shares sold that day reach 30 000?

(b) Solve the following logarithmic equation

$$2\log_2 x + \log_2 2x = \log_2 4$$

14. (13 marks)

(a) A function p is defined as $p(x) = \dfrac{x}{\ln x}$

(i) For what value(s) of x is the function undefined?

(ii) Determine $\dfrac{dp}{dx}$.

(b) The intensity, I, of an earthquake is measured using a Richter Scale. The value of the Richter Scale reading, R, is given by

$$R = \log_{10}\left(\frac{I}{I_0}\right)$$

where I_0 is the minimum average intensity.

(i) What reading on the Richter Scale would result from an earthquake 500 times the minimum average intensity, I_0?

(ii) In 1993 Los Angeles was rocked by an earthquake measuring 6.5 on the Richter Scale. What was the intensity of the earthquake, I, in terms of I_0?

(iii) How many times more intense was the 1906 San Francisco earthquake, which measured 7.5 on the Richter Scale, than the Los Angeles earthquake of 1993?

15. (17 marks)

(a) Given $\frac{dM}{dt} = \frac{4}{t}$ and when t = e, M = 2

find

(i) an expression for M in terms of t

(ii) $M(e^3)$

(b) The cost to a manufacturer (in dollars) of producing x items is given by

$$C(x) = 50\sqrt{x} + 350$$

(i) What is the cost of producing 20 items?

(ii) If 80 items are produced find the average production cost/item.

(iii) If each item is sold for $24, how many items must be sold before the manufacturer begins to make a profit?

(iv) How many items must be sold before the marginal profit exceeds $22?