SOLUTIONS INTRODUCTORY CALCULUS REVISION ASSIGNMENT 5

PART A

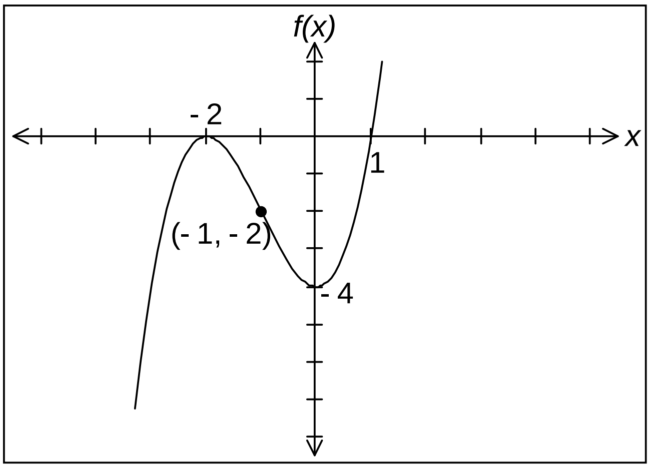
1. Graph A: $y = -4x + 4$
   Graph B: $y = \ln(x + 1)$
   Graph C: $y = \frac{1}{x}\ 1$
   Graph D: $y = x(x-2)(x+3)$
   Graph E: $y = e^{x} + 1$
   Graph F: $y = (x-3)^{2}$

2. (a) $f(x) = x^{3} + 3x^{2} - 4$
   $f'(x) = 3x^{2} + 6x$
   Stationary points where $f'(x) = 0$
   ie. $3x^{2} + 6x = 0$
   $3x(x + 2) = 0$
   ie. $x = 0,\ x = -2$
   $f''(x) = 6x + 6$
   $f''(-2) < 0 \therefore$ rel. max at $(-2, 0)$
   $f''(0) > 0 \therefore$ rel. min at $(0, -4)$

   (b) $f''(x) = 0 \Rightarrow x = -1$
   ie. point of inflection $(-1, -2)$

   (c) X intercepts
   $y = 0 \Rightarrow x = -2, 1$
   ie. $(-2, 0)\ (1, 0)$
   Y intercept
   $x = 0 \Rightarrow y = -4$
   ie. $(0, -4)$

   (d)



3. (a) $R(n) = (120 + n)(600 - 2n)$
$= 72\,000 + 360n - 2n^2$

(b) $R'(n) = 360 - 4n$
If $R'(n) = 0$
then $n = 90$
$R''(n) = -4$
$\therefore$ Revenue is a maximum when $n = 90$
ie. total number of passengers is 210

(c) Max revenue = R(90)
= \$88 200

(d) $n \geq 0$ and $n \leq 0$total carrying capacity of plane – 120

4.

function	f	g	h	k
derivative	R	P	S	Q

5. (a)

$$y = x^2 - \frac{1}{x^2}$$

$$y' = 2x + \frac{2}{x^3}$$

When $x = -1$ $y = 0$
$y' = -4$

(b) (i) $$\frac{dy}{dt} = 2e^{2t}$$

(ii) $$\frac{dx}{dt} = 8t$$

(iii)

$$\frac{dy}{dx} = \frac{dy}{dt} x \frac{dt}{dx}$$

$$= \frac{2e^{2t}}{8t}$$

$$= \frac{e^{2t}}{4t}$$

6. (a) Let t be the number of minutes after turbine is switched off and S be the turbine speed.

$$\frac{2800}{3500} \approx \frac{2240}{2800} \approx \frac{1792}{2240} \approx 0.8$$

After 12 minutes

$\therefore$ S = 3500 $(0.8)^{12}$

$\approx$ 240.5

ie. turbine speed $\approx$ 240.5 rpm

(b) If S = 100

$$100 = 3500(0.8)^t$$

$$t = \frac{\ln 100 - \ln 3500}{\ln 0.8}$$

$$t \approx 15.9$$

$\therefore$ turbine speed 100 rpm after $\approx$ 15.9 min

7. (a)

$$f(x) = \frac{4x}{2x - 1}$$

$$f'(x) = \frac{(2x - 1)4 - 4x(2)}{(2x - 1)^2}$$

$$= \frac{-4}{(2x - 1)^2}$$

$$g(x) = \ln e^{(-4x + 7)}$$

$$= (-4x + 7)\ln e$$

$$= -4x + 7$$

$$g'(x) = -4$$

(b)

If $f'(x) = g'(x)$

$$\frac{-4}{(2x - 1)^2} = -4$$

$$(2x - 1)^2 = 1$$

$$(2x - 1) = 1 \text{ or } (2x - 1) = -1$$

ie. $x = 1$ or 0

8. (a) $D(5) = -20(5)^2 + 15000$

$= 14\,500$

$\therefore$ At end of 5^{th} day donations are received at the rate of $14 500 per day.

(b) Campaign ceases when

$D(t) = 10\,500$

ie. $-20t^2 + 15000 = 10500$

$-20t^2 = -4500$

$t^2 = 225$

$t = \pm 15$ (reject $t = -15$)

$\therefore$ Campaign will last 15 days.

(c)

$$\begin{aligned}
\text{Enclosed Area} &= \int_0^{15} D(t)\,dt - \int_0^{15} 10500\,dt \\
&= \int_0^{15} \left(-20t^2 + 15000 - 10500\right) dt \\
&= \int_0^{15} \left(-20t^2 + 4500\right) dt \\
&= \left[-\frac{20}{3}t^3 + 4500t\right]_0^{15} \\
&= 45\,000
\end{aligned}$$

ie. Campaign Profit = $45 000

9. (a)

$$\begin{aligned}
A(t) &= e^{kt^2} - 1 \\
A(5) &= e^{25k} - 1 = 0.29 \\
e^{25k} &= 1.29 \\
\ln e^{25k} &= \ln 1.29 \\
k &= \frac{\ln 1.29}{25} \\
k &= 0.100 \;(3\,\text{d.p.})
\end{aligned}$$

(b)

$$\begin{aligned}
A(16) &= e^{(0.010 \times 256)} - 1 \\
&= 11.9
\end{aligned}$$

$\therefore$ After 16 days $= 11.9\ \text{km}^2$ affected

(c) If A(t) = 500

$$500 = e^{0.010t^2} - 1$$

$$501 = e^{0.010t^2}$$

$$t^2 = \frac{\ln 501}{0.010}$$

$$t = 24.9$$

Contaminated area reaches 500 km^2 after 24.9 days

(d)

$$A'(t) = 2kte^{kt^2}$$

$$= 2(0.010)t^{0.010t^2}$$

$$A'(30) = 2(0.010)(30)e^{0.010(30)^2}$$

$$= 4862$$

$\therefore$ After 30 days contamination spreading at 4862 km^2/day

10. (a)

$$V_A(20) = \frac{(-20)^2}{120} + 20$$

$$= 16.7$$

$\therefore$ After 20 sec V_A = 16.7 ms^{-1}

(b)

$$V'_A = \frac{-t}{60} + 1$$

$$V'_A = 0 \text{ when } t = 60$$

$\therefore$ train A has max velocity at t = 60 sec

V(60) = 30 ms^{-1}

(d) In k seconds train A travels

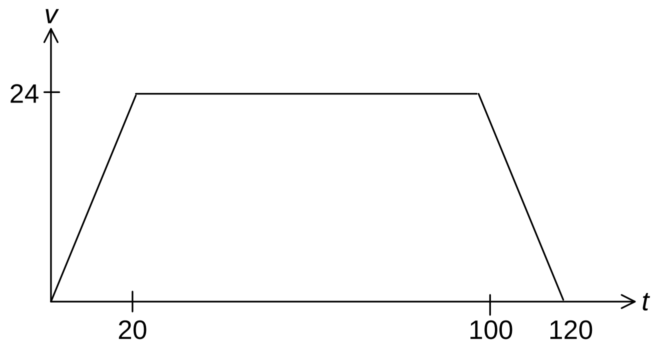
$$\int_0^k v\,dt$$

$$= \int_0^k \left(\frac{-t^2}{120} + t \right) dt$$

$$= \left[\frac{-t^3}{360} + \frac{t^2}{2} \right]_0^k$$

$$= \frac{-k^3}{360} + \frac{k^2}{2} \text{ metres}$$

(d)



(e)

$$V_B = \frac{24}{20}t \quad (0 \le t \le 20)$$

$\therefore$ For the first 20 seconds

$$V_B = \frac{6t}{5}\ \mathrm{m\,s^{-1}}$$

(f) Distance between stations $= \dfrac{-(120)^3}{360} + \dfrac{(120)^2}{2}$

$= 2400$ m

11. (a) (i)

$$\int \sqrt{5x+4}\,dx$$

$$= \frac{2(5x+4)^{\frac{3}{2}}}{15} + c$$

(ii)

$$\int \left(\frac{6}{x} - e^{6x}\right) dx$$

$$= 6\ln|x| - \frac{e^{6x}}{6} + c$$

(iii)

$$\int \left(\frac{1}{x} + x\right)^2 dx$$

$$= \int \left(\frac{1}{x^2} + 2 + x^2\right) dx$$

$$= -\frac{1}{x} + 2x + \frac{x^3}{3} + c$$

(b) (i) Area = 2f(2) + 2f(4) + 2f(6) + 2f(8)

$= 8\sqrt{2} + 8\sqrt{4} + 8\sqrt{6} + 8\sqrt{8}$

$= 69.54 \text{ m}^2$

(ii)

$$\text{Area} = \int_0^8 4\sqrt{x}\,dx$$

$$= \left[\frac{8x^{\frac{3}{2}}}{3}\right]_0^8$$

$= 60.34 \text{ m}^2$

Difference in area = 9.20 m^2

12. (a)

$$S = Ae^{kt}$$

differentiate with respect to t

$$\frac{ds}{dt} = Ake^{kt}$$

ie. $\dfrac{ds}{dt} = kS$

(b) (i) when t = 0 $16\,000 = Ae^{0}$

$\therefore$ A = 16 000

(ii) when t = 5 $19\,500 = 16\,000e^{5k}$

$$\ln\left(\frac{19500}{16000}\right) = 5k$$

$$k = 0.04 \text{ (2 d.p.)}$$

(c) At the start of year 2000, t = 12

No. of seagulls, S = $16000e^{12(0.04)}$

= 25723

ie. =25700 (nearest hundred)

(e) If population grows to 32 000

$32000 = 16000e^{(0.04)t}$

ln 2 = 0.04t

t = 17.5

13. (a) (i) ln 16, ln 4, ln 2.

ie. 4 ln 2, 2 ln 2, ln 2,

Common ratio $= \frac{1}{2}$

(ii)

$$T_8 = \ln 16\left(\frac{1}{2}\right)^7$$

$$= \frac{\ln 16}{128} \text{ or } \frac{\ln 2}{32}$$

(iii)

$$S_5 = \frac{\ln 16\left(1 - \left(\frac{1}{2}\right)^5\right)}{1 - \frac{1}{2}}$$

$$= \frac{\ln 16\left(\frac{31}{32}\right)}{\frac{1}{2}}$$

$$= \frac{31}{16}\ln 16 \text{ or } \frac{31}{4}\ln 2$$

(iv)

$$S_\infty = \frac{\ln 16}{1 - \frac{1}{2}}$$

$$= 2\ln 16$$

(b) (i)

$$\frac{dy}{dx} = e^{-x}$$

(ii)

$$\frac{d^2y}{dx^2} = -e^{-x}$$

$$\therefore \frac{d^{70}y}{dx^{70}} = -e^{-x}$$

14. $f(x) = \ln(4x + 2)$

(i)

$$h(x) = \frac{4}{4x + 2} = \frac{2}{2x + 1}$$

(ii)

$$\text{Area} = \int_1^5 h(x)\,dx$$
$$= [f(x)]_1^5$$
$$= [\ln(4x + 2)]_1^5$$
$$= \ln 22 - \ln 6$$
$$= \ln\frac{11}{3}$$

15. (a)

$$C(800) = 20\sqrt{800} + 2(800) + 900$$
$$= \$3065.69$$

(b) $$\text{Average cost/lure} = \frac{\$3065.69}{800}$$
$$= \$3.83$$

(c) $$C'(n) = \frac{10}{\sqrt{n}} + 2$$

(d)

$$\text{Cost of } 201^{\text{st}} \text{ lure} = C'(200)$$
$$= \frac{10}{\sqrt{200}} + 2$$
$$= \$2.71$$

(e) $$P(n) = 8n - C(n)$$
$$= 8n - 20\sqrt{n} - 2n - 900$$
$$= 6n - 20\sqrt{n} - 900$$

(f) $P(800) = \$3334.31$

16. (a)

$$40 = 200e^{-2t}$$
$$\frac{1}{5} = e^{-2t}$$
$$\ln\frac{1}{5} = -2t$$
$$t = 0.80 \text{ sec } (2 \text{ d.p.})$$

(b)

$$V = \int a\, dt$$
$$= -100e^{-2t} + c$$
$$\text{at } t = 0 \quad V = 0$$
$$\therefore c = 100$$
$$\therefore V = -100e^{-2t} + 100$$

(c) Distance travelled in first 2 seconds

$$= \int_0^2 V\, dt$$
$$= \left[50e^{-2t} + 100t\right]_0^2$$
$$= 50e^{-4} + 200 - 50$$
$$= \frac{50}{e^4} + 150$$
$$\approx 150.92 \text{ m}$$

PART B

1.

a = 7
b = 2
c = 3
d = 16
e = 1
f = 5
g = - 4
h = - 4
in any order
i = e or = 2.718
j = 1

2. (a) (i) $C(20) = \frac{1}{50}\left(\frac{200}{20} + 20\right)$

$= 0.6$ L/km

(ii) Fuel used in travelling 50 km

= 50 x 0.6 L

= 30 L

(iii) $C'(x) = \frac{-4}{x^2} + \frac{1}{50}$

For a min $C'(x) = 0$

i.e. $\frac{-4}{x^2} = -\frac{1}{50}$

$x = \sqrt{200}$ or $10\sqrt{2}$

It is a min since $C''(x) > 0$ for all $x > 0$

(b) (i)

$R'(x) = P(x)Q'(x) + Q(x)P'(x)$

$R'(2) = (3)(4) + (5)(-1)$

$= 7$

(ii)

$$T'(x) = \frac{Q(x)P'(x) - P(x)Q'(x)}{[Q(x)]^2}$$

$$= \frac{5(-1) - 3(4)}{25}$$

$$= -\frac{17}{25}$$

3. (a) t = 0, a = 5 cm/s^2

t = 0, v = 0 cm/s

(b)

$$v = \int a\,dt$$
$$= -t^2 + 5t + c$$
at $t = 0, v = 0 \therefore c = 0$

i.e. $v = -t^2 + 5t$

$$\text{Average velocity} = \frac{v(0) + v(3)}{2}$$
$$= \frac{0 + 6}{2}$$
$$= 3\,cm/s$$

(c) Car in motion until V(t) = 0

i.e. $-t^2 + 5t = 0$
$-t(t - 5) = 0$
$\therefore t = 0$ or 5

$\therefore$ Car in motion for 5 seconds.

(d)

$$S = \int v\,dt$$
$$= \int\left(-t^2 + 5t\right)dt$$
$$= \frac{-t^3}{3} + \frac{5t^2}{2} + d$$
At $t = 0, S = 0 \therefore d = 0$

$$S = \frac{-t^3}{3} + \frac{5t^2}{2}$$

(e)

$$\text{distance travelled} = \int_0^5\left(\frac{-t^3}{3} + \frac{5t^2}{2}\right)dt$$
$$= \left(\frac{-125}{3} + \frac{125}{2}\right) - (0)$$
$$= \frac{125}{6} \text{ or } 20\frac{5}{6}\text{ cm}$$

4. (a) (i) x – intercepts: - 5, 1

y – intercepts: 5

(ii) $y = (x-1)^2(x+5)$

$= (x^2 - 2x + 1)(x+5)$

$= x^3 + 3x^2 - 9x + 5$

$y' = 3x^2 + 6x - 9$

$= 3(x-1)(x+3)$

If $y' = 0$, $x = 1$ or -3

Hence B = (- 3, 32)

(iii) $y'' = 6x + 6$

If $y'' = 0$ then $x = -1$

$\therefore$ C = (- 1, 16)

(b)

	AB	BC	CD	DE
y'	+	-	-	+
y''	-	-	+	+

5. (a) $y' = 3(2x+3)^2(2)$

$= 6(2x+3)^2$

Since $y' \geq 0$ for all x values the function is never decreasing.

(b)

$$\text{Area} = \int_0^3 \sqrt{2x+1}\,dx$$

$$= \left[\frac{2(2x+1)^{\frac{3}{2}}}{3(2)}\right]_0^3$$

$$= 6.173 - 0.\dot{3}$$

$$= 5.84$$

(c)

$$g(x) = \ln x + \frac{1}{x}$$

$$g'(x) = \frac{1}{x} - \frac{1}{x^2}$$

$$g'(1) = 0, \quad g(1) = 1$$

$\therefore$ Eqn of tangent, $y = 1$

6. (a) (i) B
(ii) C
(iii) A
(iv) E
(v) D

(b)

$$\frac{dp}{dx} = \frac{dp}{dq}\cdot\frac{dq}{dr}\cdot\frac{dr}{dx}$$
$$= (4)(2)(10x)$$
$$= 80x$$

7. (a)

$$\frac{dv}{dt} = -\frac{1}{20}(t + 2)^2 + 50$$
$$= -\frac{t^2}{20} - \frac{t}{5} + \frac{249}{5}$$
$$v = -\frac{t^3}{60} - \frac{t^2}{10} + \frac{249t}{5} + c$$

At $t = 0$ $v = 10$ $\therefore$ $c = 10$

i.e. $v = \dfrac{-t^3}{60} - \dfrac{t^2}{10} + \dfrac{249t}{5} + 10$

(b)

$$\text{Increase in volume} = \left[\frac{-t^3}{60} - \frac{t^2}{10} + \frac{249t}{5} + 10\right]_{10}^{20}$$
$$= (832.\dot{6}) - (481.\dot{3})$$
$$= 351.3\ mm^3$$

8. (a) (i) B
(ii) F
(iii) B
(iv) C
(v) C

(b) (i)

If $A = A_0 e^{0.06t}$

then $\dfrac{dA}{dt} = 0.06A_0 e^{0.06t}$

$= 0.06A$

(ii) when $t = 0$

$54\ 000 = A_0 (1)$

$\therefore A_0 = 54\ 000\ m^2$

(iii) On Feb 15th , $t = 5$

$$A = 54\ 000 e^{0.06(5)}$$

$$= 72\ 892.4\ m^2$$

(iv) $100\ 000 = 54\ 000\ e^{0.06\,t}$

$\therefore e^{0.06\,t} = 1.851$

$t = 10.3$

$\therefore$ On Feb 21st approx $100\ 000\ m^2$ affected.

9. (a) \$15/trip = 60 customers lost

$\therefore$ Expected number of passengers 340

(b) $F(d)$ = fare charged x number of passengers

$= d(400 – 20(d – 12))$

$= d(640 – 20d)$

(c) $F(d) = 640d – 20d^2$

$F'(d) = 640 – 40d$

If $F'(d) = 0$

Then $640 = 40d$

$d = 16$ also $F''(16) < 0$

$\therefore$ Max collected when fare is \$16

(d) If $F(d) = 3\ 000$

Then $3\ 00 = 640d – 20d^2$

i.e. $20d^2 – 640d + 3\ 000 = 0$

$d^2 – 32d + 150 = 0$

$d = 26.3$

$\therefore$ Max charge = \$26/trip

10. (a)

$$f'(x) = e^x$$

$$g'(x) = 2e^{2x}$$

$$\text{If parallel } e^x = 2e^{2x}$$

$$\text{i.e. } e^x = \frac{1}{2}$$

$$x = \ln\frac{1}{2}$$

(b) $g(x) = \ln(2x + 5)$

(i) $g(2) = \ln 9$

(ii) $g(2 + h) = \ln(2(2 + h) + 5)$

$= \ln(9 + 2h)$

(iii)

$$\frac{dg}{dx} = \frac{2}{2x + 5}$$

When $x = 2$

$$\frac{dg}{dx} = \frac{2}{9}$$

11. (a) (i) 770 000 (0.97) = 746 900 barrels

(ii) $770\,000\,(0.97)^6 = 641\,388$ barrels

(b)

$$S_{12} = \frac{770\,000\left(1 - 0.97^{12}\right)}{\left(1 - 0.97\right)}$$

= 7 858 046 barrels

(c) $400\,000 = 770\,000\,(0.97)^n$

$$n = \frac{\log 0.156}{\log 0.97}$$

= 21.5

The well is expected to last 22months.

12. (a) (i) PQ : m = - 2

$y - 3 = -2(x - 1)$

$y = -2x + 5$

(ii)

$$\text{Area} = \int_0^1 \left(x^3 - 2x^2 + 4x\right)dx + \int_1^{2.5}\left(-2x + 5\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} + 2x^2\right]_0^1 + \left[-x^2 + 5x\right]_1^{2.5}$$

$= [(1.583) - (0)] + [(6.25) - (4)]$

$= 3.83\ m^2$

(b)

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{\frac{1}{e}}{1-\frac{1}{e}}$$

$$= \frac{\frac{1}{e}}{\frac{e-1}{e}}$$

$$= \frac{1}{e-1} \text{ or } = 0.58$$

13. (a) (i) $N(t) = \int \frac{dN}{dt}\,dt$

$$= \int 60e^{\frac{1}{4}}\,dt$$

$$= 60\left[\frac{e^{\frac{1}{4}}}{\frac{1}{4}}\right] + c$$

$$= 240e^{\frac{1}{4}} + c$$

$t = 0 \Rightarrow 20\ 000 = 240(1) + c$

$c = 19\ 760$

$$\therefore N(t) = 240e^{\frac{t}{4}} + 19760$$

(iii) $N(8) = 240e^{2} + 19760$

$=21\ 533$ shares

(iii)

$$30\ 000 = 240e^{\frac{t}{4}} + 19760$$

$$e^{\frac{t}{4}} = 42.6$$

$$\frac{t}{4} = \ln 42.6$$

$$t \approx 15 \text{ days}$$

(b)

$$2\log_2 x + \log_2 2x = \log_2 4$$

$$\log_2\left(x^2 . 2x\right) = \log_2 4$$

$$2x^3 = 4$$

$$x = \sqrt[3]{2}$$

14. (a)

$$p(x) = \frac{x}{\ln x}$$

(i) function undefined for $x \le 0$

(ii)

$$\frac{dp}{dx} = \frac{\ln x - x\frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln x - 1}{(\ln x)^2}$$

(b) (i) If intensity = 500 I_0

then $R = \log_{10}\left(\frac{500 I_0}{I_0}\right)$

$R = 2.7$

(ii) $6.5 = \log_{10}\left(\frac{I}{I_0}\right)$

$$10^{6.5} = \frac{I}{I_0}$$

$\therefore I = 10^{6.5} I_0$

(iii) $7.5 = \log_{10}\left(\frac{I}{I_0}\right)$

$\therefore I = 10^{7.5} I_0$

$\therefore$ San Francisco 10 times more intense.

15. (a) (i)

$$\frac{dM}{dt} = \frac{4}{t}$$

$$M = \int \frac{dM}{dt}\, dt$$

$$= 4\ln t + c$$

when t = e, M = 2

$\therefore 2 = 4(1) + c$

$\therefore c = -2$

$\therefore M = 4\ln t - 2$

(ii) $M(e^3) = 4\ln e^3 - 2$

$= 12\ln e - 2$

$= 10$

(b) (i) $C(20) = 50\sqrt{20} + 350$

= \$573.61

(ii) $C(80) = 50\sqrt{80} + 350$

= \$797.21

(iii) Profit $= 24x - (50\sqrt{x} + 350)$

Let $0 = 24x - 50\sqrt{x} - 350$

$$\therefore \sqrt{x} = \frac{50 \pm \sqrt{2500 + 4(24)(350)}}{48}$$

$$\sqrt{x} = 5$$

$\therefore$ x = 25

$\therefore$ Need to sell > 25 items before profit is made.

(iv)

$$P'(x) = 24 - \frac{25}{\sqrt{x}}$$

If $P'(x) > 22$

then $24 - \frac{25}{\sqrt{x}} > 22$

$$2 > \frac{25}{\sqrt{x}}$$

$$\sqrt{x} > \frac{25}{2}$$

x > 156.25

$\therefore$ 157 items should be sold.

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