INTRODUCTORY CALCULUS - REVISION ASSIGNMENT SERIES

Revision Assignments 4, 5 and 6 will be tested on **Wednesday 5th November 2008**.
Questions will be randomly selected from each of the three assignments. No notes will be allowed.
Solutions will be posted onto www.mathxtc.com

REVISION ASSIGNMENT FOUR

ISSUED: MONDAY 18TH AUGUST **DUE: FRIDAY 12TH SEPTEMBER 2008**

NAME:

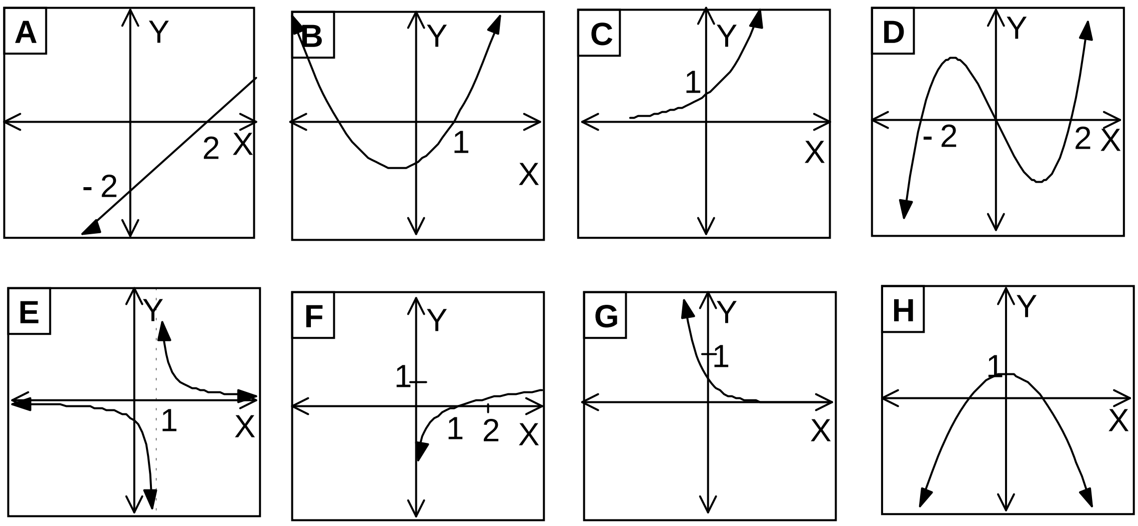
PART A

1. (2 MARKS EACH; 16 TOTAL)

Consider the following graphs.
Match each graph with its corresponding function.
Copy and complete the table.

Graph	A	B	C	D	E	F	G	H
Function								

GRAPHS



FUNCTIONS:

(i) $y = \log_{10} x$

(ii) $(x - 1)y = 1$

(iii) $y = -x^3 - 2x^2$

(iv) $y = e^x$

(v) $y = x(x^2 - 4)$

(vi) $y - x + 2 = 0$

(vii) $y = \ln x$

(viii) $1 - x^2 = y$

(ix) $y = 0.5^{x+1}$

(x) $y = (x + 1)^2 - 4$

2. (a) (4, 3 marks)

The equation of a curve is given by

$$Y = x^3 - 3x^2 + 3x + 4$$

Find (i) the slope of the tangent to the curve at $x = 2$

(ii) the equation of the tangent to the curve at the point where $x = 2$ on the curve.

(b) (6 marks)

The concentration of a drug in the bloodstream; immediately after an injection, is given by

$c(t) = 0.5t - 0.1t^2$ where t is time in minutes and $0 < t < 6$

At what time after the injection is the concentration of the drug a maximum?

3. (3, 3, 3, 3, 3 marks)

The gold output from Karramindie Mine for the first 5 years is given below:

1986	25 000 fine ounces
1987	18 500 ″
1988	13 690 ″
1989	10 131 ″
1990	7 497 ″

Output is given to the nearest fine ounce.

(i) Show that the sequence of output from the mine is geometric and state the common ratio.

(ii) Graph the values of the gold output for the years 1986 to 1990, joining the points by a smooth curve.

(iii) Estimate from your graph, the instantaneous rate of change of output with respect to time, at the end of the fourth year of mining.

(iv) Find the expected sum of the gold output from the mine up to and including the year 2000. Give your answer correct to the nearest fine ounce.

(v) Assuming the sequence goes on without bound, find the total amount of gold which could be mined. Give your answer correct to the nearest fine ounce.

4. (3, 2, 4, 1, 4, 2 marks)

The graphs shown are graphs of:

I : $y = f(x)$
II : $y = f'(x)$
III : $y = f''(x)$

By studying the graphs, answer the following questions.

(a) Describe the behaviour of $y = f(x)$ for x where

(i) $x < 0$
(ii) $0 < x < 6$
(iii) $x > 6$

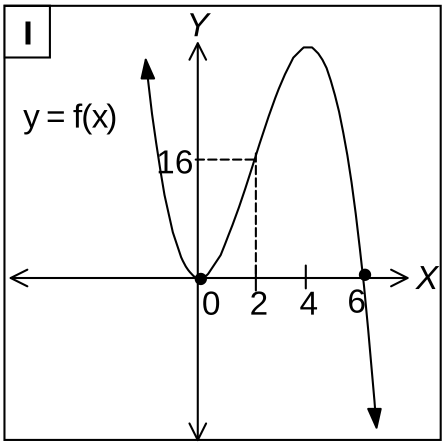
(b) Find the values of the x – intercepts of $y = f(x)$.

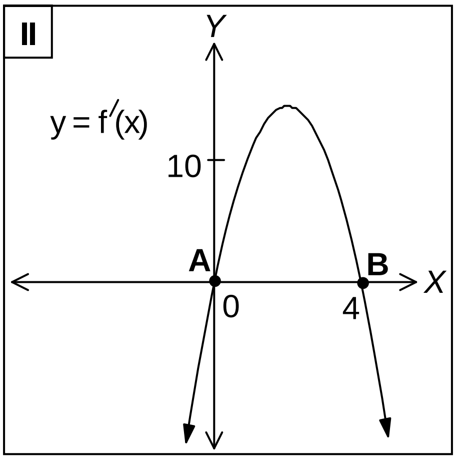
(c) Determine the expression for the function f
i.e. $y =$

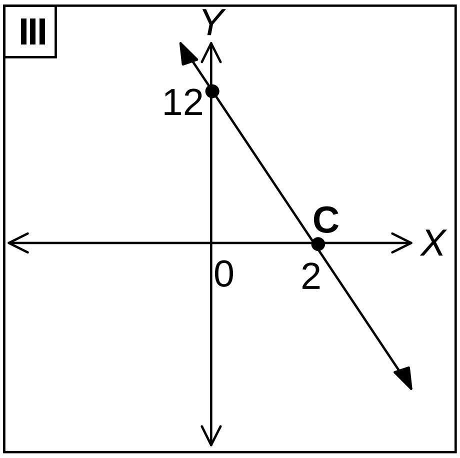
(d) What are the x-coordinates of points A and B in Graph II?

(e) Locate these x values on Graph I and state their significance.

(f) What is the x-coordinate of point C on Graph III and what is its significance on Graph I?







5. (7 marks)

The power, P, required by an experimental aircraft to achieve a velocity, v, is given by

$$P(v) = \frac{10}{v} + v^2$$

where p is in watts and v is in metres/second

Find the velocity at which the power required is a minimum.
Give your answer correct to 2 decimal places.

6. (3, 3, 3, 3, 3, 4)

Consider the function

$$y = x^2 e^{-x}$$

Investigate the local extrema of this function.

That is:

(i) find $\frac{dy}{dx}$

(ii) find $\frac{d_2 y}{dx^2}$

(iii) find x such that $\frac{dy}{dx} = 0$

(iv) find x such that y is a local minimum.
What is this minimum value of y?

(v) find x such that y is a local maximum.
What is this maximum value of y?

(vi) Using the information from parts (iv) and (v), sketch the graph of the function
For $-0.5 \leq x \leq 4$.

7. (3, 2, 3, 2, 1 marks)

The position (y metres) of an object moving in a straight line after t seconds is given by

$$y = \log_e\left(\sqrt{t}\right), \ t > 1$$

(i) Sketch, not on graph paper, the graph of y against t for $1 < t < 10$.

(ii) Give the position of the object at $t = 2.6$ secs, correct to 3 decimal places.

(iii) Find $\dfrac{dy}{dt}$

(iv) Find the velocity of the object at $t = 12$ seconds, correct 3 decimal places.

(v) Describe how the velocity varies as t becomes very large.

8. (11 marks)

Strontium 90 is a radioactive substance which occurs in the fall-out of a nuclear bomb. Strontium 90 disintegrates at a rate proportional to its mass.

That is, let m (gm) be the amount of Strontium 90 present at time t (years), then it deteriorates according to:

$$\frac{dm}{dt} = km \qquad \text{where k is a constant.}$$

(a) (3 marks)

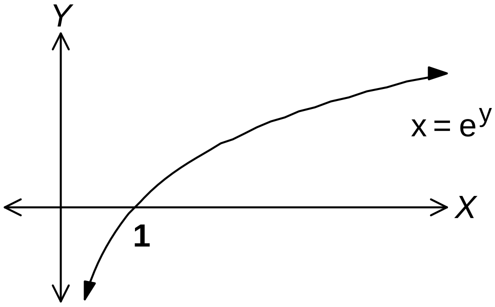
Show that $m = Ae^{kt}$ satisfies the differential equation above, where A is a constant.

(b) (2, 3, 3 marks)

Strontium 90 has a half-life of 28 years. That is, after 28 years exactly half of the original amount remains. If the amount of Strontium 90 originally present was 110 gm, find

(i) the value of A
(ii) the value of k
(iii) how much Strontium 90 would be present 200 years later? Give your answer correct to 2 decimal places.

9. (2, 2, 2, 2 marks)



(i) Find $\frac{dx}{dy}$ when $x = e^{y}$

(ii) Hence find when $x = e^{y}$

(iii) Write $\frac{dy}{dx}$ as a function of x.

(iv) Determine y as a function of x.

10. (3, 3, 2 marks)

Consider the growth function $y = 2^{t}$

(i) Find k, correct to 3 decimal places, such that $2 = e^{k}$ and hence write 2 as a power of e.

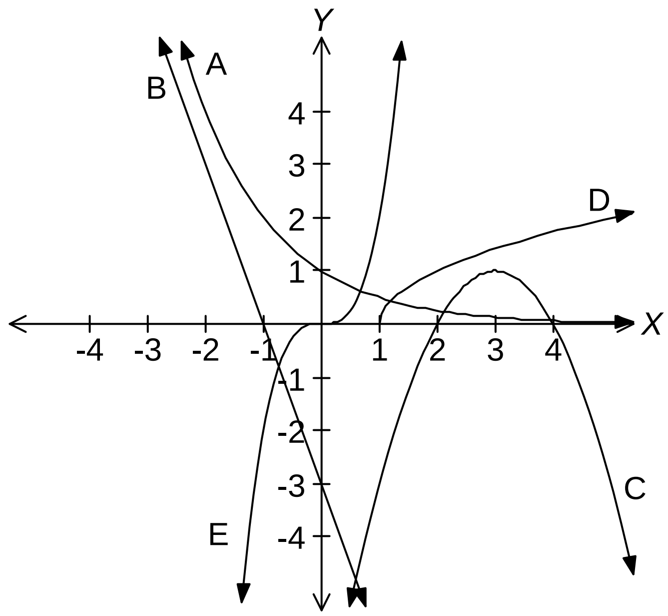
(ii) By replacing 2 with your answer in part (i) in the growth function $y = 2^{t}$, determine the marginal rate of change of y with respect to t.

(iii) Calculate the marginal rate of change of y at the instant t = 5, correct to 2 decimal places.

PART B

1. (2, 2, 2, 2, 2 marks)

On the set of axes below, the graphs of 5 functions have been placed. Below the graphs are possible functions which may match the graphs.



Match each graph with one of the functions below.

(i)	$y = \sqrt{x - 1}$	(ii)	$3x + y + 3 = 0$	(iii)	$y = -x^2 - 3$
(iv)	$y = -2^x$	(v)	$y = \log_2 x$	(vi)	$y = 2x^3$
(vii)	$y = -(x-3)^2 + 1$	(viii)	$y = 3x - 3$	(ix)	$y = 2^{-x}$

2. (4, 4 marks)

The profit P, in thousands of dollars, made by a company t months after its beginning is given by

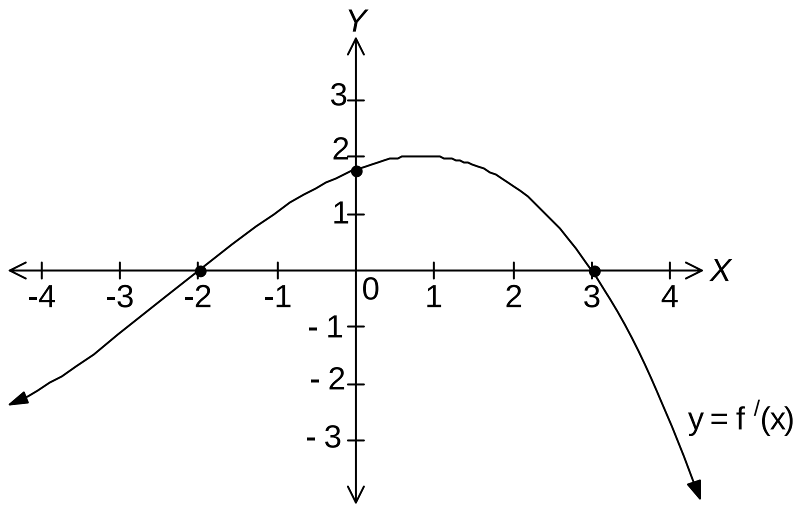
$$P(t) = 2t^3 - 15t^2 + 24t + 2 \qquad \text{where } 0 \leq t \leq 12$$

(a) Find $\frac{dP}{dt}$

(b) For which values of t is the profit decreasing?

3. (3, 3, 3 marks)

The graph of the derivative $f^{/}$ of the function f is shown below.



Use the graph to determine values of x at which f(x) has

(a) a local maximum
(b) a local minimum
(c) a point of inflection

4. (3, 2, 3, 3 marks)

Let the functions f and g be defined as

$$f(x) = \frac{x^2}{x^2 - 1} \qquad \text{and} \qquad g(x) = \frac{1}{x^2 - 1}$$

(a) Show that the functions have the same derivative.

(b) Give a geometrical interpretation of how the functions can have the same derivative.

5. (3, 2, 3, 3 marks)

During a drought on a sheep station, the number of sheep decreased at a rate proportional to the number of sheep present.

If P = population of sheep, then $\frac{dP}{dt} = -kP$

Where k is a constant and t = time in months.

(a) show that $P = Ae^{-kt}$ is a solution to the differential equation above.

At the start of the drought there were 8090 sheep in the population and 8 months later there were only 5370 animals.

(b) Use the above information to determine

(i) A

(ii) K

(c) Determine (to the nearest 10) the number of sheep alive 12 months into the drought.

6. (4, 4 marks)

(a) Find the difference between successive terms in this sequence.

$$\log_{10}\left(3^k\right),\ \log_{10}\left(9^k\right),\ \log_{10}\left(27^k\right),\ \log_{10}\left(81^k\right),\ldots\ldots,\ k > 0$$

(b) Show that the list of terms

$$3^k,\ 3^{k+1},\ 3^{k+2},\ 3^{k+3},\ldots\ldots;\ k > 0$$

form a geometric sequence. Hence find the sum of the first 6 terms.

7. (a) (3, 3, 4 marks)

(i) Let $u = \ln x$, find $\frac{du}{dx}$

(ii) Let $y = u^3$; find $\frac{dy}{du}$

(iii) Hence find $\frac{dy}{dx}$ in terms of x.

(b) (3, 2, 3 marks)

Given $y = xe^x$

(i) Determine $\frac{dy}{dx}$

(ii) Determine $\frac{d_2 y}{dx^2}$

(iii) Hence determine an expression for $\frac{d_n y}{dx^n}$

8. (4, 4, 3, 2 marks)

e^x can be written as an infinite series.

ie $e^x = 1 + x + \frac{x^2}{2.1} + \frac{x^3}{3.2.1} + \frac{x^4}{4.3.2.1} = ...$

(a) By replacing x by a number and summing the first 5 terms calculate

(i) $e^{3.5}$

(ii) e^{-1}

(b) Compare with the values of $e^{3.5}$, e^{-1} available directly from your calculator. Explain the differences between results from part (a) and your calculator.

(c) By differentiating each term with respect to x, find the derivative of e^x.

(d) What do you notice?

9. (4, 3, 4, 3, 3 marks)

A research scientist examines a bacterial culture over 7 days and records the following results in the table below.

Number of days growth	(x)	1	2	3	4	5	6	7
Number of cells	(y)	7	19	52	141	382	1039	2824

She suspects that the numbers of bacteria are growing in a geometric progression.

(a) Confirm this and find the number of cells on the 12^{th} day.

For graphing purposes, the scientist chose to transform the y values into z values by the transform $z = \log_e y$.

(b) Calculate the z values, correct to 2 decimal places, for x = 1, 2, …, 7.

(c) Express z as a function of x. what do you notice? What type of relationship exists between z and x?

(d) Using your result from (a) or (c) above, express y as a function of x.

(e) Assuming x and y are continuous variables, determine an estimate for the rate of growth of the bacteria on day 10.

10. (4, 2, 4, 2, 3, 4 marks)

A large drop of ink has hit a blotting paper (absorbent tissue). As the ink blot spreads, the area of the blot is a function of time (ie A = f(t)). The area of the blot was initially
4 cm^2 and it is increasing at the constant rate of 2 cm^2/sec for the first 5 seconds.

(a) Graph the function A = f(t). Express A as a function of t. Describe its behaviour and type of function.

(b) Determine $\frac{dA}{dt}$.

(c) If the blot is always circular, express the area of the blot A as a function or (ie A = g(r)). Graph the function A = g(r). Describe its behaviour and type of function.

(d) Determine $\frac{dA}{dr}$.

(e) By reading from the graphs of A = f(t) and A = g(r), draw the graph of radius against time (ie r = h(t)).

(f) Determine, either from the graph in part (a) or by the use of the chain rule, how fast the radius is changing when r = 2 cm.