Whole numbers

1

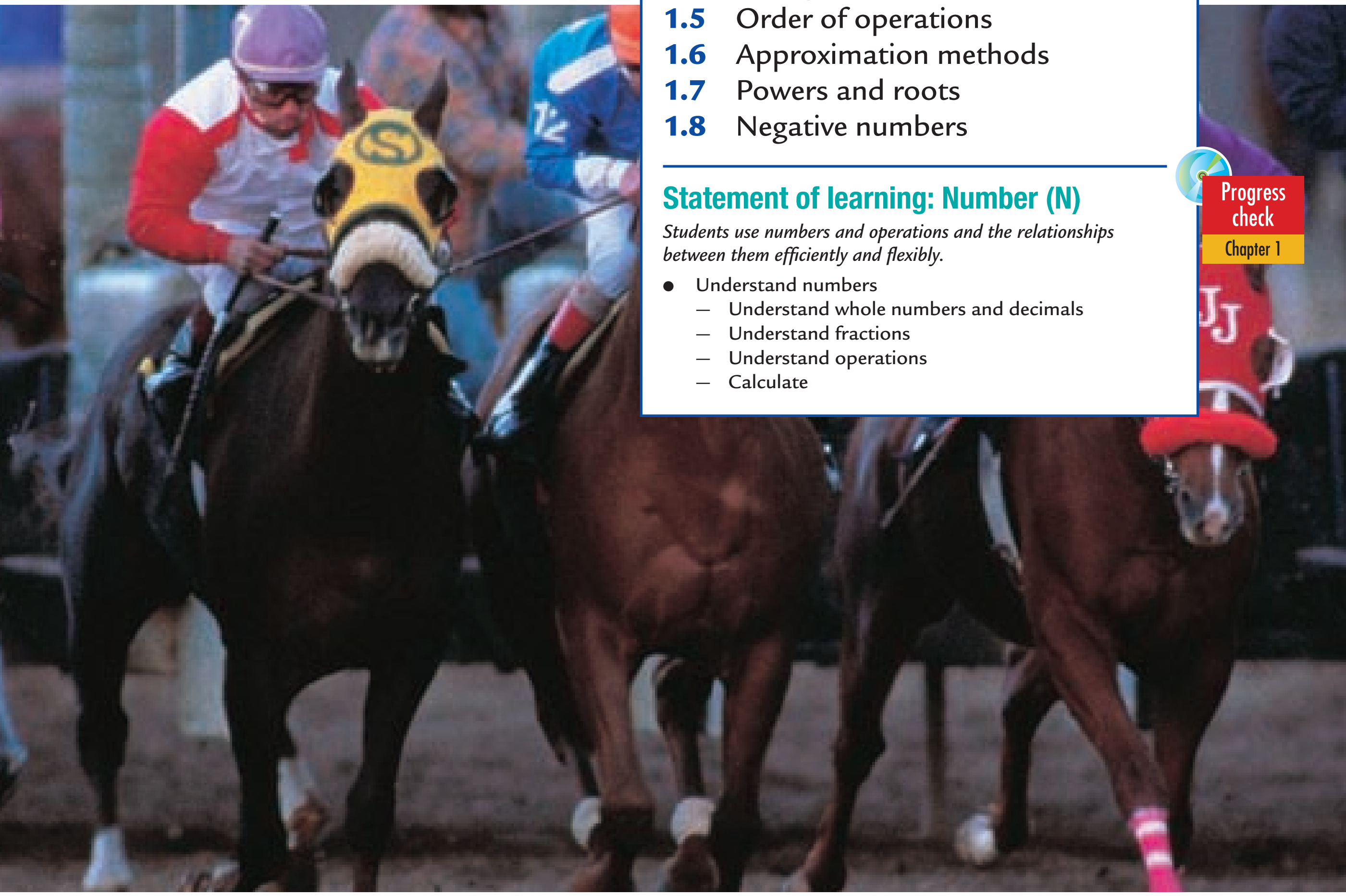
Contents

Statement of learning: Number (N)

Students use numbers and operations and the relationships between them efficiently and flexibly.

- Understand numbers
  - Understand whole numbers and decimals
  - Understand fractions
  - Understand operations
  - Calculate

Progress check
Chapter 1



Numbers were invented by people. Different groups of people invented different ways of saying, writing and using numbers. The numbers we use today are the ones that have been found to be the most useful.

The methods we use for calculation have also been invented by different people and only the best methods have been kept. Our modern methods of calculation using pen and paper date from only the 14th and 15th centuries as Hindu-Arabic numerals (1, 2, 3, …) displaced Roman numerals (I, II, III, …).

Multiplication tables were not commonly used before about AD 1300. Instead, calculations were carried out by other methods, such as counting beans or using an abacus.

1.1 Naming and writing numbers

We use a decimal number system with ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are now used worldwide to represent numbers.

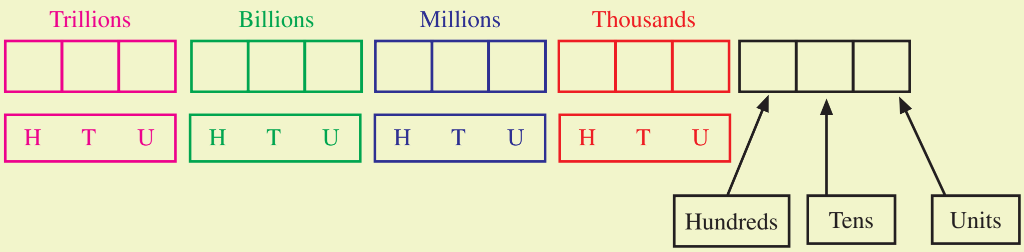
We use a **place value** system for bigger numbers.

!

Place value system

The place in which you write a digit affects its value. The larger place values are on the left. Each place is 10 times as big as the one before.

When there are more than four digits, the modern system is to separate each group of three places by a small gap. Some people use a comma between the groups of three instead of a small space.



When you write numbers in words, you should use a hyphen in the numbers from twenty-one to ninety-nine.

In Australia, we now use the system where a billion is a thousand million, a trillion is a thousand billion, and so on. Some people still use a system where each new ‘illion’ is a million times bigger than the previous one.

You can use quadrillion, pentillion, sextillion, heptillion and so on for larger numbers but this is uncommon.

You can think of each group of three numbers as a ‘house’. Inside each house the numbers go from one to nine hundred and ninety-nine. The lowest ‘house’ does not have a separate name, but all the other houses have names so you know the size of the numbers. The first big house is the ‘thousands’ house, then the ‘millions’ house, ‘billions’ house and so on.

Example 1

Write each of the following numbers in words.

a 5304 **b** 63 458 **c** 20 005 345 160

Solution

a Identify the largest group. — **The number has a 'thousands' house but only has 5 in this house.**

Write the answer. — **Five thousand, three hundred and four.**

b Identify the largest group. — **The number has a 'thousands' house, with sixty-three in this house.**

Remember the hyphens. — **Sixty-three thousand, four hundred and fifty-eight.**

c Identify the largest group. — **The number has a 'billions' house, a 'millions' house, a 'thousands' house and the lowest house.**

Write the answer, working down the houses. — **Twenty billion, five million, three hundred and forty-five thousand, one hundred and sixty.**

For historical reasons, some numbers have special names. People sometimes say numbers in different ways and you should be able to interpret what they mean. When a number is used as a code it is common to specify the number as a series of digits. Thus the personal identification number (PIN) 4502 is stated as 'four-five-oh-two' or 'four-five-zero-two'.

!

Special number names

A **dozen** means 12, a **baker's dozen** means 13, a **score** means twenty, a **ton** (in cricket) means 100, a **gross** means 144, a **k** means 1000 (or 1024 in computing), a **meg (M)** means 1000 000 (or 1024 k in computing) and a **gig** for a billion or 1024 M.

Example 2

Write the following as numbers, using the usual symbols.

a Two dozen **b** Five thousand and seven

c Nineteen eighty **d** Three billion, five hundred and thirty-two million

Solution

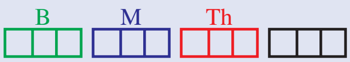
a Write as a multiplication. — **Two dozen = 2 × 12**

Write the answer. — **Two dozen = 24**

b Use zeros for the hundreds and tens. — **Five thousand and seven = 5007**

c This is a common way to say dates. — **Nineteen eighty = 1980**

d Think of the place values and houses up to billions.



Write the answer with zeros for the last two groups. — **Three billion, five hundred and thirty-two million = 3 532 000 000**

To compare the values of digits in different numbers, you can write a number in **extended** (expanded) **form** by separating each digit of the number to write its value.

Example 3

Compare the values of the 5s in the numbers 5204 and 3452.

Solution

Write the numbers in extended form.	**5204 = 5000 + 200 + 4**
	3452 = 3000 + 400 + 50 + 2
Compare the 5s.	**5000 = 100 × 50**
Write the answer.	**The 5 in 5204 is 100 times the value of the 5 in 3452.**

Investigation Sieve of Eratosthenes

Write down the numbers from 2 to 100.

A **multiple** of a number can be divided by the number.

Circle the number 2 and cross out all its multiples (4, 6, 8 and so on).

Circle the next available number (3) and cross out its multiples (6, 9, 12 and so on).

Keep going with this pattern until all the numbers are circled or crossed.

Write out the circled numbers. These are called **prime** numbers. The process of crossing out multiples is called the *Sieve of Eratosthenes.*

What are some of the properties of the prime numbers you have found?

Discuss this process as a class group. Your teacher may want you to make some notes about what you have found about prime numbers and multiples.

Exercise 1.1

Additional exercise 1.1

TLF L871 Wishball challenge

1 Write the following in words.
a 54 **b** 212 **c** 607 **d** 3142 **e** 8043

2 Write the following as numbers, using the usual symbols.
a seventy-seven
b one hundred and three
c four hundred and sixty-eight
d nine thousand, two hundred and nine
e one thousand and fifty-three

3 Write the following in extended form.
a 19 **b** 83 **c** 247 **d** 3864 **e** 9105

4 Compare the values of the nominated digits in each of the following.
 a 483, 184 (4s) b 3021, 4132 (3s) c 2058, 1246 (2s)
 d 2572, 3045 (5s) e 789, 9843 (8s)

5 Write the following in words.
 a 80 701 b 56 052 c 2 625 064 d 34 265 423 e 670 324 000 000

6 Write the following as numbers, using the usual symbols.
 a sixteen thousand and twelve
 b eighty-four thousand, one hundred and six
 c one hundred and twenty-three thousand, five hundred and seven
 d twenty-seven billion, two hundred and thirty thousand
 e thirty-eight million

7 Write the following in extended form.
 a 39 000 b 212 435 c 85 006 d 423 000 040 e 812 547

8 Compare the values of the nominated digits in each of the following.
 a 24 037, 1804 (4s) b 700 000, 21 706 (7s) c 42 375, 120 (2s)
 d 62 000 000, 47 245 (2s) e 685 476 210 (6s)

Application and problem solving

9 Write these as numbers, using the usual symbols.
 a three dozen b three score and ten c a double ton

10 Use the digits 3, 4, 5 and 6 once in each number to make:
 a different numbers with 3 at a place value of 100
 b different numbers with 3 at a place value of 1000

11 Write the following as numbers, using the usual symbols.
 a twenty-seven hundred b eighteen twenty-two c five gross

12 How many different numbers can you make using some or all of the digits 5, 6, 7, 8 a maximum of once in each number?

1.2 Size and order of numbers

The numbers 1, 2, 3, ... are known as the **counting** or **natural** numbers. Zero is not used for counting and so is not a counting number.

The numbers 0, 1, 2, 3, ... are known as the **whole** numbers. There is no *largest* whole number because there is always a next number.

There are many uses for whole numbers.

1 COUNTING. For example, there were 35 000 people at the last ‘State of Origin’ football game at Lang Park.

2 SIZING. For example, Helen wears size 8 jeans.

3 MEASURING. For example, Tully has an average annual rainfall of 4321 mm.

4 CODING. For example, you need a personal identification number (PIN) to operate an automatic teller.
5 COMPARING. For example, Wendy has twice as many stamps in her collection as Colin.
6 ORDERING. For example, in the last quiz Sally came 1st, Mario came 2nd and Jacqui came 3rd.
7 POSITIONING. For example, Western Australia's eastern border is marked by the line of longitude 129°E.
8 SCORING. For example, Brazil defeated Germany 3–2 in the World Cup.

As you can see, numbers can be used for many different purposes.

Every number has a position. You can show which number is bigger by using symbols. The symbols shown below are the standard symbols.

!

Comparison symbols

Symbol	*Meaning*	*Example*
$=$	is equal to	$4 = 3 + 1$
$\neq$	is not equal to	$4 \neq 3 + 2$
$<$	is less than	$4 < 6$
$>$	is greater than	$4 > 1$

Example 4

Complete each of the following using the symbols $<$, $=$ or $>$.

a $163 \; \square \; 167$ **b** $24 + 23 \; \square \; 44$ **c** $170 \; \square \; 173 - 3$

d five thousand $\square$ twenty-seven hundred

Solution

a The first number is smaller, so use $<$. **$163 < 167$**

b Work out the total on the left. **$24 + 23 = 47$**
The first number is bigger, so use $>$. **$24 + 23 > 44$**

c Work out the result on the right. **$173 - 3 = 170$**
The numbers are the same, so use $=$. **$170 = 173 - 3$**

d Write the numbers in digits. **$5000 \; \square \; 2700$**
The first number is larger, so use $>$. **five thousand $>$ twenty-seven hundred**

You can show groups of numbers using a **number line**.

An arrow is used on the right of the line to show that it continues in the direction of larger numbers. If only part of the line is shown, an arrowhead may also be used on the left to show it continues downward.

Particular numbers are shown by large dots marked on the line.

An arrow can be used to show that a group of numbers continues past the ones marked by large dots.

There are fractions and decimals between the whole numbers. To include only the whole numbers, it is usual to connect them by arcs. This can also be done for large groups using a few arcs along the line.

Example 5

Show the following on separate number lines.

a whole numbers less than 4

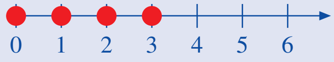
b whole numbers more than 7

c whole numbers strictly between 5 and 11

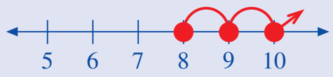
d $22 <$ whole numbers < 37

Solution

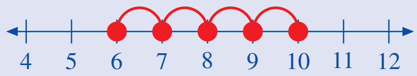
a Show the number line up to, say, 6 with an arrow.
Mark the included numbers with large dots.



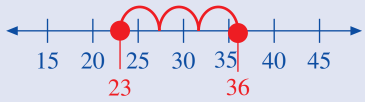
b Show the number line from, say, 5 up with arrows.
Connect the included numbers with arcs and use an arrow to show they continue upwards.



c Show the number line from 4 to 12 with arrows.
Connect the included numbers with arcs.



d Show the number line from 15 to 45 marked in 5s with arrows at the ends.
Show the start and finish numbers and connect them with arcs to show that the whole numbers between them are included.



!

Additional comparison symbols

The letter n is often used as an abbreviation for whole numbers.

Symbol	*Meaning*	*Example*
$\leqslant$	is less than or equal to	$5 \leqslant 12$, $5 \leqslant 3 + 2$
$\geqslant$	is greater than or equal to	$7 + n \geqslant 7$, $11 \geqslant 6$

The symbol $\leqslant$ can be thought of as 'not more than'. The numbers less than or equal to 15 are the same as the numbers not more than 15. Similarly, $\geqslant$ can be thought of as 'not less than'.

Example 6

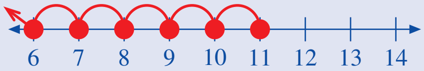
Show the following on separate number lines.

a $n < 12$

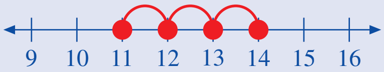
b $11 \leqslant n \leqslant 14$

Solution

a Show the number line from 6 to 14 with arrows. Connect the included numbers (less than 12) with arcs and use an arrow to show that they continue downwards.



b Show the number line from 9 to 16 with arrows. Connect the included numbers (from 11 to 14 inclusive) with arcs.



Exercise 1.2

Additional exercise 1.2

1 Write these statements in symbols using >, < and so on.

a 7 is greater than 5
b twelve is less than 30
c seventeen is equal to seven plus ten
d 50 is the same as 5 times 10
e 7 plus 20 is greater than 13

2 Complete each of the following using the symbols <, = or >.

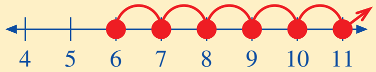
a $3092 \square 3100$
b $7 + 15 \square 20$
c $7 \times 8 \square 8 \times 8$
d $36 - 14 \square 21 - 9$
e $8 \times 10 \square 9 \times 9$

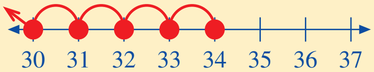
3 State whether each of the following is true or false.

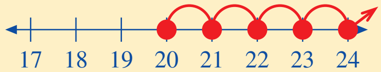
a $5 < 0$
b $17 > 12$
c $32 = 16 \times 2$
d $21 > 7 + 14$
e $40 \neq 8 \times 5$

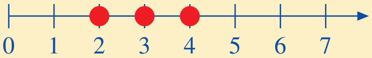
4 Describe in words each of the following graphs of whole numbers.

a


b


c


d


e


5 Show the following on separate number lines.

a whole numbers less than 6
b whole numbers more than 4
c whole numbers less than 8
d whole numbers strictly between 3 and 8
e whole numbers between 2 and 6 inclusive

6 Complete each of the following using the symbols <, = or >.

a fifteen hundred $\square$ five thousand
b three dozen $\square$ thirty-six
c four score and ten $\square$ half a gross
d fourteen thirty $\square$ sixteen hundred
e $220\,000 \square 198\,399$

7 Show the following on separate number lines.

a whole numbers strictly between 32 and 37
b $n > 15$
c $n < 17$
d $15 < n < 20$
e numbers less than 21 and greater than 12

8 Show the following on separate number lines.

a $n \geqslant 58$ **b** $n \leqslant 35$ **c** $14 \leqslant n \leqslant 17$
d $55 \leqslant n \leqslant 75$ **e** $43 < n \leqslant 62$

Application and problem solving

9 Explain how numbers are being used in each of the following.

a a telephone number
b the area of Australia
c the result of the Melbourne Cup
d the population of WA
e a Bankcard number
f a post-office box number
g the volume of a container
h a Mazda 3
i a race time of 3 minutes 2 seconds
j a street location on a map

10 Change each of the following to a mathematical sentence such as $5 < n < 20$.

a Peter is between the ages of 11 and 14 inclusive.
b The lowest price for a new computer is a little over \$800, but with few features.
c Turtles may live to be almost 200 years old.
d The largest number of cylinders in any racing car was 24, but this monster was too heavy to be competitive anyway.

11 Change each of the following to a mathematical sentence.

a A modern computer should have at least 500 Mb of memory.
b A percentage of a whole amount is a maximum of 100%.
c All my CDs have from 8 to 20 tracks.
d A millionaire has at least a million dollars worth of assets.
e The altitude of a star in the sky varies from 0° at the horizon to 90° overhead.

1.3 Addition and subtraction

To add and subtract numbers mentally or using pen and paper, you must know your number facts. You should already know by heart the answers to adding any two numbers from 1 to 9, and the corresponding subtractions. You should be able to accurately use a calculator for addition and subtraction problems and to decide on the best method for a particular problem.

Investigation Mental arithmetic methods 1

You are expected to be able to add or subtract two small numbers in your head, and to extend methods to some larger numbers. Most people use different methods for mental arithmetic than for written arithmetic. You probably already use some strategies of your own.

It is usual to do the largest place value first in mental arithmetic, because even if you make a mistake later, you will have the main part of the problem correct.

Try doing $48 + 27$ in your head.
Did you:

- add the 40 and 20 first to get 60
- add the 8 and 7 to get 15
- add the 60 and 15 to get 75 as your final answer?

Try this method:

- Change the problem to 50 + 25 by counting on 2 from 48 and back 2 from 27 to compensate.
- Add 50 + 25 to get the final answer of 75.

You can use this method in any problem where the *units digits add to more than 9*. With a little practice, you will find it a faster method. Of course, when the units digits add to less than 9, you can just add the tens and units separately in your head.

You can extend this method to larger numbers, particularly if you can 'drop zeros'. For example, you can work out 3200 + 5700 as (32 + 57) hundreds to get 89 hundreds, which is 8900.

You can use similar methods for subtraction of numbers in your head. For subtraction, it is easier to make the number to be taken away a ten number if the units digit is bigger than the units digit of the other number. Thus you can do 63 – 47 as 66 – 50 by making the 47 up to 50, and making the 63 up to 66 to compensate. If you want to, you could do 76 – 42 as 74 – 40 by decreasing both by 2.

Subtractions from 'thousands' can be done as 'adding up to 9', with the units digit adding up to 10. For example, 5000 – 2342 = 2658. In this case the hundreds and tens numbers add to 9, while the units numbers add to 10. You can use this method for any subtraction from a number ending in zeros.

Example 7

Use mental arithmetic to complete the following.

a 8 + 7
b 45 + 87
c 95 – 48
d 58 000 + 67 000
e 790 – 330
f 648 + 825
g 50 000 – 4627

Solution

a This is a basic number fact. **8 + 7 = 15**

b Do as 42 + 90 or as 50 + 82. **45 + 87 = 132**

c Do as 97 – 50 (or as 90 – 43 to get 87 – 40). **95 + 48 = 47**

d Do as (60 + 65) thousands. **58 000 + 67 000 = 125 000**

e Do as (79 – 33) tens. **790 – 330 = 460**

f Do (6 + 8) hundreds and do 48 + 25 as 50 + 23. **648 + 825 = 1473**

g Do as 'adding to 9' except the last as 'adding to 10'. **50 000 – 4627 = 45 373**

You may have your own pen and paper methods for adding and subtracting numbers. You should check your methods with your teacher to make sure they apply to all problems. The *standard written algorithms* are shown in Example 8.

Interactive tutorial
Section 1.3a

Interactive tutorial
Section 1.3b

Example 8

Use a written method to complete the following.

a 5635 + 452 + 89 + 6304 **b** 4038 – 2156
c 48 + 56 934 + 40 000 + 2347 **d** 23 000 – 5697

Solution

a Set out vertically, with the same place values under each other.
Add the units first, and put any carry number at the top.
Add the tens, including any carry number.
Add the hundreds.
Add the thousands.

$$\begin{array}{r} {}^{1}5\,{}^{1}6\,{}^{2}3\,5 \\ 4\,5\,2 \\ 8\,9 \\ +\;\;6\,3\,0\,4 \\ \hline 1\,2\,4\,8\,0 \end{array}$$

b Set out vertically, with the same place values under each other.
Take away the units first.
In the tens, you cannot take 5 from 3, so you need to change the 4000 to 3900 + 100 on the top line to make 13. Now subtract the tens.
Now do the hundreds and thousands.

$$\begin{array}{r} 3\,9\,{}_{1}\;\;\;\; \\ \cancel{4}\,\cancel{0}\,3\,8 \\ -\;\;2\,1\,5\,6 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 3\,9\,{}_{1}\;\;\;\; \\ \cancel{4}\,\cancel{0}\,3\,8 \\ -\;\;2\,1\,5\,6 \\ \hline 1\,8\,8\,2 \end{array}$$

c Set out vertically, with the same place values under each other.
Add the units first, and put any carry number at the top.
Add the tens, including any carry number.
Add the hundreds, thousands and ten-thousands.

$$\begin{array}{r} {}^{1}4\,8 \\ 5\,{}^{1}6\,{}^{1}9\,3\,4 \\ 4\,0\,0\,0\,0 \\ +\;\;2\,3\,4\,7 \\ \hline 9\,9\,3\,2\,9 \end{array}$$

d Set out vertically, with the same place values under each other.
Take away the units first, changing 3000 to 2990 + 10.
Take away the tens and hundreds.
You cannot take 5 from 2, so change 20 000 to 10 000 + 10 000 to make the 2 into 12.
Take the thousands and then do the ten-thousands.

$$\begin{array}{r} 2\,9\,9\,{}_{1}\;\;\; \\ 2\,\cancel{3}\,\cancel{0}\,\cancel{0}\,0 \\ -\;\;\;\;5\,6\,9\,7 \\ \hline 3\,0\,3 \end{array}$$

$$\begin{array}{r} 1\,{}^{1}2\,9\,9\,{}_{1}\;\;\; \\ \cancel{2}\,\cancel{3}\,\cancel{0}\,\cancel{0}\,0 \\ -\;\;\;\;5\,6\,9\,7 \\ \hline 1\,7\,3\,0\,3 \end{array}$$

Example 9

Use a calculator to complete the following.

a 280 + 5463 + 8976 + 32 + 109 **b** 3428 – 896
c 58 900 + 197 854 + 3200 + 96 500 **d** 8 290 000 – 3 785 000

Solution

a Enter as 280 [+] 5463 [+] 8976 [+] 32 [+] 109 [=] → 14860

Write the answer. **280 + 5463 + 8976 + 32 + 109 = 14 860**

b Enter as 3428 [–] 896 [=] → 2532

Write the answer. **3428 – 896 = 2532**

c Enter as 58 900 [+] 197 854 [+] 3200 [+] 96 500 [=] → 356454

Write the answer. **58 900 + 197 854 + 3200 + 96 500 = 356 454**

d Enter as 8 290 000 [–] 3 785 000 [=] → 4505000

Write the answer. **8 290 000 – 3 785 000 = 4 505 000**

Example 10

A long-distance runner has a training schedule that calls for her to run 130 km every week. On Monday, Tuesday and Wednesday she ran 14, 15 and 16 km respectively. On Thursday she got up late so could only go 10 km, but she compensated by running 15 km in the morning on Friday morning and another 15 km after work. On Saturday she ran 24 km.

a How much did she run from Monday to Saturday?

b How far should she run on Sunday?

Solution

a You need to add to get the total. **14 + 15 + 16 + 10 + 15 + 15 + 24 = 109**

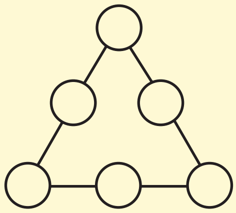
Write the answer in a sentence. **She ran 109 km from Monday to Saturday.**

b You need to subtract. **130 – 109 = 21**

Write the answer in a sentence. **She should run 21 km on Sunday.**

Investigation **Triangle Totals**

Draw the following triangle in your book.
Write the numbers 1, 2, 3, 4, 5 and 6 in the circles so that each side adds up to 10.



Can you make each side add up to 11 instead?
What is the biggest total you can get for each side?
What is the smallest total you can get for each side?
Is there more than one way to get each total? How many ways?

Try the same investigation with a triangle with four spots on each side, using the numbers from 1 to 9. What about a triangle with five spots? What about different numbers?

Exercise **1.3**

Additional exercise 1.3

TLF L102 Take-away bar

1 Use mental arithmetic to complete the following.

a 32 + 25 **b** 98 – 45 **c** 27 + 58
d 56 – 47 **e** 46 + 39 **f** 62 – 25

2 Use written methods to work out the following.

a 362 + 47 + 1873 **b** 782 – 46
c 243 + 1024 + 796 + 45 **d** 4172 – 403
e 4270 + 503 + 621 + 8472 + 25 **f** 3174 – 92

3 Use a calculator to work out these.

a 276 + 532 + 823 **b** 6213 – 420
c 6132 + 487 + 2684 + 17 **d** 2164 – 1266
e 1363 + 217 + 6092 + 306 + 2173 **f** 4002 – 1427

4 Use mental arithmetic to complete the following.

a 48 000 – 27 000 **b** 640 + 790
c 6300 + 2400 **d** 900 – 640
e 5000 – 2840 **f** 780 + 670

5 Use written methods to work out the following.

a 5357 + 3446 + 6974 **b** 10 762 – 983
c 3265 + 4746 + 8439 **d** 62 173 – 496
e 8621 + 19 374 + 834 + 21 894 **f** 25 000 – 16 248
g 283 000 + 457 000 + 64 380 + 8645 **h** 57 890 – 9847
i 647 + 24 970 + 5872 + 50 462 + 5009 **j** 24 370 – 19 684

6 Use a calculator to work out these.

a 6700 + 54 000 + 356 894 + 2874 **b** 10 340 – 5672
c 83 006 + 29 040 + 5308 + 9462 + 80 000 **d** 24 578 – 9642
e 7840 + 397 + 584 643 + 96 320 + 87 **f** 604 320 – 243 496

7 Use mental arithmetic to complete the following.

a 284 + 685	**b** 5000 – 4321
c 437 + 934	**d** 729 – 487
e 237 + 496	**f** 578 – 496
g 428 + 729	**h** 700 000 – 456 000
i 792 000 + 647 000	**j** 78 000 – 32 900

Application and problem solving

8 Calculate the answers to these.

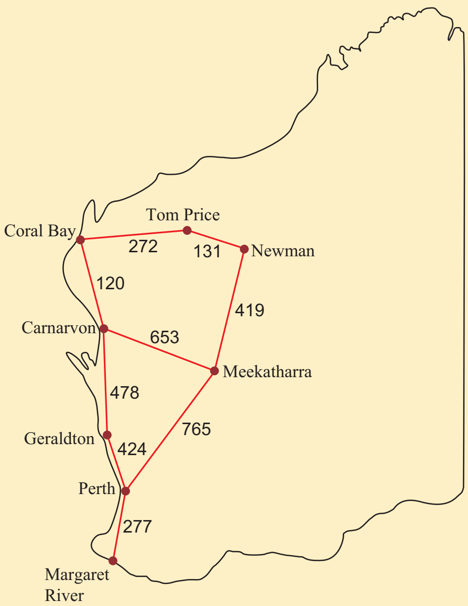
a Find the total of 472 and 3621.
b By how much does 487 exceed 299?
c Find the sum of 88, 67, 132 and 431.
d How much more than 312 is 1876?
e Find the sum of 381, 2174 and 82.

9 At Big Town High there are 376 students in Year 8, 417 in Year 9, 283 in Year 10, 216 in Year 11 and 209 in Year 12. How many students are there at the school?

10 A certain whole number was trebled and then reduced by 4 to give a result 8 more than the original number. What was the number?

11 Lee is planning a trip travelling around Western Australia in his caravan, and searching for perfect waves and adventure. Use the map to help Lee calculate the distance of various trips throughout Western Australia.

a Margaret River to Coral Bay via Geraldton and Carnarvon
b Geraldton to Tom Price via Perth and Newman
c Coral Bay to Meekatharra via Carnarvon and return via Newman
d Perth to Geraldton and return to Perth via Carnarvon and Meekatharra
e Margaret River to Tom Price via Perth, Meekatharra, and Newman, and return via Coral Bay, Carnarvon, and Geraldton



12 A science experiment started with 183 217 live bacteria cultures. It finished with 64 128 live cultures. How many died?

13 The daily attendances at an extreme sports exhibition were:

Mon.	Tues.	Wed.	Thurs.	Fri.	Sat.	Sun.
3267	3025	4162	4275	4891	8213	7921

a What was the total attendance at the exhibition over the week?
b How many more people went on Monday to Friday than on the weekend?
c If all people were charged $3 entry, how much money was collected from attendances?

14 An arcade game had a record of 756 251 points. Joshua set a new record of 874 275 points. What was the increase in the record?

15 Queensland has an area of 1 727 200 km^2 while New South Wales has an area of 801 600 km^2. Western Australia is larger than both of these states with an area of 2 525 500 km^2.

a By how much does Qld exceed NSW in area?

b How much larger in area is WA than NSW?

c By how much does the combined area of Qld and NSW exceed that of WA?

1.4 Multiplication and division

To multiply and divide numbers mentally or using pen and paper, you must know your number facts. You should already know by heart the answers to multiplying any two numbers from 1 to 10, and the corresponding divisions. You should be able to accurately use a calculator for multiplication and division problems and to decide on the best method for a particular problem.

In this book, we will use the symbol × for multiplication, even though a dot on the line is also sometimes used for multiplication. However, we will use both the symbol ÷ and the vinculum (—) for division. The solidus (/) is also used for division, so that the problem '15 divided by 3' can be written as 15 ÷ 3, $\frac{15}{3}$ or 15/3. The last form is most commonly used in computing.

When we have a remainder from a division, we can write it in either remainder form or fraction form, so we can write $13 \div 5 = 2 \text{ R } 3$ or $13 \div 5 = 2\frac{3}{5}$. The fraction form is more common.

Investigation Mental arithmetic methods 2

You are expected to be able to do some multiplications and divisions in your head.
You can use your knowledge of multiplication facts up to 10 × 10 and the shortcut for multiplying by 10, 100 or 1000 to do some multiplications.
Try doing 30 × 70 in your head.
Did you:

- think of the problem as 3 × 10 × 7 × 10
- rearrange to 3 × 7 × 100
- do 21 × 100 to get 2100?

You can extend this method of breaking up and rearranging to some other problems. Try to do 15 × 42 in your head using 2 × 5 = 10.

- Think of the problem as 3 × 5 × 2 × 21.
- Rearrange to 3 × 21 × 5 × 2.
- Then do 63 × 10 to get your final answer.

You can sometimes use the following methods for multiplication and division in your head.

- × 10 is adding a zero, × 100 is adding two zeros and so on
- × 2 is doubling
- × 4 is double double
- × 8 is double double double

- $\times 5$ is $\times 10 \div 2$, so add a zero and halve
- $\times 25$ is $\times 100 \div 4$, so add two zeros, halve and halve again
- $\times 50$ is $\times 100 \div 2$, so add two zeros and halve
- $\times 19$ is $\times (20 - 1)$, so add a zero, double and take away the number
- other 'close' multiplications can be done like $\times 19$
- $\div 10$ is taking off a zero, $\div 100$ is taking off two zeros and so on
- $\div 2$ is halving
- $\div 4$ is halve and halve again
- $\div 8$ is half half half
- $\div 5$ is $\times 2 \div 10$, so double and take off a zero

Work in small groups of two or three people to try out these methods. Use a calculator to check your attempts.

Teacher notes
Calculation shortcuts

Example 11

Do the following problems in your head.

a 48×2 **b** $5600 \div 10$

c 70×50 **d** $\dfrac{184}{4}$

e 45×64 **f** $720 \div 40$

Solution

a Double 48 to get the answer. **$48 \times 2 = 96$**

b Take off a zero. **$5600 \div 10 = 560$**

c Rearrange to $7 \times 5 \times 10 \times 10$ to get 35×100. **$70 \times 50 = 3500$**

d Halve 184 to 92 and halve again. **$\dfrac{184}{4} = 46$**

e Rearrange to $9 \times 5 \times 2 \times 32$, rearrange again to $10 \times (10 - 1) \times 32$ and do $10 \times (320 - 32)$. **$45 \times 64 = 2880$**

f Take off a zero, halve and halve again. **$720 \div 40 = 18$**

In Example 11, you could use other mental methods besides those described in the solutions. For instance, you could do 45×64 as 90×32, then 180×16, then 360×8, then 720×4, then 1440×2 to get an answer of 2880 using repeated halving and doubling. Practising such mental methods will make you fast and efficient in their use and help you to decide when it is best to use a mental method, when it is best to use a written method and when it is best to use a calculator.

You may have your own pen and paper methods for multiplying and dividing numbers. You should check your methods with your teacher to make sure they apply to all problems. The standard written algorithms are shown in Example 12.

Example 12

Use a written method to complete the following.

a 49×8 **b** $98 \div 5$

c 4594×36 **d** $\dfrac{4775}{9}$

e 3184×352 **f** $3872 \div 14$

Interactive tutorial
Section 1.4a

Interactive tutorial
Section 1.4b

Alternative method

Solution

a Set out with the larger number at the top.
Multiply $8 \times 9 = 72$ and write the units digit down.
Write the carry number, 7, above the 2.
Multiply $8 \times 4 = 32$ and add the carry number, 7.
So $32 + 7 = 39$.
There are no more to multiply, so write down the 39.
Write the answer.

$$\begin{array}{r} 4\,9 \\ \times \quad 8 \\ \hline 3\,9\,{}^{7}2 \end{array}$$

$$\mathbf{49 \times 8 = 392}$$

b Set out the division with the divisor on the left.
Divide 5 into 9, write the answer over the 9 and put the remainder above the next number to make 48.
Divide 5 into 48 to get 9 and write the answer over the 8.
There are no more to divide, so write the remainder.
Write the answer.
Write in fraction form.

$$\begin{array}{r} 1\,9\text{ R }3 \\ 5\overline{)9\,{}^{4}8} \end{array}$$

$$\mathbf{98 \div 5 = 19\text{ R }3}$$
$$\mathbf{= 19\tfrac{3}{5}}$$

c Set out with the larger number at the top.
Multiply $6 \times 4 = 24$ and write the units digit down.
Write the carry number, 2, above the 4.
Multiply $6 \times 9 = 54$ and add the carry number, 2.
So $54 + 2 = 56$.
Write down the units digit 6 and the new carry number 5.
Multiply $6 \times 5 = 30$, add the carry number, 5.
So $30 + 5 = 35$.
Write down the units digit 5 and the new carry number, 3.
Multiply $6 \times 4 = 24$ and add the carry number, 3.
So $24 + 3 = 27$.
There are no more to multiply, so write down the 27.

$$\begin{array}{r} 4\,5\,9\,4 \\ \times \quad 3\,6 \\ \hline 2\,7\,{}^{3}5\,{}^{5}6\,{}^{2}4 \end{array}$$

To start on the 3 (really 30), write a 0 on the next line.
Then proceed in the same way as before.
Add up the two working lines, being careful not to confuse the carry numbers.
Write the answer.

$$\begin{array}{r} 4\,5\,9\,4 \\ \times \quad 3\,6 \\ \hline 2\,7\,{}^{3}5\,{}^{5}6\,{}^{2}4 \\ 1\,3\,{}^{1}7\,{}^{2}8\,{}^{1}2\,0 \\ \hline 1\,6\,5\,3\,8\,4 \end{array}$$

$$\mathbf{4594 \times 36 = 165\,384}$$

d Set out the division with the divisor on the left.
Since 9 does not go into 4, divide into 47 to get 5 and put the answer over the 7. Put the remainder above the next number to make 27.
Divide 9 into 27 to get 3 and write the answer over the 7. There is no remainder to put above the 5.
Divide 9 into 5 to get 0 and put the answer over the 5.
There is no more to divide so write the remainder.

$$\begin{array}{r} 5\,3\,0 \text{ R } 5 \\ 9\overline{)4\,7\,{}^{2}7\,5} \end{array}$$

Write the answer. $\frac{4775}{9} = 530 \text{ R } 5$

Write in fraction form. $= 530\frac{5}{9}$

e Set out with the larger number at the top.
Multiply the top number by 2, writing carry numbers as you go.
To start on the 5 (really 50), write a zero on the next line.
Multiply the top number by 5, writing carry numbers as you go.
To start on the 3 (really 300), write two zeros on the next line.
Multiply the top number by 3, writing carry numbers as you go.
Add up the working lines.

$$\begin{array}{r} 3\,1\,8\,4 \\ \times \quad 3\,5\,2 \\ \hline 6\,3\,{}^{1}6\,8 \\ 1\,5\,9\,{}^{4}2\,{}^{2}0\,0 \\ 9\,5\,{}^{2}5\,{}^{1}2\,0\,0 \\ \hline 1\,1\,2\,0\,7\,6\,8 \end{array}$$

Write the answer. $3184 \times 352 = 1\,120\,768$

f Set out the division with the divisor on the left.
Since 14 does not go into 3, divide into 38. Try $2 \times$ and 3×14 to get an answer of 2. Put the answer over the 8 and the remainder above the next number to make 107.
Divide 14 into 107 to get 7 and write the answer over the 7. Write the remainder above the next number to make 92.
Divide 14 into 92 to get 6 and put the answer over the 2.
There is no more to divide so write the remainder.

$$\begin{array}{r} 2\;\;7\;\;6 \text{ R } 8 \\ 14\overline{)3\;\;8\,{}^{10}7\,{}^{9}2} \end{array}$$

$2 \times 14 = 28 \quad 3 \times 14 = 42$

$6 \times 14 = 84 \quad 7 \times 14 = 98$

$8 \times 14 = 112$

Write the answer. $3872 \div 14 = 276 \text{ R } 8$

Write in fraction form. $= 276\frac{8}{14}$

Your teacher might want you to 'cancel down'. $= 276\frac{4}{7}$

When you use a calculator for division, you may get a decimal answer. To find the remainder, you can multiply the answer by the divisor and take away from the number you were dividing into. You can also find the remainder by multiplying the decimal part by the divisor.

Example 13

Use a calculator to solve the following.

a 582×27 **b** $\frac{4126}{16}$ **c** $5820 \div 13$

Solution

a Enter as 582 × 27 = → 15714

Write the answer. **$582 \times 27 = 15\,714$**

b Enter as 4126 ÷ 16 = → 257.875

Find the remainder

Enter the multiplication as 257 × 16 = → 4112

Now find the remainder as 4126 − 4112 = → 14

OR

Subtract 257 from the answer as − 257 = → 0.875

Multiply by 16 as × 16 = → 14

Write the answer. **$4126 \div 16 = 257$ R 14**

Write in fraction form. **$= 257\frac{14}{16}$ or $257\frac{7}{8}$**

c Enter as 5820 ÷ 13 = → 447.6923077

Find the remainder

Enter the multiplication as 447 × 13 = → 5811

Now find the remainder as 5820 − 5811 = → 9

OR

Subtract 447 from the answer as − 447 = → 0.6923077

Multiply by 13 as × 13 = → 9.0000001

Round off to the correct amount. **Remainder is 9.**

Write the answer. **$5820 \div 13 = 447$ R 9**

Write in fraction form. **$= 447\frac{9}{13}$**

Example 14

A hardware store has 200 m of rope on a reel. It is to be cut into 7 m lengths, which will be sold for $4 each.

a How many lengths will there be?
b How much money will they get altogether for the rope?

Solution

a Divide the total by the length to get the number.

Number = 200 ÷ 7
= 28 with remainder 4

Write the answer in a sentence.

There will be 28 full lengths, with 4 m left over.

b Multiply the number of lengths by the price.

Total amount = $4 × 28
= $112

Write the answer in a sentence.

They will get $112 altogether.

Additional exercise 1.4

Exercise 1.4

TLF L90 The multiplier

TLF L82 The multiplier 2

TLF L2006 The divider

1 Do the following in your head.

a 0×28 **b** 2×34 **c** $68 \div 2$
d 53×100 **e** $\frac{270}{10}$ **f** $420 \div 2$
g 26×2 **h** 14×4 **i** $\frac{350}{5}$
j $0 \div 8$

2 Use written methods to complete the following.

a 97×2 **b** 78×6 **c** 8×79
d $83 \div 5$ **e** $\frac{73}{2}$ **f** 85×8
g 4×29 **h** 52×0 **i** $47 \div 2$
j $\frac{79}{5}$

3 Use a calculator to work out the following.

a 58×24 **b** $457 \div 7$ **c** 123×8
d $\frac{679}{9}$ **e** 568×65 **f** $967 \div 11$

4 Do the following in your head.

a 20×70 **b** $240 \div 8$ **c** $\frac{4500}{9}$
d 56×8 **e** 600×70 **f** $6300 \div 7$
g 168×4 **h** 280×40 **i** $\frac{500}{20}$
j $4200 \div 600$

5 Use written methods to complete the following.

a 456×37 **b** $596 \div 8$ **c** 64×309
d $\frac{619}{3}$ **e** 528×85 **f** $412 \div 7$
g 59×3120 **h** $\frac{3021}{6}$ **i** 2904×96
j $5154 \div 9$

6 Use a calculator to work out the following.

a 48×94 **b** $1977 \div 8$ **c** 587×431

d $\frac{876}{8}$ **e** 3455×68 **f** $1684 \div 12$

7 Do the following in your head.

a 48×27 **b** 42×36 **c** $2800 \div 70$

d 101×74 **e** 35×180 **f** $\frac{3900}{30}$

g $5200 \div 40$ **h** $2200 \div 8$ **i** $\frac{4200}{3}$

j 55×400

8 Use written methods to complete the following.

a 4524×283 **b** $52\,678 \div 8$ **c** $90\,674 \times 85$

d $\frac{47\,846}{15}$ **e** $207 \times 21\,394$ **f** $295 \div 12$

g 4434×517 **h** $\frac{19\,678}{9}$ **i** $478 \times 16\,045$

j $5166 \div 123$

9 Use a calculator to work out the following.

a $41\,587 \times 987$ **b** $7890 \div 23$ **c** 108×2008

d $\frac{5128}{19}$ **e** 1657×4578 **f** $2986 \div 124$

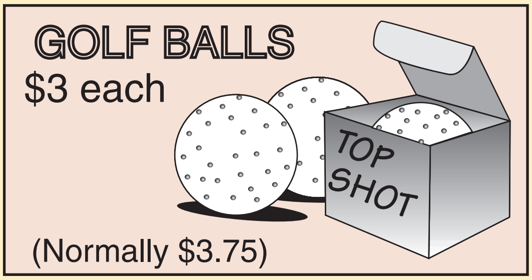
Application and problem solving

Use your calculator as directed by your teacher for these problems.

10 Rianna bought 18 golf balls.

a What was the total cost?

b How much did she save by buying them at the special price?



11 Joshua wants to count the number of words in his three-page essay. He knows that he writes about 12 words per line. He has written his essay on pages that fit 28 lines per page. How many words are in Joshua's essay?

12 Sandra organised tickets for a group of people to go to a rock concert. The tickets cost $72 each. How much did Sandra need to collect if 46 people wanted to go to the concert?

13 Kristina is paid $738 per fortnight. She pays $95 per week rent and it costs her $125 per week for food and entertainment. Her other expenses amount to $105 per week.

a How much could she save each week?

b How much could she save in a year?

14 

a How much would be repaid each year?
b How much would the car cost over the full 4 years?
c How much more was paid for the car if its cash price was $40 000?

15 Each morning, 504 people use a lift to enter an office building. The lift carries a maximum of 14 people. What would be the *minimum* number of trips that the lift would need to make to take the office workers up to their offices?

16 A plane travels 1045 km on a regular daily trip delivering mail in outback Western Australia.
a How many kilometres are travelled in a normal five-day week?
b An average of 786 letters and parcels are delivered each trip. How many would be delivered in a four-week period?

17 During one year. Jessica and Stefan saved $2041 and $1892 respectively. They wanted to pool their savings to buy a car worth $10 701.
a How much more did Jessica save than Stefan?
b How much did they need to borrow?
c They decided to borrow the extra money and repay it over three years. The interest charges were $3600. How much did they actually pay for the car?
d How much did they repay on the loan?
e How much were the monthly repayments?

18 The total quantity of diamonds mined in Western Australia over a period of six years is illustrated in the table. (Diamonds are measured in carats)

Year	Diamonds (Carats)
2002	35,486,604
2003	34,367,807
2004	22,842,000
2005	30,845,157
2006	29,300,000
2007	20,467,000

a What was the total quantity of diamonds mined in Western Australia from 2002 to 2007 (inclusive)?
b How many more carats were produced in 2002 than in 2007?
c If the diamonds sold at an average of $17 per carat, how much did the Western Australian diamond industry gross for 2005?
d If the diamonds sold for an average of $12 per carat, how much did the Western Australian diamond industry gross for 2003?

19 What would be the weekly payment for an MP3 player (160GB and holds 40,000 songs) that costs $468 and is paid off in one year?

20 Olly the orchardist has 240 apple trees. If his total production of apples was 115 200, how many apples did each tree produce on average?

21 A special double-decker bus carries 85 people. How many of these buses would be required to transport 361 760 people?

22 Spend exactly $100 on items costing $3, $5 and $13 each. You must buy at least one of each. What are the largest and smallest numbers of items you can buy? Are you sure?

1.5 Order of operations

The order of adding numbers makes no difference to the answer. So $5 + 9$ is the same as $9 + 5$. This is called **commutativity of addition**.

We can change the order of additions to make them easier. This means that we can turn around the 27 and 84 in $46 + 27 + 84$ to get $46 + 84 + 27$. This makes it easier to get 157 by choosing to add the parts that give a 'ten' first.

The order of subtraction cannot be changed. The calculation $5 - 9$ does not even have a positive answer, so is quite different from $9 - 5$. The calculation $65 - 27 - 17$ must be done *from left to right* to get the correct answer. You should think of this problem as 65 take 27 *and then* take 17 to give 21.

Brackets change the order of operations. In mathematical calculations, brackets mean 'do me first'. This means that $65 - (27 - 17)$ is a different calculation from the one above. The brackets say 'do me first', so you should think of this problem as 27 take 17 *and then* take the answer from 65 to get 55.

In primary school you started doing multiplication as repeated addition, so that 4×6 can be thought of as $(6 + 6 + 6 + 6)$ or as $(4 + 4 + 4 + 4 + 4 + 4)$. In both cases, you get 24 as the answer. Just like addition, the order of multiplication does not change the answer. This is called **commutativity of multiplication**. You can do $5 \times 7 \times 2$ as $7 \times 5 \times 2$ to get 70 more easily than the original order.

The order of division cannot be changed. $48 \div 6 \div 2$ must be done *from left to right* to get the right answer. You should think of this calculation as 48 divided by 6 *and then* divided by 2 to get 4.

The problem $48 \div (6 \div 2)$ has brackets, so you think of this as 6 divided by 2 *and then* divide the answer into 48 to get 16.

Example 15

Work out:

a $58 + 27 + 12 + 9$ **b** $60 - (12 - 8)$ **c** $12 \times 9 \times 5$ **d** $36 \div 2 \div 3$

Solution

a Turn around the 27 and 12.
Now do the additions in two parts in your head.

$$\begin{aligned} & 58 + 27 + 12 + 9 \\ &= 58 + 12 + 27 + 9 \\ &= 70 + 36 \\ &= 106 \end{aligned}$$

b Brackets say 'do me first'.

$$\begin{aligned} & 60 - (12 - 8) \\ &= 60 - 4 \\ &= 56 \end{aligned}$$

c Think of the 12 as 6×2.
Do these in your head.

$$\begin{aligned} & 12 \times 5 \times 9 \\ &= 6 \times 2 \times 5 \times 9 \\ &= 6 \times 10 \times 9 \\ &= 540 \end{aligned}$$

d You must do these from left to right.
Halve to divide by 2.

$$\begin{aligned} & 36 \div 2 \div 3 \\ &= 18 \div 3 \\ &= 6 \end{aligned}$$

Although you can do calculations like Example 15 in your head, your teacher will probably want you to explain your methods for some of the problems. When we have addition and subtraction or multiplication and division operations mixed up, we need to work from left to right, unless there are brackets.

!

Mixed operations
Operations in brackets must be done first.

Mixed multiplication and division is done from left to right.

Mixed addition and subtraction is done from left to right.

Example 16

Work out:

a $16 - (5 + 2) + 9$ **b** $20 \times 4 \div 2 \times 3 \div 2$

Solution

a Brackets say 'do me first'.
Now work from left to right.

$$\begin{aligned} & 16 - (5 + 2) + 9 \\ &= 16 - 7 + 9 \\ &= 9 + 9 \\ &= 18 \end{aligned}$$

b Work from left to right, doing 20×4 first.
Now do $80 \div 2$.
Keep working from left to right.

$$\begin{aligned} & 20 \times 4 \div 2 \times 3 \div 2 \\ &= 80 \div 2 \times 3 \div 2 \\ &= 40 \times 3 \div 2 \\ &= 120 \div 2 \\ &= 60 \end{aligned}$$

In some cases, we have all four operations mixed up, perhaps with brackets as well. In this case, we can work out the order of operations by considering the real meanings of multiplication and division.

For whole numbers, we can work out a multiplication as repeated addition, so we can think of 6×4 as $(6 + 6 + 6 + 6)$. This means we could write $35 - 6 \times 4$ as $35 - (6 + 6 + 6 + 6)$. This means we should do multiplication before addition or subtraction. The same reasoning applies to division, which is the opposite of multiplication.

We already know that when other operations are mixed up, we should work from left to right, except that brackets say 'do me first'. Obviously, if brackets have other brackets inside them, the ones most inside should be done at the very start.

This gives the following rules for the order of operations.

!

Order of operations

Work out brackets first, from innermost outwards and from left to right. A vinculum (bar) works like brackets for operations on top or underneath.

Work out multiplications and divisions next, from left to right.

Work out addition and subtraction last, from left to right.

Example 17

Interactive tutorial
Section 1.5

Evaluate

a $4 \times [9 - (5 + 2)] + 6 \div 3$ **b** $\dfrac{28 + 8}{4 - 2}$

Solution

a **Evaluate** means to work out.
Do the brackets first, inside out.
Now do times and divide, left to right.

Write the answer.

$$\begin{aligned} & 4 \times [9 - (5 + 2)] + 6 \div 3 \\ &= 4 \times [9 - 7] + 6 \div 3 \\ &= 4 \times 2 + 6 \div 3 \\ &= 8 + 2 \\ &= 10 \end{aligned}$$

b

$$\frac{28 + 8}{4 - 2}$$

Add brackets to clarify the vinculum.

$$= \frac{(28 + 8)}{(4 - 2)}$$

Work out the brackets first.

$$= \frac{36}{2}$$

Now do the division.

$$= 18$$

Additional exercise 1.5

Exercise 1.5

1 Work out the following.

a $28 + 16 + 22 + 14$ **b** $48 + 17 + 12 + 9$ **c** $40 - 16 - 3$
d $40 - (16 - 3)$ **e** $48 - 12 - 6$ **f** $48 - (12 - 6)$
g $38 + 27 + 12$ **h** $48 + (17 + 12)$ **i** $60 - 22 - 18$
j $22 - (16 - 10)$

2 Work out the following.

a $15 - 4 + 8$ **b** $15 - (4 + 8)$ **c** $4 + 16 + 8 - 7 - 3$
d $4 + 16 + 8 - (7 - 3)$ **e** $4 + 16 + (8 - 7) - 3$ **f** $48 \div 6 \times 2$
g $48 \div (6 \times 2)$ **h** $9 \times 8 \div 6 \times 3$ **i** $9 \times 8 \div (6 \times 3)$
j $36 \div 3 \times 4$

3 Work out the following.

a $\dfrac{8 \times 5}{4}$ **b** $\dfrac{9 \times 6}{3 \times 2}$ **c** $\dfrac{72}{4 \times 6}$
d $\dfrac{10 \times 9}{6} \times 5$ **e** $\dfrac{10 \times 12}{4 \times 6}$ **f** $\dfrac{10 \times 12}{4} \times 6$

4 Work out the following.

a $(8 + 4) \times 6$ **b** $17 - 6 \times 2$
c $12 + 3 \times 7$ **d** $36 \div 4 \times 3$
e $7 + 10 \times (3 \div 3)$ **f** $3 \times (16 - 4 + 2)$
g $6 \times 9 - 2 \times 7$ **h** $21 + 7 \times (36 - 29)$
i $7 + [4 \times (37 - 26)]$ **j** $[45 - (81 \div 9)] \div [(3 + 2 \times 4) + 7]$

5 Work out the following.

a $\dfrac{6 \times 5}{4 + 6}$ **b** $\dfrac{9 + 6 \times 2}{1 + 3 \times 2}$
c $\dfrac{19 + 3 \times 2}{7 \times 6 - 4 \times 4 - 1}$ **d** $8 + \dfrac{5 + 27}{6 + 2} \times 3$
e $\dfrac{8 \times 10}{4} \div \dfrac{20}{4}$ **f** $\left(\dfrac{16 + 12}{16 - 12} \times 3 - 11\right) \times 5$

1.6 Approximation methods

Saturn takes 15 493 248 minutes to travel once around the sun. When stated in this way, the size of the number obscures the meaning. It is much clearer to say that Saturn takes about 30 years to orbit the sun. In the same way, we might say that a crowd of 2000 people attended a local soccer match instead of giving the exact number.

When you use a calculator to divide 285 by 13, you get the answer 21.923 076 92. This answer is correct to 8 decimal places, but it is rare that we would need this accuracy. As you have already seen, we can use this answer to obtain the exact result that $285 \div 13 = 21$ with remainder 12. In other cases, we would want to write the answer as about 22. This means that the approximate answer could be given as 21 or 22, depending on the circumstances.

An **approximation** is a number that is not exact. When using a calculator, it is a very good idea to use approximations to check the entry of the calculation by seeing if the answer is reasonable. We use **rounding** to get an approximation of a number. We use the sign ≈ to mean **is approximately equal to**.

!

Rounding

There are three methods of rounding to a particular degree of accuracy.

	Rounding method	Examples
Rounding up	Take a part-number *up* to the next higher rounded number.	2347 ≈ 2400 (to 100s) 5087 ≈ 5090 (to 10s)
Rounding down	Take a part-number *down* to the next lower rounded number.	2347 ≈ 2300 (to 100s) 5087 ≈ 5080 (to 10s)
Rounding (off)	Take a part-number to the *nearest* rounded number. If it is half-way (5), then round up.	2347 ≈ 2300 (to 100s) 5087 ≈ 5090 (to 10s)

Rounding off is the most common method of rounding. Rounding down is also called truncating when decimal places are removed. When rounding, you can use the following method, or just use your common sense.

!

Rounding steps

1. Underline the digit to be rounded.
2. Circle the next digit (to the right) to the underlined digit.
3. If the circled digit is less than 5, leave the underlined digit as it is.
4. If the circled digit is 5 or more, round the underlined digit up to the next digit.
5. Replace the circled digit and those to its right by zeros.

Example 18

Round off each of the following to the stated degree of accuracy.

a 2578 (nearest thousand)

b 165 749 (nearest hundred)

c 19 150 (nearest hundred)

Solution

a Underline the thousands and circle the next digit. **2578**

The circled digit is 5, so round up the underlined digit.

Change 2 up to 3 and make 5, 7 and 8 zero. **2578 ≈ 3000**

b Underline the hundreds and circle the next digit. **165 749**

The circled digit is 4, so leave the 7.

Leave the 7 and change the digits to the right to zeros. **165 749 ≈ 165 700**

c Underline the hundreds and circle the next digit. — **19 150**
The circled digit is 5, so round up.
Change the 1 to 2 and change the digits to the right to zeros. — **19 150 ≈ 19 200**

In Example 18 you could use common sense as follows. 2578 is between 2000 and 3000, but closer to 3000, so, to the nearest thousand, 2578 ≈ 3000. Similarly, 165 749 is between 165 700 and 165 800, but is closer to 165 700. 19 150 is exactly halfway between 19 100 and 19 200, and in this case we round to the higher number.

Example 19

A lifeguard counted 285 people on a 20 m section of Scarborough Beach, including the grass verge, sand and water. The whole beach is 420 m long. If people were spread evenly along the whole beach, how many people were on the beach?

Solution

Divide to find the number of sections.	**Number of sections = 420 ÷ 20 = 21**
Multiply to find the number of people.	**Number of people = 21 × 285 = 5985**
Round to appropriate accuracy.	**≈ 6000**
Write the answer in a sentence.	**There were about 6000 people on the beach.**

Estimation of answers can be done mentally by **leading digit calculation**. This allows you to quickly check if the result on a calculator is reasonable. The method is as follows.

!

Leading digit calculation

1 Round off the numbers in the problem to just the first (leading) digit.
2 Do the problem with the rounded numbers.
3 Use your result to check the exact answer (from your calculator).

Example 20

Luke used a calculator to get the bracketed answers in the following. Use estimates to check that his answers are reasonable.

a 4252 + 2482 (4734) **b** 763 × 42 (32 046) **c** 7428 ÷ 34 (218.4705882)

Solution

a

	4252 + 2482
Round to the leading digits.	**≈ 4000 + 2000**
Work out 4 + 2 thousands in your head.	**= 6000**
The estimate suggests that Luke made a mistake in entering the calculation.	**Luke's answer is not reasonable.**

b

Round to leading digits.
Do 8×4 and 100×10 in your head.
Put 32 and 1000 back together.

$$\begin{aligned} & 763 \times 42 \\ \approx\ & 800 \times 40 \\ =\ & 32\,000 \end{aligned}$$

Luke's answer is reasonable.

c

Round to leading digits.
Do as $700 \div 3$, which is 7 hundreds $\div$ 3.
This is about 2 hundreds (but a bit more).

$$\begin{aligned} & 7428 \div 34 \\ \approx\ & 7000 \div 30 \\ \approx\ & 200 \end{aligned}$$

Luke's answer is reasonable, but should be rounded.

Exercise 1.6

Additional exercise 1.6

1 Round to the nearest 10.

a 47 **b** 63 **c** 197
d 264 **e** 2738 **f** 6732

2 Round to the nearest 100.

a 936 **b** 438 **c** 592
d 6658 **e** 7426 **f** 2973

3 Round to the place value shown in brackets.

a 374 (nearest 10) **b** 374 (nearest 100)
c 6947 (nearest 10) **d** 6947 (nearest 100)
e 6947 (nearest 1000) **f** 2496 (nearest 1000)

4 Round to the place value shown in brackets.

a 14 891 (nearest 10) **b** 12 973 (nearest 100)
c 58 117 (nearest 10) **d** 58 117 (nearest 100)
e 14 405 (nearest 1000) **f** 14 405 (nearest 10 000)
g 531 603 (nearest 10 000) **h** 531 603 (nearest 100 000)
i 2621 (nearest 1000) **j** 2621 (nearest 10 000)

5 Estimate the answers to each of the following by leading digit calculation.

a $823 + 469$ **b** $758 - 328$ **c** $3847 + 4915$
d $8834 - 6515$ **e** $6254 + 8102$ **f** $6027 - 2263$

6 Estimate the answers to each of the following by leading digit calculation.

a 46×41 **b** 386×32 **c** 4873×24
d 8376×673 **e** 518×962 **f** 8376×698

7 Estimate the answers to each of the following by leading digit calculation (note that some figures may be omitted altogether).

a $7369 + 48 + 5862 + 496$ **b** $8372 + 2163 + 968 + 21 + 7138$
c $8497 - 7722$ **d** 397×512
e 289×690 **f** 921×968
g 9621×3714 **h** 4269×7865
i $20\,628 \times 3794$ **j** $58 \times 470 \times 342$

8 Estimate the answers to each of the following by leading digit calculation.

a $461 \div 23$

b $\frac{1826}{169}$

c $5821 \div 18$

d $58 \times 45 \div 17$

e $(2194 + 5872) \times 68$

f $\frac{4570 + 2196 + 8570}{48}$

g $6894 + 2196 - 98 \times 46$

h $\frac{6736 + 1683 - 5268 + 9693}{27} \times 53$

i $67\,894 \div 2845$

j $45\,789\,000 \div 54\,370$

Application and problem solving

9 A student used a ruler end over end to measure a length to be 192 cm. How should the answer be given?

10 What is 93 kg to the nearest 10 kg?

11 Express the length 437 cm to the nearest metre.

12 What is 76 °C to the nearest 10 °C?

13 A large car weighs 2060 kg. Express this to the nearest **a** 100 kg, **b** tonne.

14 A grazier estimates she has 3.5 sheep per hectare on her property. About how many sheep are there in the back paddock, which has an area of 485 ha?

15 A computer has 72 chips on the mother board. In one day, 173 computers are made on a production line. About how many chips would be needed for a day's production?

16 The area of each state and territory in Australia is shown below. Use leading digit calculation to estimate the area of Australia.

State/Territory	Area (square kilometres)
Australian Capital Territory	2 430
New South Wales	801 640
Northern Territory	1 346 250
Queensland	1 727 240
South Australia	984 100
Tasmania	67 810
Victoria	227 630
Western Australia	2 525 500

17 A dripping tap wastes about 6600 litres of water in a year. About how much water is wasted each day?

18 A strawberry plant produces an average of about 0.5 strawberries per day in season. A strawberry farm has 5274 plants in production. Each punnet of strawberries needs about 18 strawberries to fill it. About how many punnets will be produced by this farm in one month of the high season?

1.7 Powers and roots

You already know that we can think of multiplication as repeated addition and we can think of division as the opposite of multiplication. It is also possible to repeat multiplication. Since we have already used the symbols $\times$, $+$, $\div$ and $-$, we need another way to write repeated multiplication. Instead of writing it with a symbol, we write the number of repetitions as a small number above the number that is multiplied. This is called a **power**.

!

Powers

A repeated multiplication is written as a **power**.

The number that is multiplied, the **base**, is written in normal size.

The number of times it is in the product, the **index**, is written as a small number at the top right of the base. The index is also called the **exponent**.

A power may be written in **extended form** by writing out the product in full.

Index or exponent

Power form $4^5 = 4 \times 4 \times 4 \times 4 \times 4$

Extended form

Base

The power shown is said as 'four to the fifth' or 'the fifth power of four'.

A number raised to the power 2 is called a **square**. 6^2 is said as 'six squared'.

A number raised to the power 3 is called a **cube**. 8^3 is said as 'eight cubed'.

Example 21

Work out the following powers by hand.

a 4^3 **b** 2^5 **c** five to the fourth **d** 7^1

Solution

a

	4^3
Write in extended form.	$= 4 \times 4 \times 4$
Multiply the first two.	$= 16 \times 4$
Now multiply by the last one.	$= 64$

b

	2^5
Write in extended form.	$= 2 \times 2 \times 2 \times 2 \times 2$
Multiply the first two.	$= 4 \times 2 \times 2 \times 2$
	$= 8 \times 2 \times 2$
Keep going.	$= 16 \times 2$
	$= 32$

c $\quad 5^4$

	5^4
Write in extended form.	$= 5 \times 5 \times 5 \times 5$
Multiply the first two.	$= 25 \times 5 \times 5$
Keep going.	$= 125 \times 5$
	$= 625$

d $\quad 7^1$

	7^1
Write just the one seven.	$= 7$

The last power shown in Example 21 is a special case without an actual multiplication. Any number raised to the power 1 is equal to itself.

Scientific calculators have special buttons for doing powers.

Example 22

Use a calculator to find:

a 7^4 **b** 15^2

Solution

The power button on your calculator should be shown as [$x^{\blacksquare}$]. Many calculators also have a square button shown as [x^2].

a Enter as 7 [$x^{\blacksquare}$] 4 [=] $\longrightarrow$ 2401

Write the answer. $\quad$ **$7^4 = 2401$**

b Enter as 15 [x^2] [=] $\longrightarrow$ 225

Write the answer. $\quad$ **$15^2 = 225$**

We have opposites for addition and multiplication. In the same way, the opposite of finding the square (known as *squaring*) is to find the **square root**. The square root of a number is written as $\sqrt{\ }$.

Example 23

Find $\sqrt{81}$.

Solution

Think of the number that multiplies by itself to give 81.	$9 \times 9 = 81$
Write in square root form.	$\sqrt{81} = 9$

In later work in mathematics, you may find it useful to know the squares (and corresponding square roots) up to $20^2 = 400$. Since $31^2 = 961$ and $32^2 = 1024$, there can only be 32 whole numbers including zero below 1000 that have an exact square root. For other numbers, such as $\sqrt{120}$, we normally use a calculator to find an approximation for a square root. Most calculators have a [$\sqrt{\blacksquare}$] button we can use for this purpose.

Example 24

Use a calculator to find an approximation for $\sqrt{120}$, correct to 2 decimal places.

Solution

Enter as [√■] 120 [=] $\longrightarrow$ 10.95445115

Round and write the answer. $\sqrt{120} \approx 10.95$

Exercise 1.7

1 Write down how you would say each of the following.

a 6^2 **b** 7^3 **c** 6^7
d 2^{10} **e** 8^4 **f** 24^4

2 Work out the following by hand.

a 5^2 **b** 6^3 **c** 7^2
d 9^2 **e** 2^7 **f** 4^5
g 3^6 **h** 9^3 **i** 5^3
j 8^1

3 Use a calculator to find each of the following.

a 2^{12} **b** 7^4 **c** 6^5
d 14^4 **e** 4^6 **f** 15^3
g 3^{10} **h** 9^5 **i** 6^4
j $2^4 \times 3^4$

4 Work out each of the following by hand.

a $\sqrt{25}$ **b** $\sqrt{196}$ **c** $\sqrt{81}$
d $\sqrt{64}$ **e** $\sqrt{576}$ **f** $\sqrt{121}$
g $\sqrt{16}$ **h** $\sqrt{49}$ **i** $\sqrt{225}$
j $\sqrt{900}$

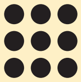
5 Use a calculator to find each of the following correct to 2 decimal places.

a $\sqrt{26}$ **b** $\sqrt{24}$ **c** $\sqrt{120}$ **d** $\sqrt{54}$
e $\sqrt{13}$ **f** $\sqrt{150}$ **g** $\sqrt{600}$ **h** $\sqrt{75}$
i $\sqrt{726}$ **j** $\sqrt{164}$

Application and problem solving

6 **a** Examine the powers of 2, 4 and 8. Make a conclusion.
b Examine the powers of 3, 9 and 27. Make a conclusion.
c What is likely to be true of the powers of 5, 25 and 125? Can you make a general rule?

7 Draw a square of dots as shown below.



a How many dots are in a square with side length of 3 dots?
b Try some other squares and make a general conclusion.

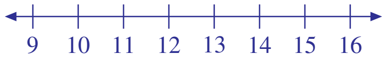
8 Work out the values of each of the following pairs of powers.

a 5^2 and 2^5 **b** 3^2 and 2^3
c 8^6 and 6^8 **d** 5^{12} and 12^5

Can you find a pattern to work out which answer of such a pair of powers is bigger?

1.8 Negative numbers

Sometimes we show part of a number line, with arrows showing that the line continues in both directions.



So far, you have looked at number lines that start from 0, because numbers start from 0. Sometimes it is convenient to have numbers below zero. A map of the Middle East has to show that the Moab plateau rises to a height of 1340 m but that the Dead Sea is 408 m below sea level. Marine charts have to show heights of landmarks above the water as well as depths of the bottom of the sea. In these cases it is natural to introduce numbers below zero.

Mathematically, we use **negative numbers** to extend the number line back from zero. Negative numbers are shown with a negative sign ($^{-}$) and are obtained by counting back on the number line.

!

Directed numbers

Directed numbers have a sign. **Negative** numbers have a $^{-}$ sign. **Positive** numbers have a $^{+}$ sign. The $^{+}$ sign is often left off positive numbers.

All positive numbers are greater than zero. All negative numbers are less than zero.

Directed numbers are shown on the **number line** with the negative numbers to the left and the positive numbers to the right. The directed number line continues indefinitely in both directions, without a beginning or end. The directed numbers $^{+}7$ and $^{-}7$ are both 7 away from 0, but in opposite directions.

Example 25

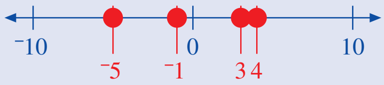
Show on separate number lines.

a $^{-}5, 3, ^{-}1, 4$ **b** whole numbers $n > {}^{-}3$ **c** whole numbers $n \leqslant 2$

Solution

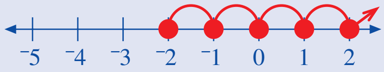
a Show the number line from, say, $^{-}10$ to 10 with arrows.

Mark the included numbers with large dots, making room as necessary.



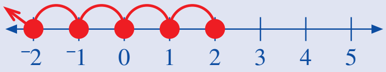
b Show the number line from $^{-}5$ to 2 with arrows.

Connect the included numbers (more than $^{-}3$) with large dots and use an arrow to show they continue upwards.



c Show the number line from $^{-}2$ to 5 with arrows.

Connect the included numbers (from 2 down) with large dots and use an arrow to show they continue downwards.



Example 26

Insert $>$, $<$ or $=$ to make true statements.

a $^{-}6 \;\square\; 5$ **b** $^{-}3 \;\square\; ^{-}7$ **c** $^{+}4 \;\square\; ^{-}4$

Solution

a Any negative is less than any positive. $\mathbf{^{-}6 < 5}$

b $^{-}3$ is further to the right on the number line, so is greater than any number to its left. $\mathbf{^{-}3 > ^{-}7}$

c Any positive is more than any negative. $\mathbf{^{+}4 > ^{-}4}$

Example 27

Choose the biggest and smallest numbers from the following.

a $6, ^{-}5, 4, ^{-}7, 0, ^{-}4, 2$ **b** $^{-}1, ^{-}3, ^{-}8, ^{-}2$

Solution

a The biggest will be the most positive. **The biggest number is 6.**

The smallest will be the most negative. **The smallest number is $^{-}7$.**

b Since all are negative, the biggest will be the least negative. **The biggest number is $^{-}1$.**

The smallest will be the most negative. **The smallest number is $^{-}8$.**

Additional exercise 1.8

TLF L2001 Scale matters

Spreadsheet Negative no. puzzles

Exercise 1.8

1 Show on separate number lines.

a 4, ⁻3, 6, ⁻1, 0
b 3, ⁻2, ⁻1, 0, 5
c ⁻3, ⁻4, ⁻6, ⁻1, ⁻8
d 15, ⁻15, ⁻20, ⁻5
e ⁻52, ⁻54, ⁻49, ⁻51
f ⁻100, ⁻200, 0, 200

2 Show on separate number lines, for whole numbers n.

a $n > {}^-2$
b $n \leqslant {}^-2$
c $n \geqslant {}^-4$
d $n < {}^-20$
e $n \geqslant {}^-16$
f $n < 45$

3 Insert <, > or = to make a true statement.

a 0 ☐ ⁻5
b ⁻3 ☐ 0
c ⁻6 ☐ ⁻7
d ⁻2 ☐ 2
e ⁻40 ☐ ⁻20
f 23 ☐ ⁻25

4 In each of the following, choose the smallest number.

a 3, ⁻6, 7, ⁻1, 0
b 2, 4, 7, 0
c ⁻3, ⁻5, ⁻7, ⁻1, 3
d ⁻6, ⁻9, ⁻4, 0, ⁻3
e ⁻4, ⁻7, 7, 4, 0
f ⁻5, ⁻23, 34, ⁻4, 2

5 In each of the following, choose the biggest number.

a 3, ⁻6, 7, ⁻1, 0
b 2, 4, 7, 0
c ⁻3, ⁻5, ⁻7, ⁻1, 3
d ⁻6, ⁻9, ⁻4, 0, ⁻3
e ⁻4, ⁻7, 7, 4, 0
f ⁻5, ⁻23, 34, ⁻4, 2

Application and problem solving

6 Which one is colder in each of the following?

a ⁻15 °C or 8 °C
b ⁻2 °C or ⁻5 °C
c ⁻12 °C or ⁻13 °C
d 16 °C or ⁻3 °C
e ⁻48 °C or ⁻17 °C
f 2 °C or ⁻40 °C

7 The *potential energy* in Joules (J) of two similar objects is given in the following. In each, decide which object has the higher potential energy.

a A = 120 J B = 80 J
b A = 40 J B = ⁻20 J
c A = ⁻100 J B = ⁻180 J
d A = ⁻60 J B = 0 J
e A = ⁻400 J B = ⁻380 J

8 The lowest point in Australia is at Lake Eyre. It is about 15 m below sea level. How should this be shown on the map?

9 Near Lake Eyre, one point (A) has an elevation of ⁻5 m, while a second point (B) has an elevation of ⁻9 m. Would you be walking uphill or downhill when going from A to B?

10 The ground floor of a building is floor 0, the first floor is floor 1 and the fifth floor is floor 5 on the elevator buttons. The car park has six underground floors below the basement. What numbers should the car park have on the elevator buttons?

Chapter **summary**

- Our decimal system is a **place value** system of numbers. The value of each digit depends on its place. In writing numbers and giving their names, the digits are grouped in threes. Each group of three digits has hundreds, tens and units. The lowest three place values have no other name, but then the groups go up as thousands, millions, billions, trillions, quadrillions and so on.
- Some numbers have special names. A **dozen** means 12, a **baker's dozen** means 13, a **score** means twenty, a **ton** (in cricket) means 100, a **gross** means 144 and a **k** means 1000 (or 1024 in computing).
- We compare the sizes of numbers using the symbols for less than ($<$), greater than ($>$), equal to ($=$), less than or equal to ($\leqslant$), greater than or equal to ($\geqslant$) and not equal to ($\neq$). Numbers and groups of numbers may be shown on a **number line** using large dots and arrows.
- When required to add, subtract, multiply or divide numbers you should use the most suitable method—mental arithmetic, written method or calculator. Division may be written using the ÷ or / symbols with the divisor last, or by writing the divisor under the other number.
- Addition and subtraction are opposite operations. Similarly, multiplication and division are opposite.
- Addition and multiplication are **commutative** but the order of subtraction and division cannot be changed.
- The order of mixed operations is:
  1 Work out brackets first, from innermost outwards and from left to right. A vinculum (bar) works like brackets for operations on top or underneath.
  2 Work out multiplications and divisions next, from left to right.
  3 Work out addition and subtraction last, from left to right.
- **Rounding** is used to write **approximations** for numbers to a particular degree of accuracy. The ≈ symbol means **is approximately equal to**. It is usual to round to the nearest approximation, with '5' being rounded up.
- Calculation by rounding to the first digit is called **leading digit calculation**. It is usually used to check answers by an approximation.
- A repeated multiplication may be written as a **power**. The number multiplied is written once as the **base**. The number of times it is in the multiplication is written as the small **index (exponent)** at the top right. The **extended form** of a power has the multiplication written out in full. For example, $4^3 = 4 \times 4 \times 4$.
- A **square** is a power with index 2, while a **cube** has index 3.
- The **square root** of a number is written like $\sqrt{49}$. It is the new number that must be squared to give the original number. $\sqrt{49} = 7$ because $7^2 = 49$.
- **Directed numbers** have a sign. Numbers above zero may have a $^+$ sign written in for emphasis. **Negative numbers** are used to extend the number line back past zero. They have a negative sign, $^-$, and are all less than zero. Negative numbers are used to represent quantities less than zero, such as $^-40$ °C.

Chapter review

Ex 1.1

1 **a** Write 784 in words.
b Write five thousand, six hundred and seven as a number, using the usual symbols.
c Compare the values of the 3s in the numbers 3408 and 234.

Ex 1.2

2 **a** Insert $<$, $>$ or $=$ to make a true statement for $6 \;\square\; 5 + 3$.
b Show the numbers less than 4 on a number line.

Ex 1.3, 1.4

3 Calculate each of the following using mental arithmetic.
a $45 + 27$ **b** $68 - 29$ **c** 67×2 **d** $5900 \div 10$

Ex 1.3, 1.4

4 Complete each of the following using written methods.
a $2455 + 462 + 1350 + 86$ **b** $2600 - 436$
c 7×69 **d** $\frac{83}{5}$

Ex 1.6

5 Round each of the following to the place value shown in brackets.
a 5347 (nearest 100) **b** 596 (nearest 10)

Ex 1.1

6 **a** Write 92 607 000 in words.
b Write 784 000 000 000 in words.
c Write 86 045 in extended form.

Ex 1.2

7 Show the following on a number line: $64 < n < 69$.

Ex 1.3, 1.4

8 Calculate each of the following using mental arithmetic.
a $480 + 560$ **b** $2600 - 1200$ **c** 400×24 **d** $\frac{720}{90}$

Ex 1.3, 1.4

9 Complete each of the following using written methods.
a $32\,567 + 7890 + 6058 + 579$ **b** $26\,458 - 19\,709$
c 4126×37 **d** $\frac{4560}{8}$

Ex 1.5

10 Work out:
a $24 - 7 + 9 - (12 - 10)$ **b** $10 \times 6 \div 5 \times 2$

Ex 1.6

11 Estimate the answers to each of the following by leading digit calculation.
a $698 - 231$ **b** 48×32

Ex 1.2

12 Show $16 < n \leqslant 35$ on a number line, for whole numbers n.

Ex 1.3, 1.4

13 Calculate each of the following using mental arithmetic.
a $248 + 763$ **b** $940 - 637$ **c** 52×35 **d** $4200 \div 70$

Ex 1.5

14 Calculate each of the following.
a $16 \times 4 - [4 + (12 - 3 \times (5 - 2))]$ **b** $28 - \frac{5 \times 12}{6 + 9} \times 3$

Ex 1.6

15 Estimate the answers to each of the following by leading digit calculation.
a $5347 + 985 + 6134 + 497$ **b** $230 \times 57 \times 679$
c $\frac{687 - 293 + 956}{71}$

Ex 1.7

16 Work out each of the following correct to 2 decimal places if necessary.
a 5^4 **b** 16^7 **c** $\sqrt{64}$ **d** $\sqrt{19}$

Ex 1.8

17 Show on separate number lines.
a $^{-}6, 2, {}^{-}3, 3, {}^{-}1$ **b** Whole numbers n so $^{-}3 \leqslant n < 4$

Application and problem solving

18 Write five score as a number, using the usual symbols. Ex 1.1

19 What use is being made of the number in a PIN? Ex 1.2

20 **a** Mary has \$45, Joan has \$63 and Maria has \$52 to spend at a theme park. How much do the three friends have altogether? Ex 1.3

b At the end of the day they are left with \$7, \$9 and \$16 respectively. Altogether, how much did they spend?

21 Jason used very accurate industrial scales to measure his mass as 54 389 grams. How should he give his mass on a medical form? Ex 1.6

22 Write 'forty-two hundred' in symbols. Ex 1.1

23 The match fee for a game of Rugby in the under-14s is \$8. Hassan played 14 games in the season. How much did he pay during the season? Ex 1.4

24 It takes about 17 minutes for an officer in the Australian Tax Office to process a handwritten tax return. About how long should be allowed to process 347 tax returns? Ex 1.6

25 Change to a number sentence: 'The pass mark for a learner's licence is 27 out of 30.' Ex 1.2

26 A book is 14 mm thick. How many books would be needed to make a stack 120 cm high? Ex 1.4

27 A landscaping company estimates that about 12 paving bricks are wasted from every pallet of 250 bricks. Roughly how many bricks will be wasted for a large paving job that is estimated to need 7780 bricks? Ex 1.6

28 To find your body mass index, you divide your mass (in kg) by your height (in m) squared. What is the body mass index of a 78 kg man of height 170 cm? Ex 1.7

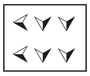
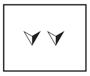
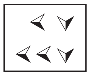
29 The heights of several places are given as $^{-}120$ m, 85 m and 39 m. Arrange them in order from the lowest to the highest points. Ex 1.8

Project Ancient number systems

Cuneiform characters on this ancient clay tablet from Mesopotamia tally the number of sheep and goats.

Different civilisations developed different number systems. However, all the number systems were based on similar principles: a small number of symbols were used to represent the numbers. It is believed that, originally, people just used scratches to represent numbers. So, to represent 15 cattle, 15 scratches would be used. As people started to trade with one another, more sophisticated number systems were needed.

The Babylonians developed a very early number system using marks made in clay tablets, which were then baked to preserve the results. The marks were made by pressing a wedge-shaped *stylus* into the clay. Vertical marks were used for units and placed horizontally for tens. The later Babylonian system was based on groups of 60 by separating the marks into groups.

For example,  would mean 24, but   would mean 2 lots of 60 and 32, so altogether it would be 152.

The Egyptian system of writing numbers could have been influenced by the Babylonians, but was adapted for carving in stone, wood and so on. It was also used with pen and ink on papyrus. The early Egyptian symbols repeated the following symbols to represent numbers.

1	10	100	1000	10 000	100 000	1 000 000
						
Vertical staff	Heel mark	Coiled rope	Lotus flower	Pointing finger	Fish	Astonished man

In later times it appears that the Egyptians replaced some repeated strokes by symbols to represent a digit. For example, |||| was replaced by — to represent the number 4. The use of symbols for digits makes the writing of numbers much quicker and easier to understand.

The Greek and Roman civilisations used symbols for 1, 5, 10, 50, 100, 500 and 1000. In later Roman times, particular letters from the alphabet were used. Both the Greeks and the Romans used repetition, although the Romans later used subtraction as well as addition to simplify the writing of numbers.

Greek numerals

1	5	10	50	100	500	1000	10 000
I	Γ	Δ	𐅄	H	𐅅	X	M

Roman numerals

1	5	10	50	100	500	1000
I	V	X	L	C	D	M

In early Roman times, the number 4 was represented as IIII, while 6 was shown as VI. In the later Roman system, the number 4 was changed to IV (‘one before five’), but 6 was still shown as VI (‘one after five’).

Task 1

a Show how large and small numbers like 47 and 3258 would be written by different ancient civilisations.

b In each system, find which numbers needed the greatest numbers of symbols to represent them (in the Roman system, 68 = LXVIII needs six symbols).

c Use the library to find out more about the later Egyptian numbers.

The ancient civilisation had to work out addition, multiplication, subtraction and division problems with their number systems. In Roman times, a boy would spend many hours learning his numbers and how to work out simple problems. In our system, you only need to learn number facts up to 9s and can use these to work out other problems. In Roman times, you needed to learn many more tables to solve simple arithmetic problems.

The Romans, and many other civilisations, used the counting of beads of some kind in order to solve problems. These systems developed into the *abacus*, similar to a modern child’s counting frame. In some cases, beans were moved backwards and forwards across a line.

Task 2

a Construct a simple abacus that can be used to add four-digit numbers.
b Show how two numbers are added on your abacus.
c Show how two numbers are subtracted on your abacus.

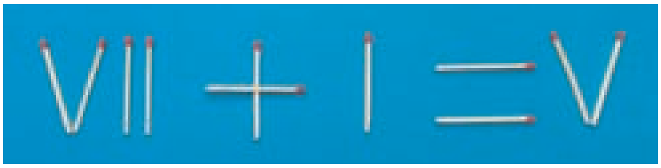
Multiplication and division are very difficult to perform using the Roman system of numbers. They did not use multiplication tables, but instead used either the abacus or the system of 'duplation and mediation', which means halving and doubling.

Task 3

a Show how to multiply using an abacus.
OR
b Show how two numbers are multiplied using duplation and mediation.

Since many of the Roman numbers can be shown using matchsticks, a large number of modern puzzles have been made using Roman numbers.

The puzzle below has to be solved by shifting one match to make a true statement.



By shifting one match, we can make VII – I = VI

We can make up many such puzzles, with instructions such as 'move one match', 'move two matches' and so on.

Task 4

a Make up some matchstick puzzles using Roman numbers.
b You must also supply the answers!

Report

Your teacher will tell you which task/s you should undertake for this research activity. You may also have to write a report about your research.