The distributive law: Removing brackets 1A

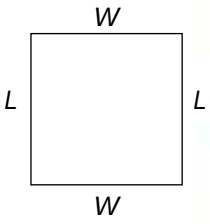
The rule for calculating the perimeter of the rectangle shown is:

Perimeter = $2L + 2W$

Alternatively, we could add the length L to the width W and take twice the total. As an expression this is $2 \times (L + W)$.

The two expressions are the same, and so

$2 \times (L + W) = 2L + 2W$



This illustrates the **distributive law**, in which the term outside the bracket multiplies each term inside the bracket:

$a(b + c) = ab + ac$ and $a(b - c) = ab - ac$

The term a can also come after the bracket so that we would have:

$(b + c)a = ba + ca$ and $(b - c)a = ba - ca$

Example

Expand the following expressions (i.e. remove the brackets):

a $5(x + 4)$

Solution

$5(x + 4)$
$= 5 \times x + 5 \times 4$
$= 5x + 20$

b $6x\ (2x - 5)$

$6x\ (2x - 5)$
$= 6x \times 2x - 6x \times 5$
$= 12x^2 - 30x$

c $7x + 5(x + 2)$

$7x + 5(x + 2)$
$= 7x + 5x + 10$
$= 12x + 10$

Exercise 1A

1 Complete the following:

a $4 \times (7 + 3)$
$= 4 \times$ .....
= .....

b $4 \times 7 + 4 \times 3$
= ..... + ...
= .....

c $8 \times (12 - 4)$
$= 8 \times$ .....
= .....

d $8 \times 12 - 8 \times 4$
= ..... + ...
= .....

2 Expand (i.e. remove brackets) each of the following:

a $3(x + y)$ **b** $7(a + b)$ **c** $7(m + n)$
d $9(x + 4)$ **e** $6(y + 7)$ **f** $10(p + 8)$
g $(a + 4)8$ **h** $(c + 5)9$ **i** $(b + 9)5$
j $(6 + p)6$ **k** $(12 + q)7$ **l** $(5 + n)8$

3 Expand each of the following:

a $5(x - y)$ **b** $7(a - b)$ **c** $9(m - n)$
d $8(x - 7)$ **e** $3(x - 9)$ **f** $9(x - 12)$
g $2(13 - m)$ **h** $8(7 - q)$ **i** $4(9 - p)$
j $(7 - x)8$ **k** $(18 - b)3$ **l** $(15 - m)4$

4 Expand each of the following:

a $x(y + z)$ **b** $m(p + q)$ **c** $a(b + c)$
d $r(s - t)$ **e** $p(q - r)$ **f** $l(m - n)$
g $m(n - 8)$ **h** $a(b - 12)$ **i** $z(y - 8)$
j $c(9 - b)$ **k** $p(14 - q)$ **l** $m(9 - n)$

5 Expand each of the following:

a $8(3x + 2)$ **b** $5(4y + 6)$ **c** $7(9b + 4)$
d $5(7q - 6)$ **e** $9(6p - 12)$ **f** $8(3s - 7)$
g $4(2x + 5y)$ **h** $5(6m + 9n)$ **i** $7(8a + 12b)$
j $(3a - 7b)6$ **k** $(8m - 6n)11$ **l** $(2m - 3n)9$

6 Expand each of the following and simplify as far as possible:

a $3(2x + 6) + 9$ **b** $7(7y - 1) + 10$
c $4 + 5(3m + 6)$ **d** $30 + 9(b - 2)$
e $4(2a + 3b + 4c) + 5b$ **f** $4x(3x + 2)$
g $2x(3x + 2) + 3(3x + 2)$ **h** $6y(5y - 8)$
i $3x(2x + 5) + 4x(3x - 2)$ **j** $5p(4p + 2m) - 20p^2 - 8mp$

7 Write an algebraic expression for the following using brackets, then remove the brackets.

a I think of a number n, add 3 and then multiply the total by 6.
b I think of a number n, subtract 4, then multiply the result by 7.
c I think of a number n, double it, add 3, then multiply the result by 4.
d I think of a number n, multiply it by 4, subtract 6, then multiply the result by 3.
e I think of a number n, halve it, subtract 3, then multiply the result by 4.

8 On a particular day a adults and c children attend a swimming pool:

a Write an expression for the total number of people attending the pool.
b If adults pay \$3 each to enter and children pay \$1·50, write an expression for the total amount collected by the pool.
c Assuming the attendance pattern is the same each day, write an expression using brackets for the pool's takings over a 5-day period.
d Remove the brackets from you expression in part c.

Rules and formulas 1B

When we use an identifying symbol and an equals sign before an expression, the result becomes a formula. A **formula** explains the process for working out a mathematical result.

For example, we have developed $2L + 2W$ as the expression for finding the perimeter of a rectangle.

$P = 2L + 2W$ is the formula that works out the perimeter of a rectangle.

P (perimeter) is the **subject** of the formula. The pronumerals in any formula are often referred to as **variables** because the values we substitute for them usually vary, depending on the situation.

Example

1 Use the formula to find the perimeter of this rectangle:

Solution

$P = 2L + 2W$
$P = 2 \times 4 + 2 \times 3$
$P = 8 + 6$
$P = 14$ metres

2 The cost C ($) of hiring a surfboard for t hours is given by the formula:

$C = 10 + 3t$

where there is a $10 initial charge, plus a charge of $3 for each hour the board is in use.

Find the cost of hiring the board for:

a 1 hour

$C = 10 + 3 \times 1$
$= \$13$

It costs $13 to hire the board for 1 hour.

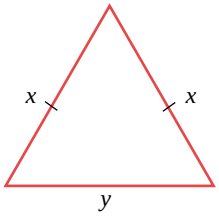
b 4 hours

$C = 10 + 3 \times 4$
$= \$22$

It costs $22 to hire the board for 4 hours.

Exercise 1B

1 The perimeter of the isosceles triangle shown is given by the formula $P = 2x + y$:



a Find the perimeter where $x = 5$ cm and $y = 3$ cm.
b Find the perimeter if $x = 8$ m and $y = 4$ m.

2 The area A of a rectangle of length L and width W is given by the formula $A = LW$:

a Find the area of a rectangle where $L = 12$ cm and $W = 9$ cm.
b Find the area of a rectangle where $L = 6$ m and $W = 7$ m.

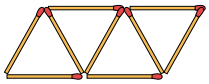
3 The number of points P gained by a football team is given by the formula $P = 6g + b$, where g is the number of goals kicked and b is the number of behinds kicked.

a Find the points score for a team that has scored:

i 5 goals and 16 behinds **ii** 8 goals and 9 behinds

b If a team scored 20 points, what possible numbers of goals and behinds could have been kicked?

4 The number of matches N needed to tessellate T equilateral triangles in a straight line is given by $N = 1 + 2T$:

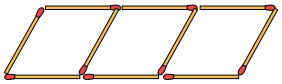


a How many matches are needed to tessellate 5 triangles?

b How many matches are needed to tessellate 20 triangles?

c How many triangles can be tessellated with 33 matches?

5 The diagram shows a linear tessellation of rhombi.



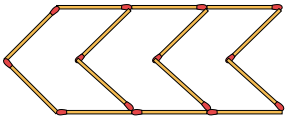
a How many matches are needed to tessellate:

i four rhombi? **ii** five rhombi?

b List as a sequence the number of matches needed for 1, 2, 3 … 10 rhombi.

c Can you find the formula that works out N the number of matches needed to tessellate R rhombi? Test your formula using your results to part a above.

6 The diagram shows a linear tessellation of an arrow.



a How many matches are needed to tessellate:

i four arrows? **ii** five arrows?

b List as a sequence the number of matches needed for 1, 2, 3 ... 10 arrows.

c Can you find the formula that works out N the number of matches needed to tessellate A arrows? Test your formula using your results to part a above.

7 The cost in dollars of printing programs for the school production is made up of a fixed 'set up' cost of \$50 plus \$0·50 for each program printed.

a Find the cost of printing:

i 10 programs **ii** 25 programs

iii 50 programs **iv** n programs

b Write a formula which works out the cost C of printing n programs.

c Check your formula by using it to find the cost of 10 programs.

d Use your formula to find the cost of printing 150 programs.

8 A student starts saving for a new skateboard. She opens an account with a \$30 birthday gift and adds \$5 to this each week from her pocket money.

a How much has she saved after:

i 2 weeks **ii** 5 weeks

iii 8 weeks **iv** n weeks

b Write a formula which works out A the amount saved after n weeks.

c Check your formula by using it to find A when n equals 5.

d How long will it take her to save \$135, the cost of the skateboard?

Solving equations with flow charts 1C

This method displays the steps used to compile an equation. It is used where the unknown pronumeral appears *only once* in the equation. The solution is obtained by reversing the steps.

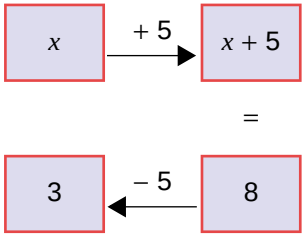
Example

Solve the following:

1 $x + 5 = 8$

Solution

Display the flow chart:

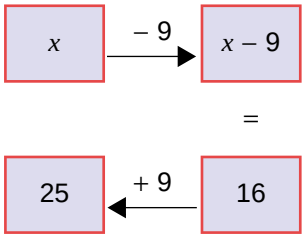


Reverse the flow and use 'inverse' or opposite operations

The solution is then $x = 3$, which is easily checked by inspection.

2 $x - 9 = 16$

Display the flow chart:



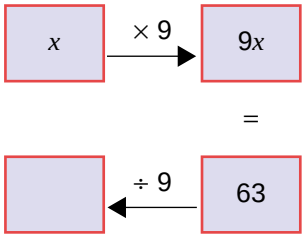
Reverse the flow and use 'inverse' or opposite operations

The solution is then $x = 25$, which is easily checked by inspection.

Exercise 1C

For questions 1–3, complete the details to solve the equations.

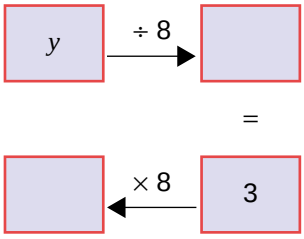
1 $9x = 63$ Complete the flow chart:



Reverse the flow and use 'inverse' or opposite operations

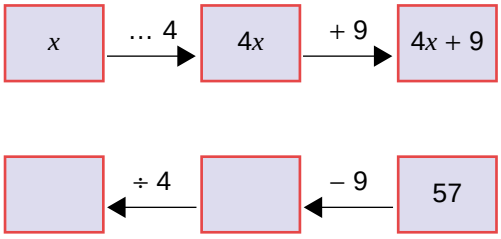
The solution is then $x =$ ……

2 Solve $\frac{y}{8} = 3$ Complete the flow chart:



The solution is then $y =$ ……

3 $4x + 9 = 57$ Complete the flow chart:



The solution is then x = ……

4 Solve each of the following equations by first drawing a flow chart and then reversing the flow:

a $p + 8 = 19$
b $10 + a = 26$ (hint: rewrite as $a + 10 = 26$)
c $x + 10 = 17$
d $y + 15 = 31$
e $r - 9 = 13$
f $a + 8 = 20$
g $b - 14 = 10$
h $m - 6 = 9$

5 Solve each of the following equations by drawing a flow chart and then reversing the flow:

a $x \times 6 = 18$
b $y \times 8 = 40$
c $m \times 12 = 108$
d $9a = 54$
e $7b = 35$
f $8c = 32$
g $\frac{p}{7} = 4$
h $\frac{q}{9} = 3$
i $\frac{r}{12} = 5$
j $\frac{m}{9} = 4$
k $\frac{n}{3} = 13$
l $\frac{p}{8} = 12$
m $\frac{c}{4} = 3$
n $\frac{a}{5} = 11$
o $\frac{p}{7} = 13$

6 Solve each of the following equations, which have several steps, by drawing a flow chart and reversing the flow:

a $2x + 9 = 15$
b $4y + 6 = 30$
c $7a + 4 = 67$
d $3m - 5 = 13$
e $8n - 10 = 78$
f $9p - 8 = 28$
g $2x + 4 = 10$
h $2y + 6 = 0$
i $6a + 5 = 17$
j $5m + 1 = 11$
k $2n - 10 = 12$
l $9p - 2 = 34$
m $2x + 5 = 7$
n $4y + 6 = 46$
o $7a + 4 = 39$

7 Write equations for the following statements and then solve them by using a flow chart:

a I think of a number x, multiply it by 5 and the result is 20.
b I think of a number y, divide it by 8 and the result is 6.
c I think of a number n, multiply it by 8, then add 5 to get a result of 37.
d I think of a number q, multiply it by 9 then subtract 8 and get a result of 55.
e I think of a number m, multiply it by 6, then subtract 2 to get a result of 40.
f I think of a number s, multiply it by 9 then subtract 1 and get a result of 80.

Solving equations by inverse operation 1D

The inverse operations used in solving an equation when using a flow chart can also be applied directly to solve it using algebra.

Example	Solution
1 I think of a number x and add 5 to get a result of 11. The equation is $x + 5 = 11$	We subtract 5 from *both sides* to keep the 'balance': $x + 5 = 11$ $-5 \quad -5$ Solution is $x = 6$
2 I think of a number n and subtract 7 to get a result of 13. The equation is $n - 7 = 13$	We add 7 to both sides: $n - 7 = 13$ $+7 \quad +7$ Solution is $n = 20$
3 A certain number y is multiplied by 8 to get a result of 32. The equation is $8y = 32$	We divide both sides by 8: $\frac{8y}{8} = \frac{32}{8}$ Solution is $y = 4$
4 A certain number q divided by 12 gives a result of 8. The equation is $\frac{q}{12} = 8$	We multiply both sides by 12: $\frac{q}{12} \times 12 = 8 \times 12$ Solution is $q = 96$

Exercise 1D

1 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples:

a $x + 4 = 16$ **b** $13 + y = 24$ (hint: rewrite as $y + 13 = 24$) **c** $m - 7 = 15$ **d** $n - 19 = 6$ **e** $18 = p - 5$ **f** $p - \frac{1}{2} = 3\frac{1}{4}$ **g** $q + 2{\cdot}5 = 4{\cdot}75$ **h** $t - 3{\cdot}25 = 1{\cdot}75$

2 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples. Note that part g onwards will involve answers which are simple fractions:

a $4x = 12$ **b** $6y = 72$ **c** $9n = 45$ **d** $3g = 33$ **e** $11p = 44$ **f** $5m = 60$ **g** $2r = 5$ **h** $3s = 7$ **i** $7t = 11$ **j** $4z = 10$ **k** $8q = 44$ **l** $6h = 22$ **m** $10x = 5$ **n** $12y = 8$ **o** $24z = 18$

3 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples:

a $\frac{x}{9} = 8$ **b** $\frac{a}{13} = 3$ **c** $\frac{b}{7} = 12$

d $\frac{p}{6} = 16$ **e** $\frac{q}{15} = 7$ **f** $\frac{r}{18} = 3$

g $\frac{m}{4} = 2\frac{1}{2}$ **h** $\frac{n}{3} = 1{\cdot}5$ **i** $\frac{p}{4} = 2\frac{3}{4}$

4 Six popcorns at Luna Park cost me \$19·50.

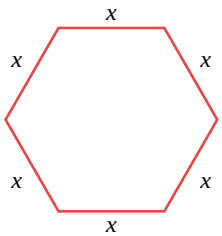
a Let p be the cost of one popcorn, then write an equation and solve it for p.

b Let c be the change obtained from \$50, then write an equation and solve it for c.

5 When the price of a movie ticket P is divided by 11 to calculate the amount of GST paid, the result is \$1·05. Write an equation for P and solve it to find the price of the movie ticket.

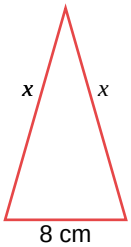
6 Using the perimeters of the following figures, write equations and solve to find x.

a



Perimeter is 216 cm

b



Perimeter is 38 cm

Solving two- and three-step equations 1E

Equation with more than one operation can be solved by using the same method of opposite operations.

Example | **Solution**

1 I think of a number x, multiply it by 3, then add 2 to get a result of 14.

The equation is $3x + 2 = 14$

Note the order of the steps: Working from x, we multiplied by 3 then added 2.

$3x + 2 = 14$

Subtract 2 from both sides: $-2 \quad -2$

$3x = 12$

Divide both sides by 3: $\frac{3x}{3} = \frac{12}{3}$

$x = 4$

2 I think of a number y , multiply it by 3, then divide the result by 4 to get an answer of 6.

The equation is: $\frac{3y}{4} = 6$

or, alternately, $\frac{3}{4}y = 6$

$\frac{3y}{4} = 6$

Multiply both sides by 4: $\frac{3y}{4} \times 4 = 6 \times 4$

$3y = 24$

Divide both sides by 3: $\frac{3y}{3} = \frac{24}{3}$

$y = 8$

3 I think of a number x, multiply it by 5, divide the result by 4 and finally add 6 to get an answer of 16.

The equation is: $\frac{5x}{4} + 6 = 16$

or alternately: $\frac{5}{4}x + 6 = 16$

$\frac{5x}{4} + 6 = 16$

Subtract 6 from both sides $-6 \quad -6$

$\frac{5x}{4} = 10$

Multiply both sides by 4: $\frac{5x}{4} \times 4 = 10 \times 4$

$5x = 40$

Divide both sides by 5: $\frac{5x}{5} = \frac{40}{5}$

$x = 8$

Exercise 1E

1 Solve the equations below:

a $3x + 4 = 31$ **b** $8y - 9 = 31$ **c** $9m - 3 = 60$

d $7 + 3n = 19$ **e** $10 + 5q = 35$ **f** $15 + 12a = 39$

g $18 + 6b = 36$ **h** $45 + 5c = 80$ **i** $5x - 7 = 33$

k $2r + 3 = 4$ **l** $6s + 5 = 14$ **m** $7a - 6 = 14$

2 Solve these two-step equations:

a $\frac{5x}{4} = 10$ **b** $\frac{3y}{4} = 9$ **c** $\frac{2z}{3} = 4$

d $\frac{3}{5}a = 6$ **e** $\frac{2}{9}b = 8$ **f** $\frac{5}{2}c = 10$

g $\frac{2}{3}m = 5$ **h** $\frac{4}{5}n = 2$ **i** $\frac{3p}{2} = 4$

3 Solve the following equations:

a $\frac{x}{6} + 5 = 7$ **b** $\frac{y}{7} + 9 = 12$ **c** $\frac{z}{2} - 6 = 8$

d $\frac{1}{8}m + 7 = 9$ **e** $\frac{1}{4}n - 2 = 4$ **f** $\frac{1}{12}p - 1 = 5$

g $\frac{q}{2} + \frac{3}{4} = 2$ **h** $\frac{1}{4}r - \frac{1}{2} = 1$ **i** $\frac{1}{3}s + 2 = 3\frac{2}{3}$

4 Solve these equations:

a $\frac{2x}{3} + 1 = 5$ **b** $\frac{3a}{4} + 2 = 8$ **c** $\frac{4n}{3} + 3 = 15$

d $\frac{5n}{3} - 1 = 9$ **e** $\frac{4p}{5} + 1 = 9$ **f** $\frac{2q}{7} - 3 = 1$

g $\frac{5}{6}m - 7 = 3$ **h** $\frac{2}{5}z + 4 = 10$ **i** $\frac{3}{8}t - 4 = 2$

5 Write an equation for each of the following statements and then solve it:

a I think of a number x, multiply it by 5, then subtract 15 to get a result of 50.

b Four times a certain number x divided by 5 gives a result of 8.

c A certain number y is divided by 5, then 7 is added to give a result of 12.

d Three times a certain number y is divided by 9, then 3 is subtracted to give a result of 2.

6 Ski-pass day tickets on Mount Buller normally sell for d dollars each. On a particularly bleak day, the company decided to give a school group of 10 students a group discount of $75. If the total amount paid is $375, write an equation involving d and solve it to find the normal price d of a ski ticket.

7 At the kiosk near the ski run I bought a drink, which cost $3 dollars, and four doughnuts for a total of $8. If p is the price of a doughnut, write an equation involving p and solve it to find the cost of a single doughnut.

8 For a group of students to stay at a ski lodge, a special deal is negotiated so that the group pays only three-quarters of the usual price $\$P$. If the group pays $126, write an equation involving P and solve it to find the usual price.

9 At the bottom of the ski slope there are 20 people in the queue waiting to catch the ski tow. Each minute the tow clears 12 people from the queue and 15 people join it. If t is the time required for the queue to grow to 71 skiers, write an equation involving t and solve it.

Inequations 1F

An **inequation** occurs where two algebraic expressions are related by one of the following four **inequality signs**:

$<$ meaning 'less than'	$\leq$ meaning 'less than or equal to'
$>$ meaning 'greater than'	$\geq$ meaning 'greater than or equal to'

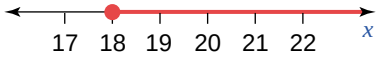
Some simple inequation statements are $3 < 7$ and $8 > 6$. Note that the inequality signs always point to the smaller of the two quantities, or equivalently the wider end opens up beside the larger of the two quantities.

Example

1 If x is the age at which a person can obtain a driving licence in Victoria, write an inequation for x and illustrate the solution on a number line.

Solution

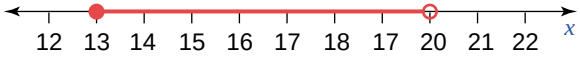
$x \geq 18$



Note that a 'filled in' or 'closed' end circle indicates that the end number is included.

2 If x is the age at which you are called a teenager, write an inequation for x and show the solution on a number line.

x has to be 13 or more but less than 20. Reading from the centre to the left and then to the right, we have $13 \leq x < 20$



Note that an 'empty' or 'open' end circle indicates that the end number is not included.

3 A number y is multiplied by 3, then 2 is subtracted to give a result which is 10 or more.

Write the inequation and solve it, showing the solution on a number line.

We apply the previously covered algebraic steps:

Start with $3y - 2 \geq 10$

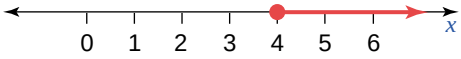
$+2 \quad +2$

$3y \geq 12$

$\frac{3y}{3} \geq \frac{12}{3}$

$y \geq 4$

That is, any number which is 4 or more satisfies the inequation.



Exercise 1F

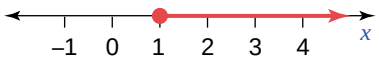
1 Use the symbols $<$, $>$ or $=$ to correctly complete the following statements:

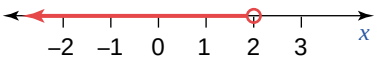
a $15 + 12 \ldots\ldots 2 \times 5$
b $3 \times 6 \ldots\ldots 40 \div 2$
c $4 + 8 \ldots\ldots 6 \times 2$
d $16 - 5 \ldots\ldots 24 \div 3$
e $4 \times 0{\cdot}5 \ldots\ldots 20 \div 10$
f $36 + 27 \ldots\ldots 6 \times 7$
g $9 \times 12 \ldots\ldots 216 \div 2$
h $4^2 + 1 \ldots\ldots 20 - 2$

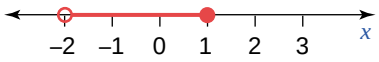
2 Draw a number line to represent the set of possible values for x:

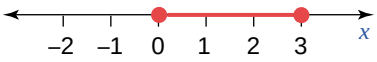
a $x > 3$ **b** $x < -1$ **c** $3 < x \leq 6$ **d** $-1 \leq x < 5$

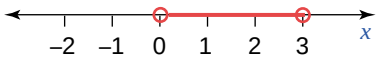
3 Write down the correct inequality for x which is represented by the following number lines:

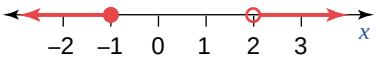
a


b


c


d


e


f


4 Write each of the following statements as an inequation, and show the solution on a number line:

a x is less than 10
b y is more than 8
c z is 5 or less
d m is 7 or more
e x is greater than 3 but less than 6
f x is between 4 and 8
g x is 7 or more but less than 10
h y is greater than 5 but less than or equal to 11
i The price of petrol p varies from 90 cents per litre to 98 cents per litre inclusive.
j In NSW the age a at which you can get a driving licence is 17 years or more.
k The age a of the people attending the blue light disco on a particular Saturday night was more than 13 but less than or equal to 20.
l The time taken to get to school t varies from 20 to 30 minutes inclusive.
m The speed s at which I can ride my bike varies from zero to 35 kilometres per hour inclusive.
n The speed at which I may drive my car in the suburbs is any speed greater than zero but not more than 50 kilometres per hour.
o The discount d I receive when I pay cash can be anything from 5 to 10 per cent inclusive.

5 Solve the following inequations:

a $x + 5 < 9$ **b** $y - 8 > 10$ **c** $z - 15 \geq 16$
d $m + 7 \leq 12$ **e** $n + 19 < 21$ **f** $p - 17 \leq 3$
g $4x \geq 20$ **h** $12y > 72$ **i** $6z < 33$
j $9m \leq 36$ **k** $5p < 45$ **l** $10q > 55$
m $\frac{x}{7} \geq 5$ **n** $\frac{y}{13} \leq 4$ **o** $\frac{z}{15} > 4$
p $\frac{1}{4}m < 6$ **q** $\frac{1}{2}n \geq 9$ **r** $\frac{1}{3}p < 8$
s $\frac{2x}{3} \leq 12$ **t** $\frac{3y}{5} > 10$ **u** $\frac{4z}{3} < 8$
v $\frac{2}{5}a \leq 4$ **w** $\frac{7}{6}b < 14$ **x** $\frac{3}{4}c \geq 6$

6 Solve the following inequations by using the setting out of earlier sections:

a $4x + 2 \geq 8$	**b** $5 + 2y < 13$	**c** $6z - 2 \leq 22$
d $2a + 5 < 12$	**e** $10b + 9 \geq 24$	**f** $5c - 8 < 15$
g $\frac{m}{7} + 5 \geq 8$	**h** $\frac{n}{6} - 9 < 2$	**i** $\frac{p}{15} - 3 \geq 2$
j $\frac{1}{4}x + 2 < 3$	**k** $\frac{1}{8}y - 9 \geq 1$	**l** $\frac{1}{10}z - 5 \leq 4$
m $\frac{2x}{3} + 4 \leq 14$	**n** $\frac{3y}{5} - 2 > 7$	**o** $\frac{4z}{3} - 7 < 5$
p $\frac{2}{5}a + 3 \leq 5$	**q** $\frac{7}{6}b - 2 < 12$	**r** $\frac{3}{4}c - 9 \geq 3$

7 Write an inequation for each of the following statements and then solve it. Illustrate your solution on a number line:

a I think of a number x, multiply it by 5 and then subtract 12 to get a result greater than 18.

b Four times a certain number x divided by 5 gives a result less than 16.

c A certain number y is divided by 4, then 7 is added to give a result of 9 or more.

d Three times a certain number y is divided by 4, and 1 is subtracted to give a result greater than 8.

8 When I withdraw \$100 from my bank account, the balance is still in excess of \$175. Letting b be the original balance, write an inequation for b and solve it.

9 I have d dollars in my pocket and my friend has 6 dollars more than me. Together we have less than 16 dollars. Write an inequation for d and solve it.

10 The time t minutes taken for me to get to school is usually greater than 15 minutes, but is no more than 25 minutes. Write an inequation for t and show its solution on a number line.

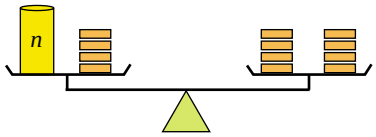
11 Free tickets for the carnival ride are given to people who are either less than 15 years of age or at least 60 years of age. Let a be a person's age in years, and write an inequality for a. Display the solution on a number line.

Applications and Activities

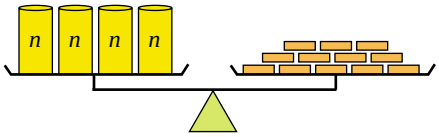
In this section we will solve equations using 'balance diagrams'. Balances of this type have been used in banks to manually count large numbers of coins.

Example

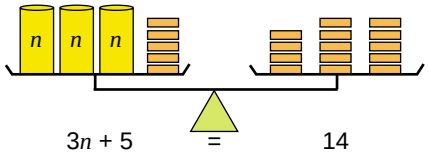
1 The equation $n + 4 = 8$ can be represented by a lightweight container of n dollar coins plus four separate coins on the left, balanced by eight coins on the right.



2 The equation $4n = 12$ can be represented with four lightweight containers of n dollar coins on the left, balanced by 12 coins on the right.

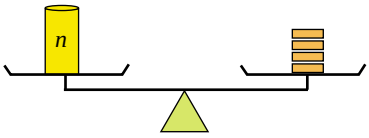


3 The quation $3n + 5 = 14$ can be repesented by the following balance diagram:

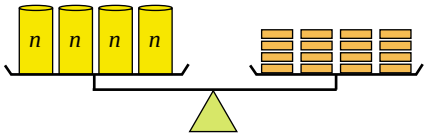


Solution

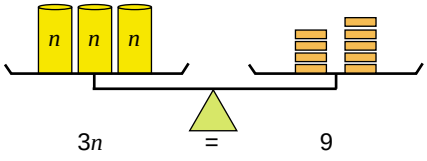
Subtracting 4 (coins) from both sides, just as you would with the equation, gives the solution $n = 4$.



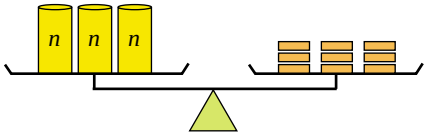
Dividing both sides into groups of 4, just as you would divide both sides of an equation by 4, gives the solution $n = 3$.



Subtracting 5 from both sides:



and then dividing each side into groups of 3:



gives the solution $n = 3$.

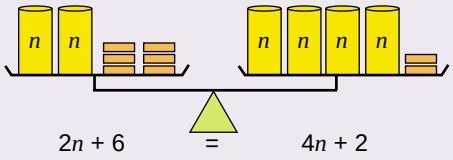
1 Solve the following equations by drawing balance diagrams:

a $n + 7 = 13$ **b** $n + 8 = 12$
c $6 + x = 15$ **d** $4y = 16$
e $5x = 35$ **f** $8n = 24$

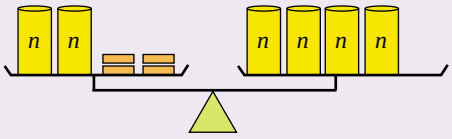
2 Solve the following equations by drawing balance diagrams:

a $3x + 4 = 22$ **b** $2n + 3 = 17$
c $6n + 4 = 16$ **d** $5 + 2n = 11$
e $9 + 4x = 13$ **f** $10 + 5n = 20$

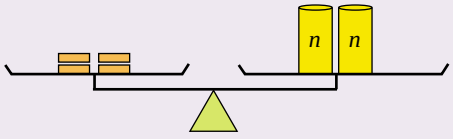
3 Write equations to represent the following sequence of balance diagrams, then solve the equation $2n + 6 = 4n + 2$



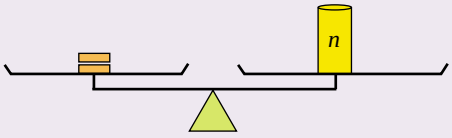
Take 2 from both sides:



Take $2n$ from both sides:



Divide both sides into groups of 2:



4 Solve the following equations using balance diagrams:

a $n + 6 = 3n + 2$ **b** $2n + 1 = n + 5$ **c** $2n + 6 = 3n + 2$
d $4n + 6 = 2n + 12$ **e** $4n + 8 = 6n$ **f** $6n = 5n + 6$

5 Write an equation for each of the following statements and solve it:

a Ten more than a certain number is equal to 3 times that number.
b If I add 8 to a certain number the value is equal to 5 times that number.
c I think of a number n, multiply it by 7 and then add 5. The answer is equivalent to 8 times the number I thought of plus 2.
d Six times a certain number x plus 5 gives the same answer as 3 times the same number plus 17.

Enrichment and Extension

1 Solve the following equations and verify your solutions by substitution:

a $10 - 3x = 4$ **b** $45 - 2y = 9$ **c** $52 - 8m = 12$

d $45 - 4n = 25$ **e** $39 - 6p = 15$ **f** $125 - 17q = 74$

g $90 - 5n = 35$ **h** $20 - 6p = 8$ **i** $185 - 10q = 105$

2 Solve the following equations and verify your solutions by substitution:

a $55 - 6(x + 3) = 1$ **b** $38 - 3(y - 5) = 20$ **c** $67 - 7(z - 5) = 18$

d $23 - 2(2a + 1) = 9$ **e** $35 - 3(3b - 2) = 5$ **f** $100 - 7(3c + 4) = 9$

g $15 - 3(2a - 1) = 0$ **h** $50 - (5b - 2) = 17$ **i** $60 - (4c + 1) = 47$

3 Solve the following equations and verify your solutions by substitution:

a $40 - 3(10 - 2a) = 34$ **b** $30 - 5(20 - 4b) = 10$ **c** $55 - 9(34 - 6c) = 19$

d $\frac{3(20-4m)}{2} - 3 = 12$ **e** $\frac{4(30-6y)}{3} + 5 = 13$ **f** $\frac{3(28-4z)}{10} + 8 = 14$

g $\frac{3(50-4m)}{2} - 3 = 42$ **h** $\frac{2(45-5y)}{3} + 2 = 22$ **i** $\frac{3(75-5z)}{5} + 5 = 20$

4 Solve the following equations and verify your solutions by substitution:

a $\frac{28}{2m+1} + 6 = 10$ **b** $\frac{70}{4n-1} + 9 = 11$ **c** $\frac{128}{9p-4} - 3 = 1$

d $\frac{16}{20-3x} + 3 = 5$ **e** $\frac{18}{12-y} - 2 = 0$ **f** $\frac{44}{33-2z} + 5 = 7$

5 Solve the following equations and verify your solutions by substitution:

a $8 - \frac{18}{12-3p} = 5$ **b** $12 - \frac{27}{18-5q} = 3$ **c** $17 - \frac{50}{15-2m} = 7$

d $\frac{16}{22-3x} + 4 = 5$ **e** $\frac{27}{12-y} + 1 = 4$ **f** $\frac{40}{30-2z} + 5 = 9$

6 Write equations for the following statements and solve them to find the unknown pronumeral:

a A certain number x is multiplied by 7 and the result is subtracted from 50. This result is then divided into 32 and 6 is added to get a final result of 10.

b A certain number y is multiplied by 2 and the answer is subtracted from 15. This result is multiplied by 2, then divided by 3 and finally 8 is subtracted to get an answer of 2.

c A certain number z is multiplied by 4, the result is subtracted from 12, and this answer is divided into 8. This value is then subtracted from 9 to give an answer of 7.

Exercise 1C

1 Solve the following equations:

a $x + 6 = 13$ **b** $9 + y = 14$ **c** $z - 7 = 11$

d $m - 9 = 21$ **e** $16 - q = 10$ **f** $20 - p = 12$

g $7m = 56$ **h** $n \times 5 = 45$ **i** $\frac{p}{4} = 3$

j $\frac{q}{7} = 8$ **k** $\frac{36}{r} = 3$ **l** $\frac{28}{s} = 4$

2 Check by substitution whether the given solutions to the following equations are correct:

a $3x - 4 = 11$ $[x = 5]$ **b** $\frac{y}{4} + 5 = 10$ $[y = 20]$

3 Solve the following equations by using a flow chart:

a $m + 6 = 11$ **b** $p - 13 = 7$ **c** $8r = 48$

d $\frac{t}{7} = 3$ **e** $4q + 3 = 19$ **f** $6p - 7 = 41$

Exercise 1D

4 Solve the following equations, this time setting out the appropriate algebraic steps:

a $x + 12 = 21$ **b** $y - 17 = 18$ **c** $9z = 72$

d $8m = 44$ **e** $\frac{m}{5} = 4$ **f** $\frac{n}{8} = \frac{1}{2}$

Exercise 1E

5 Solve the following equations setting out the appropriate algebraic steps:

a $6x + 2 = 50$ **b** $20 + 3y = 32$ **c** $8z - 4 = 68$

d $\frac{5x}{2} = 10$ **e** $\frac{2y}{3} = 6$ **f** $\frac{3}{4}q = 6$

g $\frac{z}{3} + 5 = 7$ **h** $\frac{1}{5}m + 7 = 9$ **i** $\frac{1}{4}n - 1 = 3$

j $\frac{3p}{5} + 8 = 14$ **k** $\frac{4}{5}q + 5 = 13$ **l** $\frac{3}{7}r - 2 = 4$

6 Write an equation for each of the following statements, then solve it:

a I think of a number x, multiply it by 7, and add 4 to get a result of 60.

b Three times a certain number y divided by 4 gives a result of 6.

c A certain number y divided by 2 then added to 5 gives a result of 14.

d Three times a certain number n divided by 2 when 3 is subtracted from it gives a result of 12.

7 At the cinema, a group of 10 students was given a total discount of $20 when they each bought an ice cream. If they paid a total of $25, find the price of an individual ice cream before any discount was applied.

Exercise 1F

8 Use the symbols <, >, or = to correctly complete the following statements:

a $16 + 13 \ldots 4 \times 6$ **b** $4 \times 5 \ldots 80 \div 2$

c $12 \times 6 \ldots 144 \div 2$ **d** $5^2 - 4 \ldots 6 \times 4$

9 Write each of the following statements as an inequation and show the solution on a number line:

a x is less than 13 **b** z is greater than 8

c y is 10 or more but less than 15 **d** m is between 20 and 30

10 Write each of the following statements as an inequation:

a The number of goals g by which I expect my football team to win next weekend is at least 5 but no more than 8.

b The mark m that I expect to get on my next Mathematics test is more than 60% but no more than 100%.

11 Solve the following inequations by using the appropriate setting out:

a $x + 9 < 12$ **b** $z - 13 \leq 8$ **c** $5z > 15$

d $7m > 42$ **e** $\frac{p}{5} \geq 4$ **f** $\frac{1}{4}q \leq 11$

g $\frac{3n}{4} \geq 6$ **h** $\frac{5}{3}t \leq 10$ **i** $\frac{2}{5}s \geq 4$

12 Solve the following inequations employing appropriate setting out:

a $3x + 2 > 17$ **b** $5y - 7 \leq 8$ **c** $\frac{z}{5} - 4 \geq 0$

d $\frac{1}{4}m + 3 > 9$ **e** $\frac{2p}{5} + 9 < 13$ **f** $\frac{3}{4}q - 8 \leq 7$

13 Write an inequation for each of the following statements and then solve it. Illustrate your solution on a number line:

a I think of a number n, multiply it by 6, then add 2 to get a result greater than 50.

b Four times a certain number y divided by 12 gives a result greater than 3.

c A certain number q divided by 9, plus 8 gives a result of 10 or more.

d Twice a certain number p divided by 8, minus 1 gives a result of 1 or less.

14 When I add d dollars to my bank account, which had a balance of $100, the new balance is between $125 and $148. Write an inequation for d and solve it. Show your solution on a number line.