

Mathematics 3A

Sample Semester One Examination Questions

Each sample question includes information linking it to the syllabus. Each is designated as suitable for Section 1 – calculator-free, Section 2 – calculator-assumed, or either.

Question number Section 1 or 2	Syllabus entries Content area	Content sub-heading	Topic description
Question 1 Section 1	1.1.1, 1.1.4, 1.1.7, 1.3.1 Number and algebra	Estimation and calculation Equations and inequalities	Mental strategies, algebraic expressions
Question 2 Section 1	1.1.7 Number and algebra	Estimation and calculation	Simplifying and solving equations
Question 3 Section 1	1.2.2, 1.2.3 Number and algebra	Functions and graphs	Transformations
Question 4 Section 1	1.2.2 Number and algebra	Functions and graphs	Transformations
Question 5 Section 1 or 2	1.2.4 Number and algebra	Functions and graphs	Distinguishing graphs
Question 6 Section 1 or 2	1.3.2 Number and algebra	Equations and inequalities	Solving graphically
Question 7 Section 1 or 2	1.3.5 Number and algebra	Equations and inequalities	Inverse proportion
Question 8 Section 2	1.4.2 Number and algebra	Patterns	Growth and decay
Question 9 Section 2	1.4.1 Number and algebra	Patterns	Recursive functions
Question 10 Section 2	1.4.2 Number and algebra	Patterns	Growth and decay
Question 11 Section 1 or 2	1.5.1, 1.5.2, 1.5.3 Number and algebra	Finance	Knowledge of loan spreadsheets
Question 12 Section 2	1.5.3, 1.5.5 Number and algebra	Finance	Reducible interest
Question 13 Section 1 or 2	2.1.1, 2.1.2, 2.1.3, 2.1.4 Space and measurement	Rate	Distance and displacement
Question 14 Section 1 or 2	3.1.1, 3.1.3 Chance and data	Quantify chance	Sample space, simple probabilities
Question 15 Section 2	3.1.7 Chance and data	Quantify chance	Normal distribution
Question 16 Section 1	3.1.7 Chance and data	Quantify chance	Normal distribution
Question 17. Section 2	3.2.2 Chance and data	Interpret chance	Capture-recapture
Question 18 Section 2	3.3.1 Chance and data	Collect and organise data	Sampling
Question 19 Section 1	3.4.2, 3.4.3, 3.4.9 Chance and data	Represent data	Box-plots, central tendency
Question 20 Section 2	3.4.1, 3.4.4, 3.4.7 Chance and data	Represent data	Grouped data
Question 21 Section 2	3.4.3, 3.4.5, 3.4.7, 3.5.8 Chance and data	Represent data	Comparison of data
Question 22 Section 2	3.4.3, 3.4.5, 3.4.7, 3.5.8 Chance and data	Represent data Interpret data	Comparing data sets

Question 1 (4 marks) Section 1	1.1.1, 1.1.4, 1.1.7, 1.3.1 Number and algebra	Estimation and calculation Equations and inequalities	Mental strategies, algebraic expressions
--------------------------------------	---	--	---

The acceleration due to gravity, g metres/sec² is related to the mass of the earth, M kg, the radius of the earth, R metres, and the gravitational constant, G , according to the formula

$$g = \frac{GM}{R^2}.$$

Estimate M correct to the first significant figure, given that

$$g = 9.8, \quad G = 6.7 \times 10^{-11}, \quad \text{and} \quad R = 6.4 \times 10^6.$$

Question 2 (6 marks) Section 1	1.1.7 Number and algebra	Estimation and calculation	Simplifying and solving equations
--------------------------------------	-----------------------------	-------------------------------	--------------------------------------

Solve the equations:

(a) $\frac{3x^7}{x^9} = 12$ (3 marks)

(b) $3 \times 2^{7x} = 96$ (3 marks)

Question 3 (6 marks) Section 1	1.2.2, 1.2.3 Number and algebra	Functions and graphs	Transformations
--------------------------------------	------------------------------------	----------------------	-----------------

(a) Describe the transformation of the graph of $y = \sqrt{x}$ so the resulting function of the graph is:

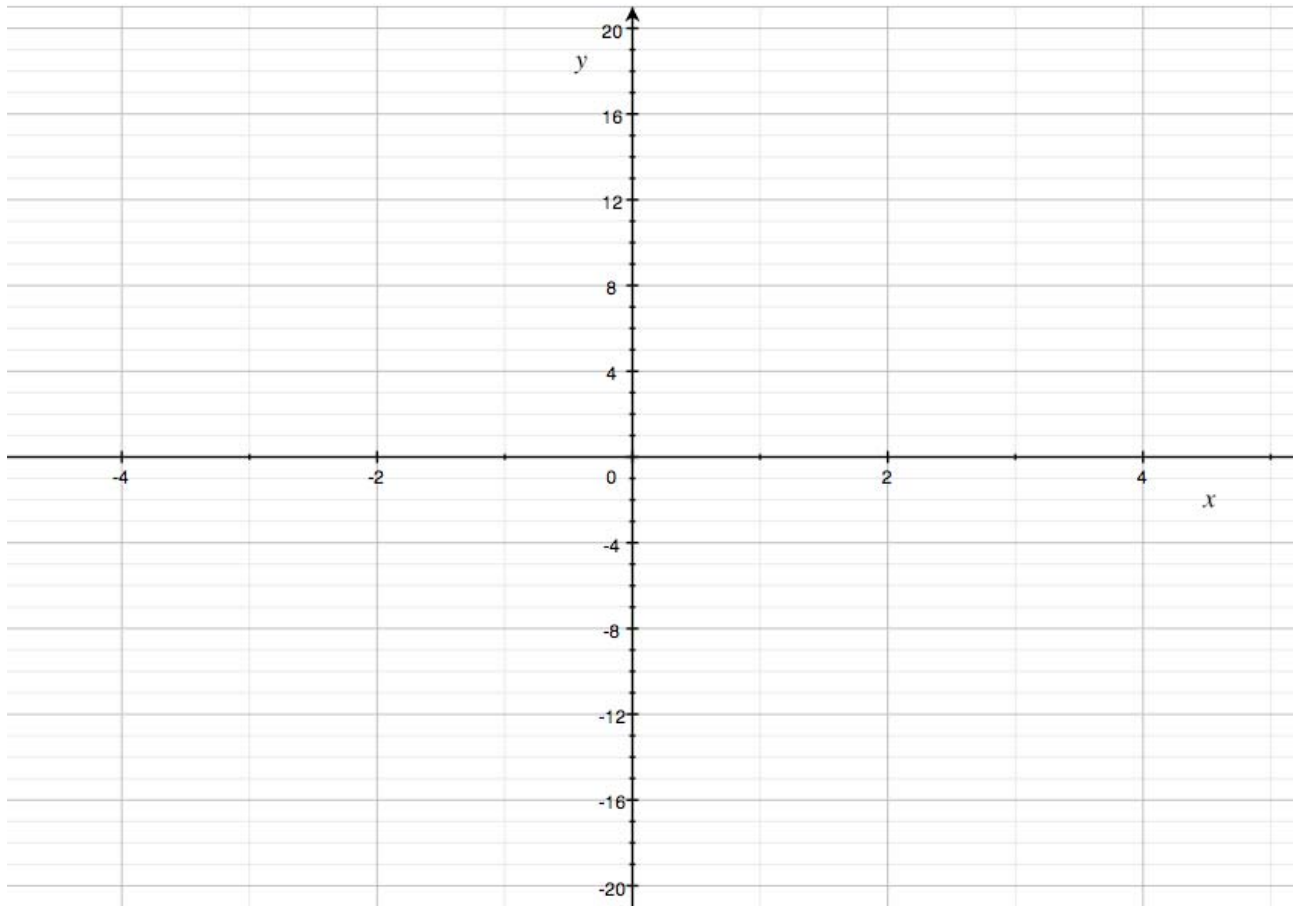
(i) $y = 2\sqrt{x-1}$ (2 marks)

(ii) $y = \sqrt{2x} + 4$ (2 marks)

(b) What are the domain and range of $y = 2\sqrt{x-1}$? (2 marks)

Question 4 (7 marks) Section 1	1.2.2 Number and algebra	Functions and graphs	Transformations
--------------------------------------	-----------------------------	----------------------	-----------------

Draw a neat sketch of the graph of the function, $f(x) = x^2 - 4$ on the axes below.



Find the coordinates of the points where;

- (a) $y = -f(x) + 2$ cuts the y -axis
- (b) $y = f(x + 1)$ cuts the x -axis
- (c) $y = f(1 - x)$ cuts the x -axis
- (d) $y = -f(x) + 3$ has its local maximum

Question 5 (5 marks) Section 1 or 2	1.2.4 Number and algebra	Functions and graphs	Distinguishing graphs
---	-----------------------------	----------------------	-----------------------

Given that $a, b, c, \dots, k$ are all positive real numbers, match each graph with one of the equations shown below:

$$y = a^x$$

$$y = \frac{-b}{x}$$

$$y = c(x-d)^2$$

$$y = fx + g$$

$$y = h(x-i)(x-j)(x+k)$$

$$y = a^{-x}$$

$$y = \frac{b-1}{x}$$

$$y = c(x+d)^2 - e$$

$$y = fx - g$$

$$y = h(x-i)(x+j)^2$$

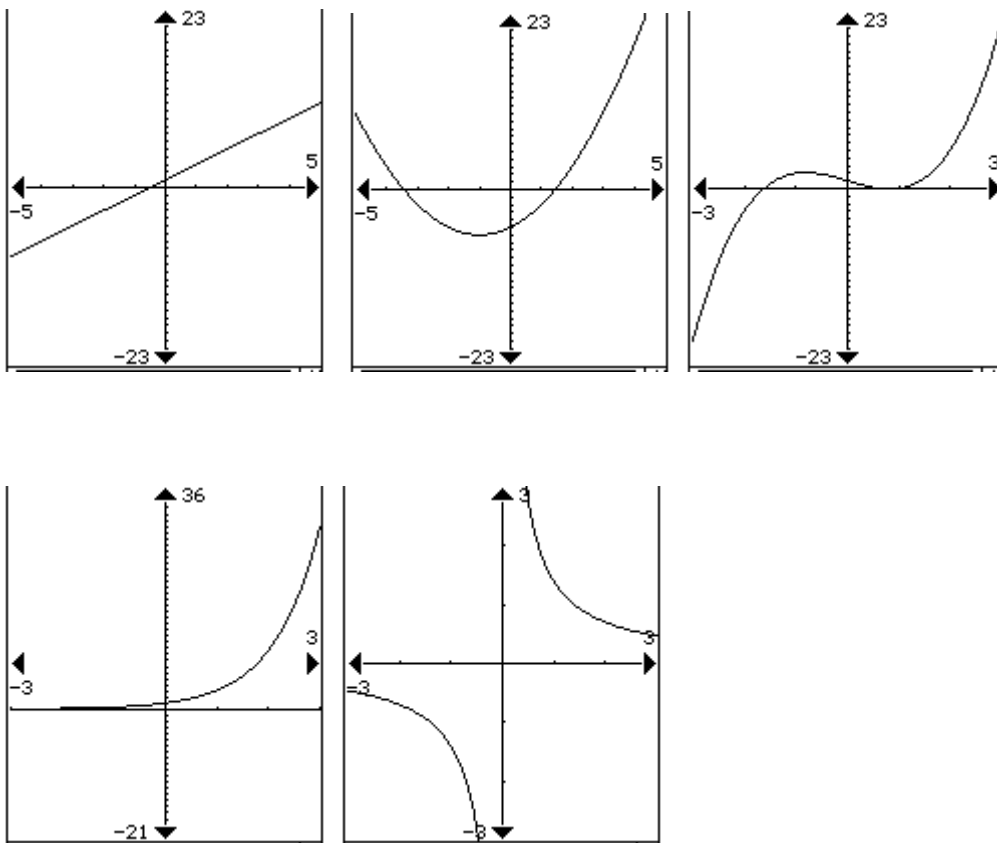
$$y = \left(\frac{1}{a}\right)^x$$

$$xy = b$$

$$y = c(x-d)^2 - e$$

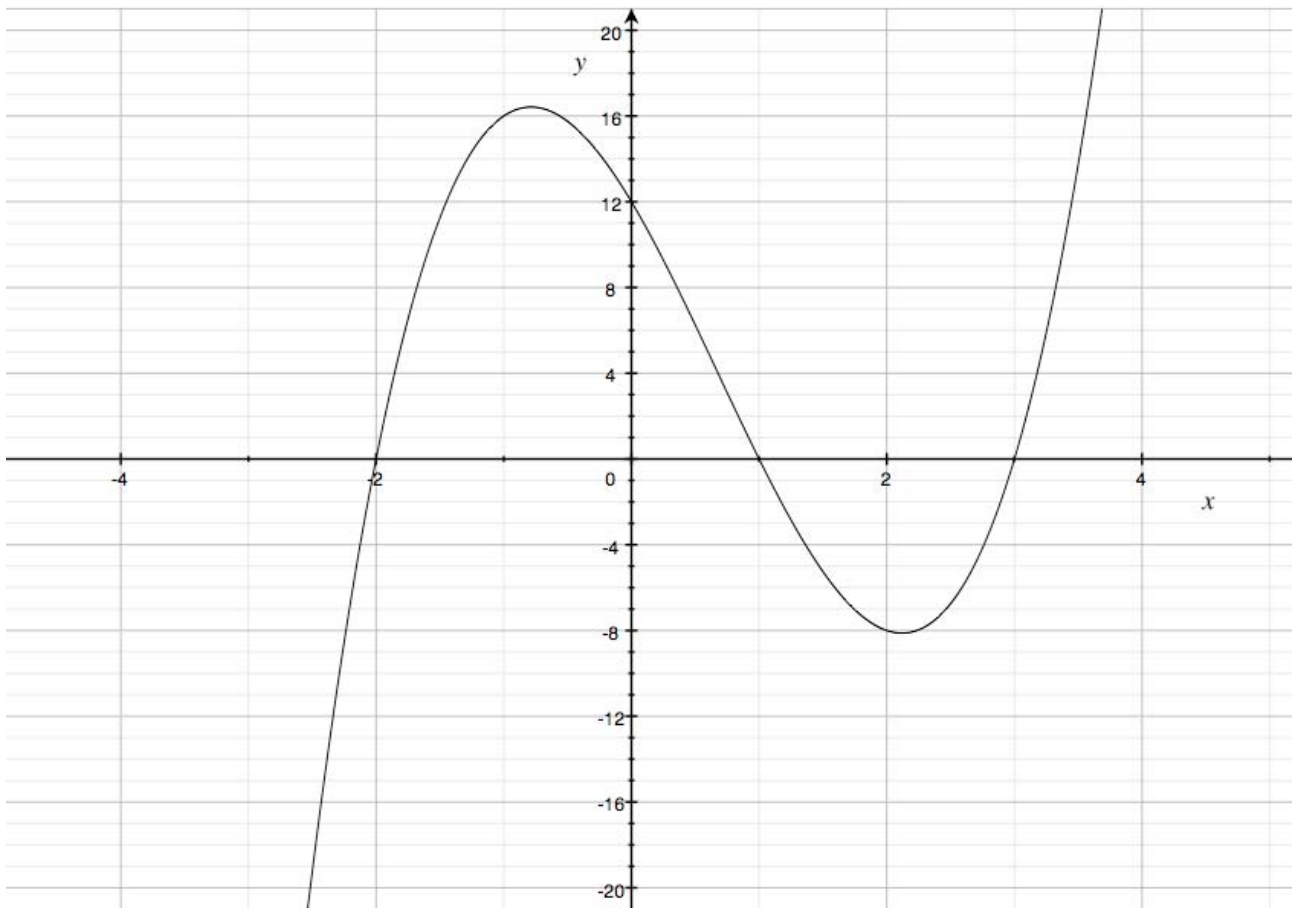
$$y = -fx + g$$

$$y = h(x-i)^2(x+j)$$



Question 6 (4 marks) Section 1 or 2	1.3.2 Number and algebra	Equations and inequalities	Solving graphically
---	-----------------------------	-------------------------------	---------------------

- (a) Given that the function, $y = f(x)$, shown below, has a y -intercept of 12, and x -intercepts of -2, 1 and 3, determine the value of $f(4)$.



Question 7 (6 marks) Section 1 or 2	1.3.5 Number and algebra	Equations and inequalities	Inverse proportion
---	-----------------------------	-------------------------------	--------------------

A cafeteria at a remote tourist resort has enough food for 320 people for 35 days.

- (a) How long will the same amount of food last 530 people, assuming that each person eats at the same rate as if there were 320 people? (2 marks)
- (b) An Australian wide business group wants to book out the resort for intense training of its staff. No contact is to be allowed with the outside world. The minimum duration of the training camp is 14 days and, for safety reasons, food cannot be kept on site for more than 100 days. Considering only the time constraints stated above, what is the lowest and highest number of people that can be accommodated by the tourist resort, assuming that the rate of consumption of each individual is the same, and is consistent, regardless of the circumstances? (4 marks)

Question 8 6 marks Section 2	1.4.2 Number and algebra	Patterns	Growth and decay
------------------------------------	-----------------------------	----------	------------------

The value of a certain stock changes daily. On odd-numbered days of the month it increases by 10%, and on even numbered days of the month it decreases by 10%.

- (a) By how much will the value of the stock change over consecutive two days? (2 marks)
- (b) At the beginning of November 1st the value of the stock is \$50000. What will the value be at the end of November 30th? (2 marks)
- (c) Is there any month in which the value of the stock increases? Justify your answer. (2 marks)

Question 9 4 (marks) Section 2	1.4.1 Number and algebra	Patterns	Recursive functions
--------------------------------------	-----------------------------	----------	---------------------

Any term of a sequence can be found by adding three to double the previous term.

- (a) Which rule below matches the description given above. (1 mark)

$$T_n = 3T_{n-1} + 2$$

$$T_n = 2T_{n-1} + 3$$

$$T_n = (n-1)T_2 + 3$$

$$T_n = T_{n-1} + 3$$

- (b) If $T_1 = 7$ find the value of

- (i) the seventh term (2 marks)

- (ii) n , such that the sum of the first n terms exceeds 2 000. (1 mark)

Question 10 (4 marks) Section 2	1.4.2 Number and algebra	Patterns	Growth and decay
---------------------------------------	-----------------------------	----------	------------------

The population of the Lake Swan is under threat due to urban development. The population of the species, n years since data was first recorded, is given by the formula, $P_n = P_{n-1} \times 0.92$. If the population, 5 years after the data was initially taken, is approximately 5 680, determine:

- (a) what the population was when the data was initially recorded (2 marks)
- (b) the year and month that the population is expected to fall below 4 000, given that the data was initially recorded on January 1st, 2000. (2 marks)

Question 11 (10 marks) Section 1 or 2	1.5.1, 1.5.2, 1.5.3 Number and algebra	Finance	Knowledge of loan spreadsheets
---	---	---------	-----------------------------------

Mike and Mary are planning to buy a house and are exploring loan options.

They have set up a spreadsheet as shown below. Cell R53 is for the interest rate per annum. Cell R54 is for the number of compounding periods per annum. Cell R55 is for the monthly repayment. Cell R56 is for the amount that Mike and Mary will borrow.

Row 59 has headings for the: Month, Starting amount at the beginning of the month, Interest for the month, Repayment for the month, and Amount owing at the end of the month.

	P	Q	R	S	T
51	Loan with reducible interest				
52					
53	% Interest rate p.a.				
54	Number of compounding periods p. a.				
55	Monthly loan repayment				
56	Starting amount				
57					
58					
59	Month	Starting amount	Interest	Repayment	Amount owing
60	1				
61	2				
62	3				
63	4				
64	5				

Mike and Mary are considering a loan of \$250 000 at 9% p.a. compounded monthly with monthly repayments of \$2000.

- (a) Enter the figures in the appropriate cells in the spreadsheet. (1 mark)

- (b) Write appropriate formulae in cells Q60, R60, S60 and T60 to calculate the amount owing for any values that are entered in cells R53 to R56. (7 marks)
- (c) What other actions would you perform on the spreadsheet to calculate the amount owing until the loan is paid off? (2 marks)

Question 12 (6 marks) Section 2	1.5.3, 1.5.5 Number and algebra	Finance	Reducible interest
---------------------------------------	------------------------------------	---------	--------------------

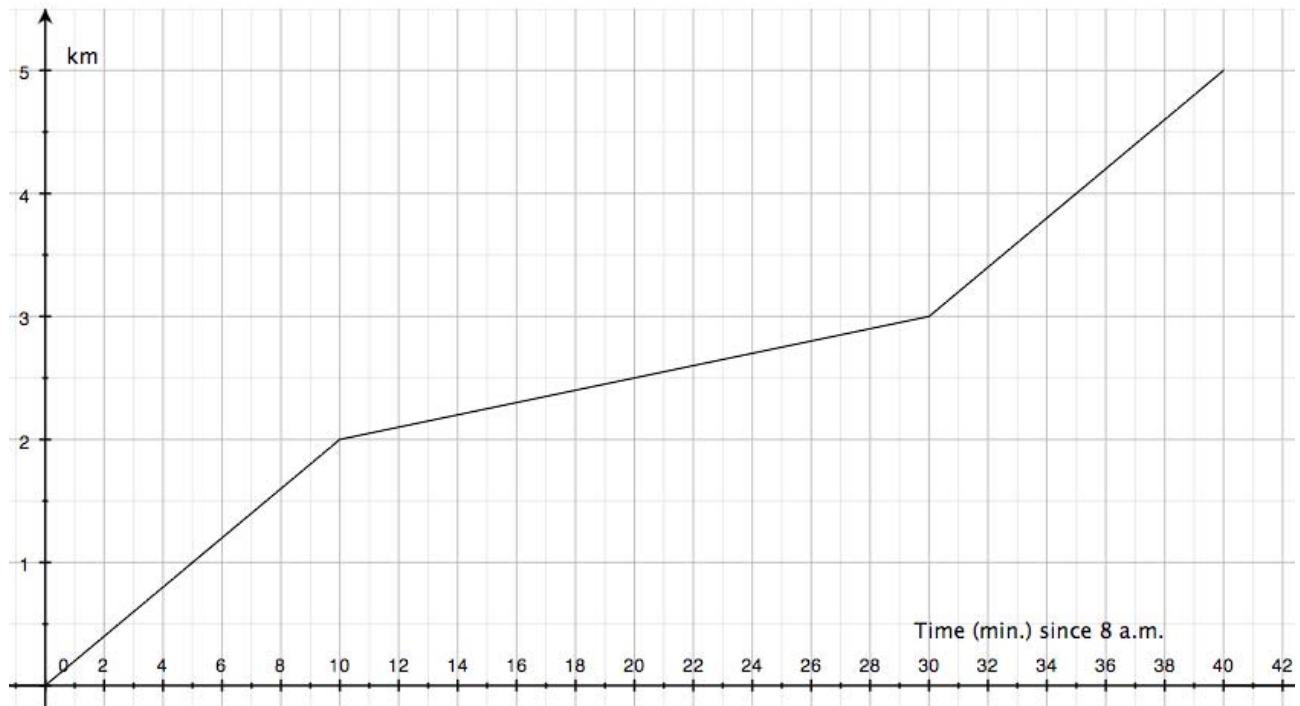
Susan borrowed \$15 000 to purchase a car. She decides to repay \$400 at the end of each month. The table below is incomplete.

Amount Owing	Interest	Repayment	Balance
15 000		400	14 675
14 675		400	
		400	
		400	
		400	
		400	

- (a) What was the annual rate of interest for Susan to borrow her money? (2 marks)
- (b) How much money will she owe at the end of the first year, if the repayments stay at \$400 per month for the first 6 months, and then increase to \$600 for the last 6 months? (4 marks)

Question 13 (11 marks) Section 1 or 2	2.1.1, 2.1.2, 2.1.3, 2.1.4 Space and measurement	Rate	Distance and displacement
---	---	------	---------------------------

Cedric cycled from home to school on a regular basis. On one morning, he cycled to a flooded area of road and had to walk his bike through the low lying water. Once through this section, he was able to get back on his bike and complete his journey to school.



- (a) Determine the speed, in km/h, that Cedric travelled on his bike, before he got to the flooded section of the road? (2 marks)
- (b) Cedric's sister, Celia, arrived at the school by car. She got to the school gate at the same time as Cedric. If the car Celia was in was travelling at an average of speed of 50 km/h, at what time did she leave the house? (2 marks)
- (c) Cedric's friend, Ashtar, was always slow to get moving in the morning, but was quite an athletic student. He started with a slow walk, had a rest for 5 minutes, then jogged, before resting again for 5 minutes, and then finally running all the way to school. Ashtar left Cedric's place at 8 am and then arrived at school at 8.40 am. He started walking at 1km/h, and his slow walk, jog and run were each of the same time duration. Each change of pace increased his speed by the same amount. Draw Ashtar's journey on the graph above. (7 marks)

Question 14 (5 marks) Section 1 or 2	3.1.1, 3.1.3 Chance and data	Quantify chance	Sample space, simple probabilities
--	---------------------------------	-----------------	---------------------------------------

Which is more likely:

- getting exactly 2 heads when 3 fair coins are tossed, or
- getting exactly 1 six when 2 fair die are tossed?

Justify your answer.

Question 15 (6 marks) Section 2	3.1.7 Chance and data	Quantify chance	Normal distribution
---------------------------------------	--------------------------	-----------------	---------------------

The Aussie Snack company produces small packets of fruit and nuts. The label on each packet states that it contains 150 grams of wholesome goodness. A machine is used to fill the packets. The amount of fruit and nuts delivered by the machine varies from packet to packet.

Let C denote the weight of the contents of a packet. The weight of the packets can be modelled by a normal distribution with a mean of 151.9 grams and a standard deviation of 0.44 grams.

- (a) Determine the proportion of packets that will contain less than 152 grams of fruit and nuts. (1 mark)
- (b) Aussie Snack boasts that less than 10% of its fruit and nut packets ever have less than the advertised amount of 150 g. Is this a valid statement? Justify your answer (2 marks)
- (c) Aussie Snacks wants to keep the mean weight of the packet at 152 g and only have 0.5 % of packets having a weight of less than 150 g. What must the new standard deviation of the weight of the packets be to ensure that the packing machine keeps to this new standard? (3 marks)

Question 16 (2 marks) Section 1	3.1.7 Chance and data	Quantify chance	Normal distribution
---------------------------------------	--------------------------	-----------------	---------------------

The scores of 1000 students taking a test are approximately normally distributed, with a mean of 65 and a standard deviation of 15.

- (a) How many students would you expect to score 50 or less? (1 mark)
- (b) How many students would you expect to score 95 or more? (1 mark)

Question 17. (2 marks) Section 2	3.2.2 Chance and data	Interpret chance	Capture-recapture
--	--------------------------	------------------	-------------------

Fiona went to study the number of native mice on a small remote island off the Pilbara coast and on the first night set a trap for the mice. The trap was harmless and allowed them to stay alive and return to the wild the next day. A small group of 35 mice were caught and given a small marking, so as to assist Fiona estimate the total population on the island. A month later Fiona set the same trap as before and caught a total of 63 mice, of which 17 had the same small marking which Fiona placed on them from a month ago. Estimate the total population of mice on the island. Show all of your working.

Question 18 (5 marks) Section 2	3.3.1 Chance and data	Collect and organise data	Sampling
---------------------------------------	--------------------------	---------------------------	----------

The Newbrook Community College consists of 244 Year 8 students, 252 Year 9 students, 286 Year 10 students, 224 Year 11 students and 177 Year 12 students. If a stratified survey of 80 students was taken, how many students from each year group took part in the survey?

Question 19 (10 marks) Section 1	3.4.2, 3.4.3, 3.4.9 Chance and data	Represent data	Box-plots, central tendency
--	--	----------------	-----------------------------

For the following set of data (given in ascending order)

m 5 5 7 n p 10 12 12 q 17 18

the mean is 10; the range is 16; the median is 9.5; the interquartile range is 7.

(a) Find the values of m , n , p and q . (5 marks)

(b) Draw a box-plot to display this data. (3 marks)

(c) Do any scores within this group of data fit the description of an outlier? Justify your answer. (2 marks)

Question 20 (10 marks) Section 2	3.4.1, 3.4.4, 3.4.7 Chance and data	Represent data	Grouped Data
--	--	----------------	--------------

The table below shows the number of times, over a period of 30 working days, people at a city financial firm, purchased a newspaper on the way to work.

Score	Frequency
1 - 5	3
6 - 10	5
11 - 15	12
16 - 20	24
21 - 25	33
26 - 30	21

- (a) Find the following:
- (i) the mean number of newspapers purchased,
 - (ii) the median class for the table above,
 - (iii) the standard deviation of the newspapers bought.
- (b) Draw a neat sketch of a frequency histogram for the data displayed in the table above.
- (c) Describe the spread of data shown in the graph and comment as to possible reasons for this result.

Question 21 (11 marks) Section 2	3.4.3, 3.4.5, 3.4.7, 3.5.8 Chance and data	Represent data Interpret data	Comparing data sets
--	--	----------------------------------	---------------------

The annual rainfall, measured in millimetres, at 2 sites in the Western Australian wheat belt in the last 9 years, is shown below:

Site A	287	353	319	308	285	294	257	341	296
Site B	194	793	256	125	216	227	617	239	293

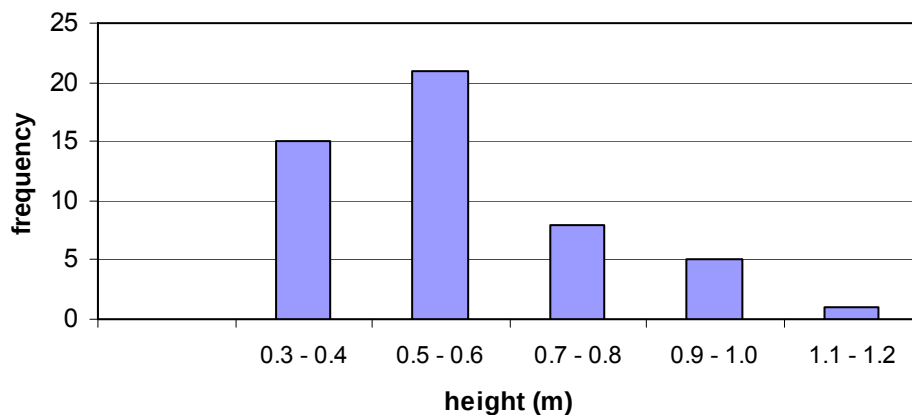
- (a) Compare the annual rainfalls at the two sites. (6 marks)
- (b) Based on these figures, what are your predictions for next year's rainfall at these two sites? (3 marks)
- (c) Which of these predictions would you expect to be more reliable? Justify your answer? (2 marks)

Question 22 (11 marks) Section 2	3.4.3, 3.4.5, 3.4.7, 3.5.8 Chance and data	Represent data	Comparison of data
--	--	----------------	--------------------

As part of a study on tree growth, an environmental scientist planted 50 trees on a plot of land and monitored their height for five years.

After one year, all 50 trees were alive and their heights are summarised in the graph below. Mean height was 0.54 m with a standard deviation of 0.20 m.

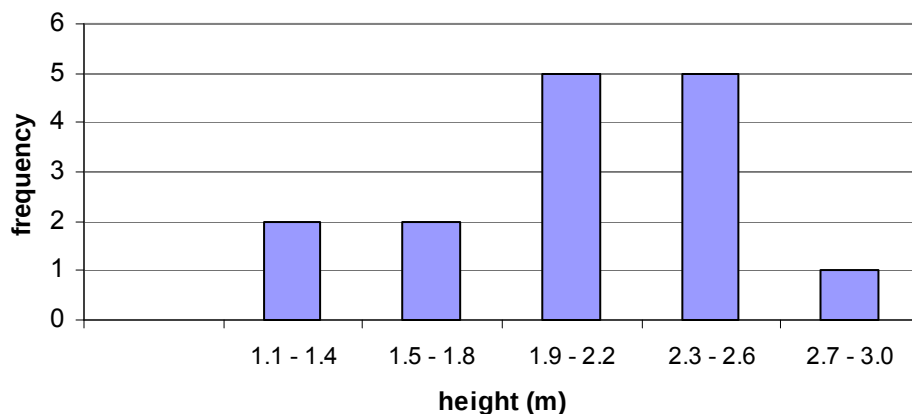
Tree height after 1 year of growth



After five years, height was measured for a representative sample of 15 of the trees. The results in metres are listed and graphed below:

1.32	1.4	1.64	1.77	1.87	1.88	2.02	2.11	2.16	2.41	2.50	2.52	2.56	2.60	2.95
------	-----	------	------	------	------	------	------	------	------	------	------	------	------	------

Tree height after 5 years of growth



Compare height for the trees after one year of growth and after five years of growth.

Question 1

(4 marks)

$$M = \frac{gR^2}{G} \quad \checkmark = \frac{9.8 \times (6.4 \times 10^6)^2}{6.7 \times 10^{-11}} = \frac{9.8 \times 6.4 \times 10^6 \times 6.4 \times 10^6}{6.7 \times 10^{-11}} \quad \checkmark$$

$$\approx \frac{10 \times 6.4 \times 10^{12}}{10^{-11}} \quad \checkmark = 6.4 \times 10^{1+12+11} \approx 6 \times 10^{24} \quad \checkmark$$

Question 2

(6 marks).

(a) $\frac{3x^7}{x^9} = 12$

$$\frac{3}{x^2} = 12 \quad \checkmark$$

$$\frac{3}{12} = x^2 \quad \checkmark$$

$$x^2 = \frac{1}{4}$$

$$x = \pm 0.5 \quad \checkmark$$

(b) $3 \times 2^{7x} = 96$

$$2^{7x} = 32 \quad \checkmark$$

$$2^{7x} = 2^5 \quad \checkmark$$

$$7x = 5$$

$$x = \frac{5}{7} \quad \checkmark$$

Question 3

(6 marks)

(a) (i) Translate $y = \sqrt{x}$ one unit to the right $\checkmark$ and then double the y coordinate of each ordered pair i.e. transform (x_1, y_1) to $(x_1, 2y_1)$ causing an enlargement by a factor of 2 parallel to the y -axis. $\checkmark$

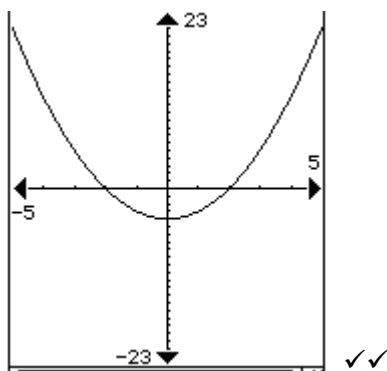
(ii) Halve the x co-ordinate of each ordered pair i.e. transform (x_1, y_1) to $(0.5x_1, y_1)$ causing an enlargement by a factor of $\sqrt{2}$ parallel to the y -axis. $\checkmark$
and then a translation of the resulting graph four units in the positive y direction $\checkmark$

(b) Domain $x \geq 1$ $\checkmark$, range $y \geq 0$ $\checkmark$

Question 4

(7 marks)

$$f(x) = x^2 - 4$$



- (a) $(0, 6)$ $\checkmark$
- (b) $(-3, 0), (1, 0)$ $\checkmark$
- (c) $(-1, 0), (3, 0)$ $\checkmark$
- (d) $(0, 7)$ $\checkmark\checkmark$

Question 5. (5 marks)

Line graph $y = fx + g$ ✓

Quadratic $y = c(x + d)^2 - e$ ✓

Cubic $y = h(x - i)^2(x + j)$ ✓

Exponential $y = a^x$ ✓

Reciprocal $xy = b$ ✓

Question 6. (4 marks)

(a) $f(x) = a(x + 2)(x - 1)(x - 3)$ ✓
 $12 = a(2)(-1)(-3)$ ✓
 $12 = 6a$
 $2 = a$ ✓

$f(4) = 2(6)(3)(1)$
 $f(4) = 36$ ✓

Question 7 (6 marks)

(a) Let x be the number of 'food days' for 530 people.
 $320 \times 35 = 530 \cdot x$ ✓
 $11\,200 = 530x$
 $x = 21$
 There is enough food for 21 days (for 530 people) ✓

(b) Let y be the least number of people at the resort
 $320 \times 35 = y \times 100$ ✓
 $11\,200 = 100y$
 $112 = y$ ✓
 Let m be the most number of people at the resort
 $320 \times 35 = y \times 14$ ✓
 $11\,200 = 14y$
 $y = 800$ ✓

i.e. The resort can accommodate between 112 and 800 people.

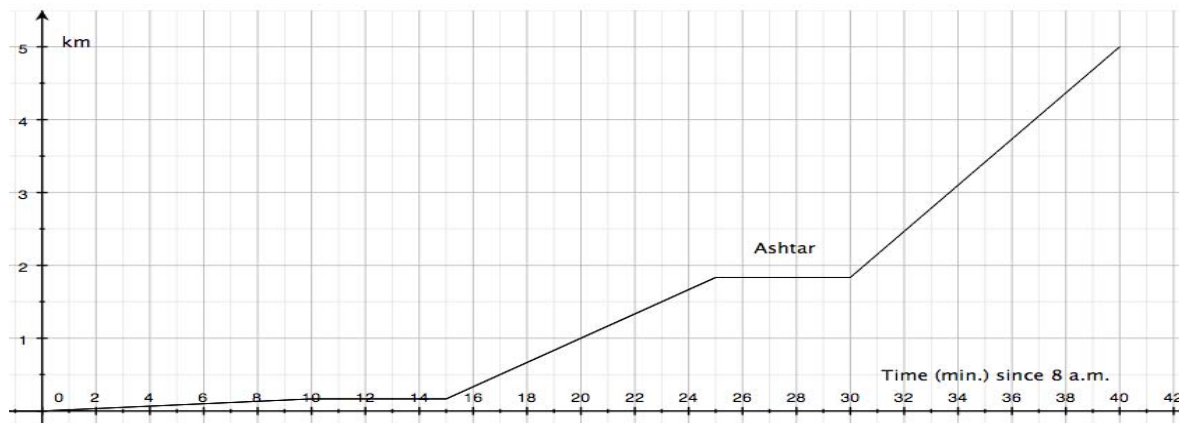
Question 8 (6 marks)

(a) $110\% \times 90\% = 1.1 \times 0.9 = 0.99$ ✓ So the value of the stock decreases by 1%. ✓

(b) $V = 5000 \times (1.1)^{15} \times (0.9)^{15} \approx 43\,003$ ✓
 So at the end of November the value is \$43 003. ✓

(c) The best months are months with 31 days, because there is an extra day when the stock increases in value. ✓ But $(1.1)^{16} \times (0.9)^{15} \approx 94.6\%$. ✓ So even in these months the stock decreases in value. So the answer is no.

Question 9. (4 marks) (a) $T_n = 2T_{n-1} + 3$ ✓	(b) (i) 637 ✓✓ (ii) $n = 8$ ✓
Question 10 (4 marks) (a) $5\,680 = P(0.92)^5$ ✓ $P = 8\,618$ ✓	(b) $4\,000 = 8\,618(0.92)^x$ $x = 9.21$ ✓ $0.21 \times 12 = 2.46$ i.e. The population is expected to fall below 4 000 in March, 2009. ✓
Question 11 (10 marks) (a) 9, 12, 2000, 250000 correctly entered in cells R53, R54, R55 and R56 ✓ (b) cell Q60 =R56 ✓ cell R60 =Q60*\$R\$53/100/\$R\$54 ✓✓✓ cell S60 =\$R\$55 ✓✓ cell T60 =Q60+R60-S60 ✓ (same marks can be allocated for other formulae that achieve the same results) (c) cell Q61 =T60 ✓ fill down columns Q to T ✓	
Question 12. (6 marks) (a) $I = Prt$ $75 = 15000 \times r \times \frac{1}{12}$ ✓ $r = 0.06$ i.e. Interest Rate was 6% p.a. ✓	(b) After 6 months Susan owes \$ 13 025.46 ✓✓ After 12 months Susan owes \$ 9 775.84 ✓✓
Question 13. (11 marks) (a) 2 km during 10 min. = 12 km/h ✓✓	(b) Time in car = $5 \div 50$ $= 0.1$ h $= 6$ min. ✓ Arrival time is 8.34 am ✓
(c) Let Ashtar's speed increase by s km/h each time. Speed x Time = Distance $1 \times \frac{1}{6} + (1 + s) \times \frac{1}{6} + (1 + 2s) \times \frac{1}{6} = 5$ ✓ $s = 9$ km/h ✓ So distance covered over each 10 minute 'moving interval' were For the first 10 minutes, $\frac{1}{6}^{th}$ of a km ($\frac{1}{6} \times 1$) ✓ In the second interval, $1\frac{2}{3}^{rd}$ of a km ($\frac{1}{6} \times 10$) ✓ In the third interval, $3\frac{1}{6}^{th}$ of a km ($\frac{1}{6} \times 19$) ✓	



✓✓

Question 14.

(5 marks)

For the coin tossing the sample space is $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$. ✓

These are equally likely outcomes so the probability of getting exactly 2 heads is $3/8$. ✓

For the die tossing the sample space is

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

✓

These are equally likely outcomes so the probability of getting exactly 1 six is $11/36$. ✓

Since $3/8 > 11/36$, getting 2 heads is more likely than getting 1 six. ✓

Question 15

(6 marks)

$$C \sim N(151.9, 0.44)$$

(a) $P(C < 152) = 0.59$ ✓

(b) $P(C < 150) = 0.000\ 008$ ✓
Statement is valid, as less than 1% of packets have less than 150 g in the packet. ✓

(c) $P(C < 150) = 0.005$
If $Z \sim N(0, 1)$
Let $P(Z < w) = 0.005$
i.e. $w = -2.575$ ✓

$$\frac{X - \mu}{\sigma} = -2.575$$

$$\text{i.e. } \sigma = \frac{150 - 152}{-2.575} \checkmark$$

$$\sigma = 0.777 \text{ (to 3 decimal places)}$$

$$\checkmark$$

Question 16. (2 marks)

- (a) 50 is 1 standard deviation below the mean, $\checkmark$
so we would expect about 16% of 1000, i.e. 160 students to score 50 or less. $\checkmark$
- (b) 95 is 2 standard deviations above the mean, $\checkmark$
so we would expect about 2.5% of 1000, i.e. 25 students to score 95 or more. $\checkmark$

Question 17. (2 marks)

Let p be the population of mice.

$$\frac{35}{p} = \frac{17}{63} \checkmark$$

$$p = 63 \times 35 \div 17$$

$$p \approx 129.7 \checkmark$$

i.e. There are approximately 130 mice on the island.

Question 18. (5 marks)

$$\text{Total number of students} = 244 + 252 + 286 + 224 + 177$$

$$= 1183 \checkmark$$

$$\text{Students from Year 8 in sample} = \frac{244}{1183} \times 80 \approx 16.5$$

$$\text{Students from Year 9 in sample} = \frac{252}{1183} \times 80 \approx 17.0$$

$$\text{Students from Year 10 in sample} = \frac{286}{1183} \times 80 \approx 19.3$$

$$\text{Students from Year 11 in sample} = \frac{224}{1183} \times 80 \approx 15.1$$

$$\text{Students from Year 12 in sample} = \frac{177}{1183} \times 80 \approx 12.0 \quad \text{Calculations } \checkmark \checkmark$$

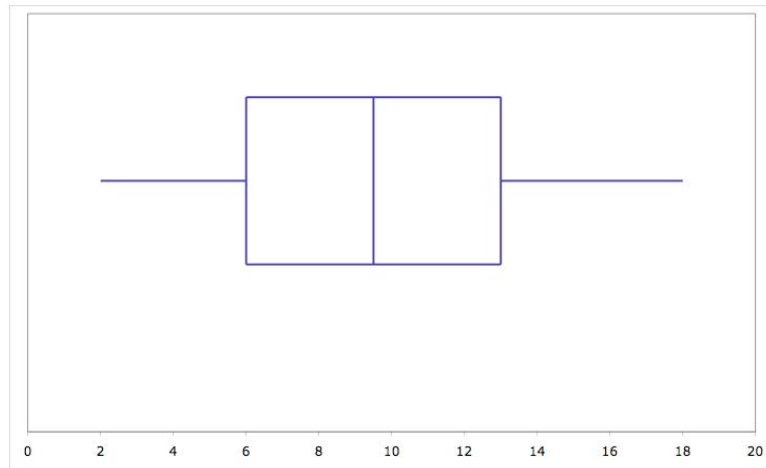
i.e. The number of students taking part in the survey would be 17 (Year 8), 17 (9), 19 (10), 15 (11) and 12 (12). $\checkmark \checkmark$

Question 19 (10 marks)

- (a) Range = 16, so $m = 2 \checkmark$
Median = 9.5, so $p = 9 \checkmark$

Inter quartile range = 7, so $q = 14$ ✓
 Mean = 10, so sum of scores = 12×10
 $= 120$ ✓
 $2 + 5 + 5 + 7 + n + 9 + 10 + 12 + 12 + 14 + 17 + 18 = 120$
 $n + 111 = 120$
 $n = 9$ ✓

(b)



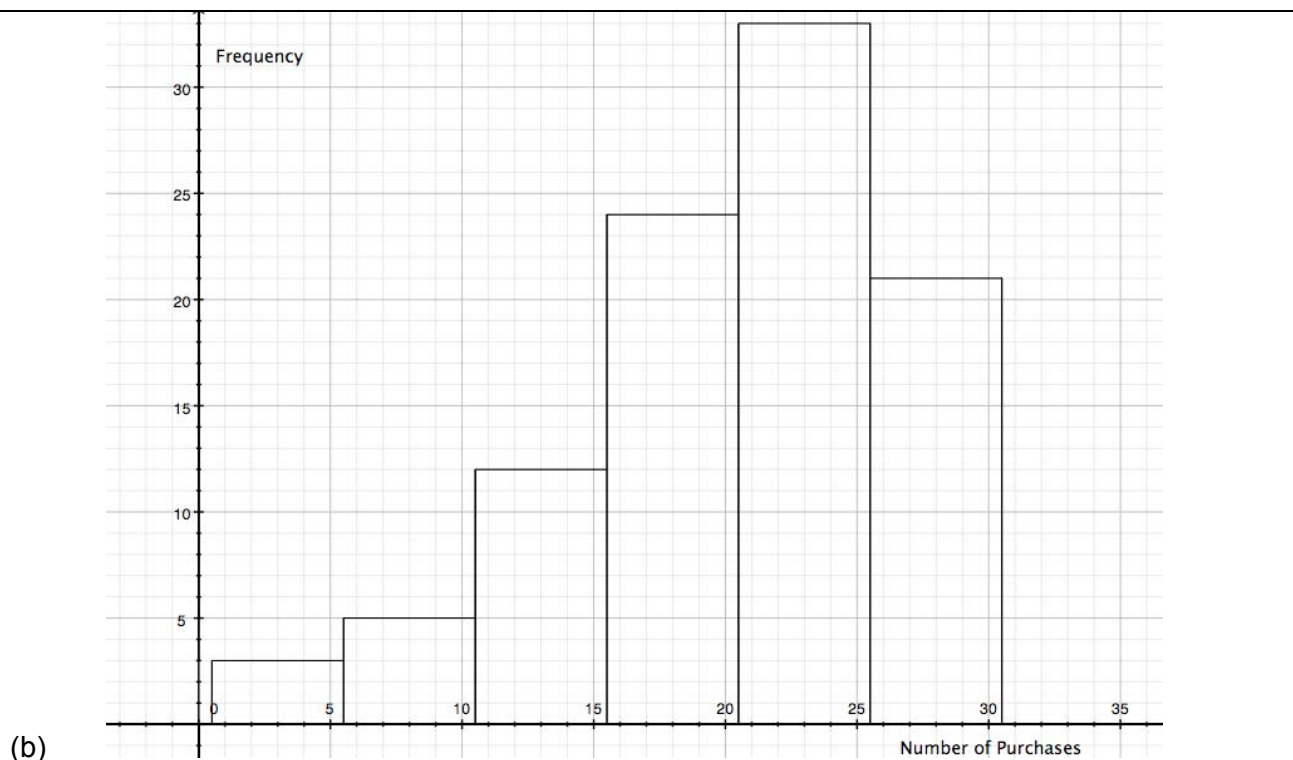
✓✓✓

(c) inter-quartile range = 7
 upper quartile + $1.5 \times 7 = 23.5$
 lower quartile - $1.5 \times 7 = -4.5$
 No scores exist beyond the values -4.5 or 23, so there are no outliers.

Question 20

(10 marks)

- (a) (i) mean = 20.24 (to 2 decimal places) ✓✓
 (ii) median class = 21 to 25 ✓
 (iii) $s = 6.31$ (to 2 decimal places) ✓

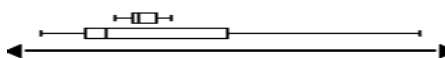


- (c) The data is not symmetrical and is skewed to the left. ✓
 It could be that most people that work in this firm are regular readers of the newspaper, or have a need to keep up to date with news about financial matters. Those that only occasionally purchase a newspaper are in the minority. ✓

Question 21

(11 marks)

- (a) For site A the mean annual rainfall is 304.4 mm and the standard deviation is 28.0 mm. ✓✓ For site B the mean annual rainfall is 328.9 mm and the standard deviation is 209.8 mm. ✓✓



So clearly, the rainfall at site A is much more consistent, and generally higher than at site B. ✓

The higher average value at site B is due to the two outliers. ✓

- (b) In the absence of other information the most reasonable predictions are measures of central tendency. ✓
 A strong case could be made for using the median values, 296 for site A and 239 for site B, ✓✓ since these are less affected by outliers, average values. However it would also be reasonable to use the means, 312 mm for site A and 329 for site B.
- (c) The prediction for site A is more reliable because the standard deviation is much smaller for site A. ✓✓

Question 22 (11 marks)

mean height after 5 years = 2.11m ✓ (appropriate rounding) ✓ (2.114... before rounding)

standard deviation = 0.46 m ✓ (0.4578... before rounding)

The average height of the trees after 5 years was about 4 times the average height of the trees after one year
or

The average height of the trees after 5 years was approximately 1.6 m more than the average height of the trees after one year).

✓ (descriptive comparison)

✓ (quantification 4 times, or 1.6 m more)

The spread in height after five years was about twice the spread after one year as indicated by the standard deviation of 0.46 m (5 years) and 0.20 m (1 year). (and range $2.9 - 1.3 = 1.6$ m (5 years) and $1.2 - 0.3 = 0.9$ m (1 year))

or

The spread in height after five years was much greater than the spread after one year as indicated by the standard deviation of 0.46 m (5 years) and 0.20 m (1 year)

✓ (descriptive comparison—judgement that spread after 5 years is twice the spread after 1 year is optional)

✓ (comparison of standard deviation or inter-quartile range—comparison of range optional)

After five years the height of more than half the trees (10 out of 15) was between 1.9 m and 2.6 m, in the middle to top half of the height range, whereas after one year the height of the majority of trees was between 0.3 m and 0.6 m, at the bottom of the height range. That is, the height data after five years tends to be skewed to the left whereas height data after one year is skewed to the right.

✓ (descriptive comparison)

✓ (figures that back up the description)

✓ (technical comparison of skewness)

✓ units (m) mentioned appropriately throughout