St Stephen's School – Carramar Campus
Mathematics EPW 2: Posting Packages
Due Date: Friday March 27th 2009

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Name: ______________________ Mark:

Teacher: SOLUTIONS

Parent Signature:

This EPW (Extended Piece of Work) comprises two parts:
Part A is to be answered at home and is due on **Friday 27th March 2009**
Part B is to be answered in class under test conditions on **Friday 27th March 2009.** You will ***not*** be allowed the take home part in the test, it must be handed in before you start. Calculators are allowed.

PART A

USA Post Office - Package Restriction

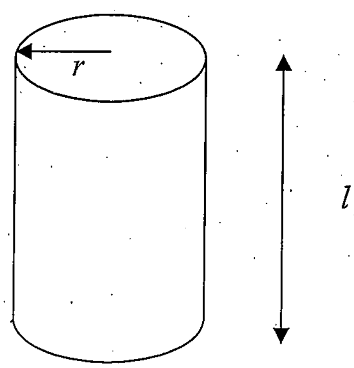
The United States Post Office will not accept a package which has a **combined length and girth *(perimeter of the cross-section perpendicular to the length)*** in excess of **84 inches**.

[Throughout this EPW we will consider the postage of packages in the USA and so will work in **inch** units.]

Task 1

We wish to determine the dimensions of the cylindrical tube of the largest volume that can be posted.

Consider a cylinder of radius *r* and length *l*.



(a) Write, in terms of *r* and *l*,

(i) an expression for the girth

$Girth = 2\pi r$

(ii) an expression for the cross sectional area of the cylinder

$A = \pi r^2$

(iii) an equation to represent the maximum length and girth allowed by the USA Post Office.

$2\pi r + L = 84$

(iv) rearrange this equation to make *l* the subject

$L = 84 - 2\pi r$

(b) Hence determine the equation of the volume of the cylinder V(*r*), expressed in terms of *r* alone. **Leave any brackets in your expression, do not expand.**

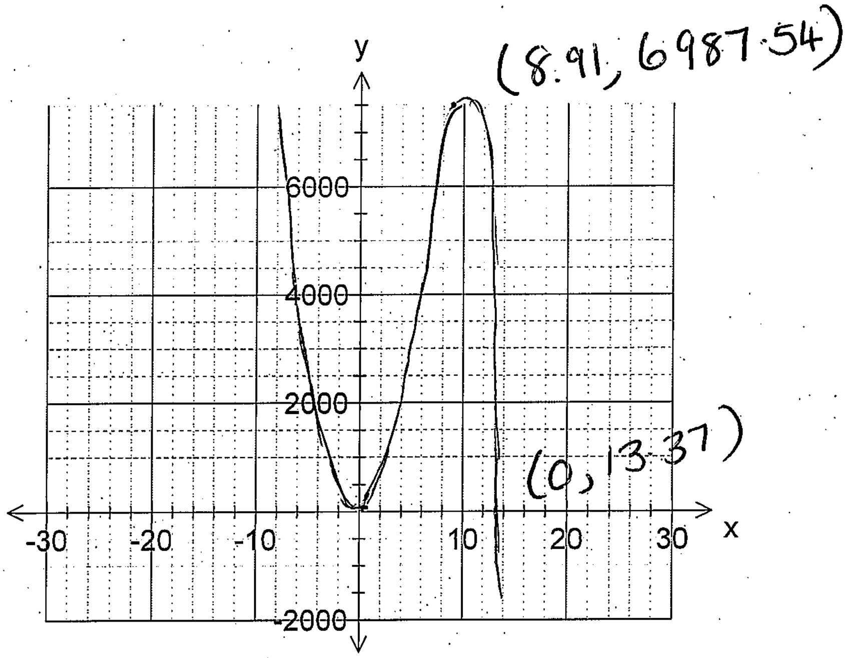
$V = \pi r^2 \times L$

$V = \pi r^2 (84 - 2\pi r)$

(c) Use your graphical calculator to determine the dimensions of the cylinder of largest volume that can be posted.

Let V(r) be y and r be x and plot the graph using a view window of $-30 \le x \le 30$ and $-2000 \le y \le 7500$ initially.

Make a sketch of it in the space below, clearly showing the coordinates of any intercepts and turning points.



Could you modify the viewing window further to help you solve this problem?

Yes, only need positive values so change to $0 \le x < 30$ and $0 \le y \le 7000$

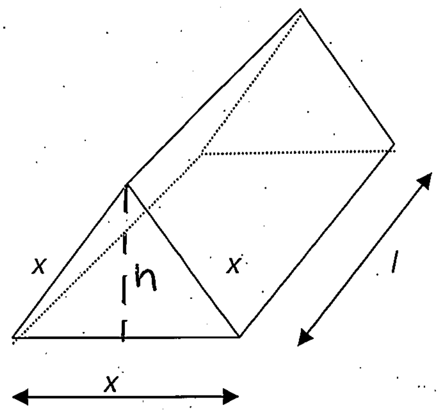
For maximum volume $r =$ 8.91 (2d.p) $l =$ 28.02 (2d.p)

(d) Explain how you used your calculator to solve this problem. Clearly state the steps involved in reaching the solution (in each case).

The maximum volume occurs at the turning point of the curve in quadrant 1. Therefore using G-Solve and then max we get an r value of 8.91. Substitute this into $L = 84 - 2\pi r$ to find L.

Task 2

We wish to send the largest possible box of Toblerone chocolate to a friend via the USA Post Office. We have influence at Toblerone and have been asked to provide the measurements for the solid block that contains the most chocolate.



(a) Show that the area (A) of the cross-section of the equilateral triangular prism of edge ***x*** units is given by the formula: $\frac{\sqrt{3}}{4}x^2$ inch^2.

$h^2 = x^2 - \left(\frac{x}{2}\right)^2$

$h^2 = x^2 - \frac{x^2}{4}$

$h^2 = \frac{3x^2}{4}$

$h = \frac{\sqrt{3}}{2}x$

$A = \frac{\sqrt{3}}{2}x \times \frac{x}{2}$

$\therefore A = \frac{\sqrt{3}x^2}{4}$

(b) Determine the formula for the volume (V) in terms of x alone. **Leave any brackets in your expression, do not expand.**

$V = A \times L \qquad L + 3x = 84 \rightarrow L = 84 - 3x$

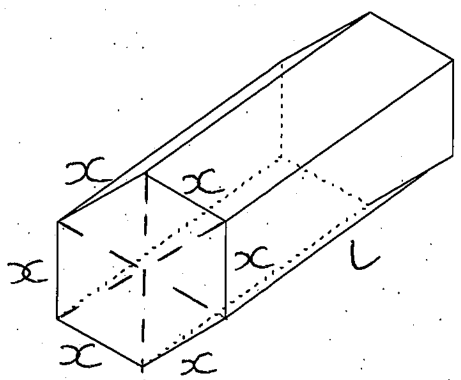
$V = \frac{\sqrt{3}}{4}x^2(84 - 3x)$

(c) Use your graphical calculator to determine the dimensions of the Toblerone of largest volume that can be posted.

x = 18·67 (2d.p) l = 27.99 (2d.p)

Task 3

The Toblerone Company chooses to manufacture special Easter boxes of its chocolates. The special boxes each contain 6 Toblerone boxes packaged together to form a regular hexagonal prism.



Label the cross-section edges each *x* units and the length *l* units.

(a) Write down a formula for the cross-sectional area of the hexagonal box.

$$A = 6 \times \frac{\sqrt{3}}{4} x^2$$

(b) State the formula for the volume function, *V*(*x*)

$$V = A \times L \qquad l = 84 - 6x$$

$$V = 6 \times \frac{\sqrt{3}}{4} x^2 (84 - 6x)$$

(c) Hence determine the dimensions of the largest box that can be posted.

$x = 9.33$ (2d.p)

$l = 28.02$ (2d.p).