

Course Outline - Introductory Calculus SEMESTER ONE 2005 St Stephen's School Carramar

Topic	Syllabus Objectives	Weeks	Assessment Items
Functions and Sequences Chapter 1		2 – 3.5 Term 1	Investigation 1 Wk 4 (8%) SC
Linear Functions Chapter 2	1.1	3.5 - 5 Term 1	Test 1 Wk 5 (5%) JB
Quadratics Chapter 3	1.2, 1.3	6 Term 1	
Cubics and Other Functions Chapter 4	1.4 to 1.6	7 Term 1	
Indices, Logarithms and Series Chapter 5	2.1, 2.4 to 2.6, 2.9	8 - 10 Term 1	Test 2 Wk 9 (5%) SC
Differentiation Chapter 6	3.1 to 3.5, 3.7 to 3.9, 3.11, 3.12	1 - 4 Term 2	Investigation 2 Wk 3 (8%) JB Test 3 Wk 5 (5%) SC
Rates of Change Applications of Differentiation Chapter 7	4.3, 4.4	5 - 6 Term 2	
Revision and Exams		7 - 8 Term 2	Semester One Exam (15%)
Applications of Differentiation Cont Chapter 7	4.3, 4.4	9 Term 2	
Turning Points and Other Features Chapter 8	4.1, 4.2, 4.5, 4.6	10 Term 2	Assignment(s) Semester One (3%)

Course Outline - Introductory Calculus SEMESTER TWO 2005 St Stephen's School Carramar

Topic	Objectives	Weeks	Assessment Items
Optimisation Chapter 9	4.7	1 - 2 Term 3	Investigation 3 Wk 2 (9%) SC
The Exponential Function Chapter 10	2.2, 2.3, 2.7, 2.8, 3.10, 4.8	3 - 4 Term 3	
Natural Logarithms Chapter 11	2.9, 2.10, 2.11, 3.6	5 - 6 Term 3	Test 4 Wk 5 (5%) JB
Integration Chapter 12	5.1 to 5.10	7 - 9 Term 3	Test 5 Wk 9 (5%) SC
Applications of Integration Chapter 13	6.1, 6.2	10 Term 3 2 - 3 Term 4 (Year 11 Retreat Wk 1 Term 4)	Test 6 Wk 2 (5%) JB
Revision and Exams		4 - 6 Term 4	Semester Two Exam (25%) Assignment(s) Semester 2 (2%)

Student Text: Introductory Calculus Second Edition by A.J. Sadler

Term 1

Functions and Sequences

Ch 1

Functions – symbols, tables, graphs

Sequences – terms, t_n notation, recursive formula

Linear Functions

Ch 2

- 1.1 Linear Functions, their graphs, essential features, parallel and Perpendicular lines.

Quadratics

Ch 3

- 1.2 Quadratic Functions, their graphs, essential features, different forms.
1.3 Quadratic Formula, Quadratic roots.

Cubics and Other Functions

Ch 4

- 1.4 x^n (n fixed) functions and their related graphs.
1.5 Explore features of polynomials and their graphs.
1.6 Matching a function with its graph.

Indices, Logarithms and Series

Ch 5

- 2.1 Index Laws.
2.4 Geometric Series.
2.5 Investigate the limiting behaviour of r^n as $n \rightarrow \infty$, (r fixed).
2.6 Infinite Geometric Series.
2.9 $y = a^x \leftrightarrow x = \log_a x$ Inverse Relationship.

Term 2

Differentiation

Ch 6

- 3.1 Average rate of change of a function.
3.2 Instantaneous rate of change of a function.
3.3 Derivative Notation.
3.4 Differentiate x^n for any real number n .
3.5 Differentiation by first principles.
3.7 Sum, Product and Quotient rules.
3.8 Chain rule.
3.9 Differentiate inverse relationships, and use the formula $\frac{dx}{dy} \times \frac{dy}{dx} = 1$.
3.11 Derivative Functions.
3.12 Second Derivative.

Rates of Change and Applications of Differentiation

Ch 7

- 4.3 Differentiate functions involving time such as velocity and acceleration.
4.4 Marginal rate of change and Instantaneous rate of change.

Turning Points and Other Features

Ch 8

- 4.1 Gradient of a curve at a point.
- 4.2 Equation of a tangent to a curve.
- 4.5 Local extreme points, sign test and 2nd derivative tests.
- 4.6 Relate features of the graph of a function to properties of its first and second derivatives.

Term 3

Optimisation

Ch 9

- 4.7 Solve optimisation problems by finding global extreme points.

The Exponential Function

Ch 10

- 2.2 Exponential function properties and their graphs.
- 2.3 Geometric growth and decay.
- 2.7 Investigate the limiting behaviour $\left(1 + \frac{a}{n}\right)^n$ as $n \rightarrow \infty$, (a fixed).
- 2.8 Define e as the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \rightarrow \infty$.
- 3.10 Differentiate e^x .
- 4.8 Identify and solve problems involving geometric growth and decay.

Natural Logarithms

Ch 11

- 2.9 $y = a^x \leftrightarrow x = \log_a x$ Inverse Relationship.
- 2.10 Properties of Logarithmic Functions and their graphs.
- 2.11 Define the natural logarithm $\log_e x$.
- 3.6 Differentiate $\ln x$.

Integration

Ch 12

- 5.1 Develop the concept of the indefinite integral as an anti-derivative.
- 5.2 Integrate x^n ($n \neq -1$).
- 5.3 Integrate x^{-1} .
- 5.4 Integrate e^x .
- 5.5 Integrate $(ax + b)^n$ and e^{ax+b} ($= Ae^{kx}$) using linear changes of variable.
- 5.6 Integrate linear combinations of powers $(ax + b)^n$ and exponentials e^{ax+b} .
- 5.7 Solve the differential equation $y' = f(x)$ by integration, and evaluate the constant of integration using $y(a) = b$.
- 5.8 Develop the concept of the definite integral as the signed area under a graph.
- 5.9 Estimate integrals of x and x^2 using approximating sums of the areas of rectangles.
- 5.10 Use the 'integral of a derivative' formula:
$$\int_a^b f'(x)dx = f(a) - f(b)$$
to calculate definite integrals.

- 6.1 Evaluate total changes from given rates of change.
- 6.2 Determine areas under and between curves.

2005 ASSESSMENT POLICY INTRODUCTORY CALCULUS

Assessment Weighting's

EPW's (x3)	25%
Tests (x6)	30%
Exams (x2)	40%
Assignments	5%

At the end of Year 11, each student will receive a mark out of 100 and a grade (A, B, C, D or E) based on the following:

1. The use of grade related descriptors for each level of achievement.
2. The final year score accumulated during the year.
3. Examination of student files by St Stephen's School staff or Curriculum Council moderators as may be necessary.

Every mathematics student is required to keep a "Student Assessment File" that must contain all assignment/project work, tests, extended pieces of work and Semester Exams.

That is, the file must contain every piece of work by the student that contributes towards the final mark.

Failure on part of a student to provide any given piece of assessment from the "Student Assessment File" may result in awarding a grade upon insufficient evidence.

St Stephen's School Assessment Policy

Refer to the Secondary School Assessment Policy contained within the Student Work Record.

A Progressive Record of Assessments is contained at the rear of the Student Work Record. Students are encouraged to use this to help monitor their academic progress across all their subjects.

**Extended Pieces of Work (EPW) Assessment Policy.
Year 11 and 12 TEE Mathematics Courses.**

To the student.

An extended piece of work is any assignment, problem solving activity, project or investigation that you are required to complete out of class. After you hand up the completed EPW to your mathematics teacher he/she will give you a validation or test to verify that the EPW is your work. Consequently an EPW consists of two parts. A “take home” component and the “in class” assessment or validation.

The out of class or take home component will be given a weighting of no more than 20% of the total EPW weighting. On the same day that the out of class component is due you will be required to sit the in class validation. When your validation has been marked, your result is added to the mark that you received for the out of class component. The mark allocated to the out of class component is predominantly an effort mark. If you complete this component in a neat, clear and concise format then you can quite easily obtain a perfect mark. While the validation aims to check that the EPW is your own work, it also tests your level of understanding of the content and processes pertained within the EPW. The validation is likely to extend you beyond the mathematical requirements of the out of class component.

For Example: Betty received a mark of 4 out of 5 for the take home part of her investigation. On the day that this was due she sat the validation and she received a mark of 16 out of 20. So her final mark for this investigation is $4 + 16 = 20$ out of a possible 25.

Your mathematics teacher may decide to give you an EPW at a designated time that does not contain a take home component. In this case the assessment is 100% weighted towards the in class component and marked similarly to a test.

If you hand up the out of class component late without an acceptable reason marks will be *deducted* at the rate stated in the St Stephen's Assessment Policy contained within your Student Work record. If you do not satisfactorily complete the out of class component you will receive a zero score for the EPW. You must show that you have rigorously attempted all questions.

These processes are in place to ensure the validity and fairness of student results for EPW in mathematics. If you have any questions please don't hesitate to ask myself or your mathematics teacher.

Stephen Corcoran
Head of Mathematics

INTRODUCTORY CALCULUS (YEAR 11) – D505

Rationale

Introductory Calculus and other subjects that lead to Year 12 TES subjects in Mathematics must fulfil a variety of needs. They should be both a satisfying continuation of the work of earlier years and an adequate preparation of tertiary studies. To do this these subjects must present mathematics as an organised body of useful knowledge and provide students with the skills and confidence necessary to apply this knowledge in practical situations.

These demands are met by offering studies in a range of topics in modern mathematics. The topics have the potential for useful application and are within the capabilities of the more mathematically able students of this age group. Students graduating from secondary school with a knowledge of these areas will appreciate the power of mathematics to provide a systematic way of understanding and interpreting the world around them.

Introductory Calculus provides an introduction to differential and integral calculus and its applications. It will be invaluable to students proceeding to tertiary studies in courses which make some use of calculus.

Educational Objectives

This subject seeks to present mathematics as an organised body of knowledge that will provide students with a sound basis for later work in mathematics and other subjects. In addition the subject attempts to develop the following skills and attitudes.

Cognitive Objectives

Students will be expected to:

- recall mathematical facts and traditional terminology
- acquire mathematical concepts
- understand mathematical relationships
- acquire manipulative and computational skills
- use mathematical facts, traditional terminology, concepts, relationships and skills in routine ways
- comprehend information in oral and written forms including graphical, diagrammatic and tabular presentations
- select and use appropriate forms for representing mathematical data and relationships
- recognise and extend patterns and make conjectures, predictions and inferences from information given in oral and written forms
- understand and use deductive reasoning
- apply suitable mathematical techniques and problem-solving strategies to both routine and non-routine situations
- select and use technologies appropriately.
- communicate mathematical ideas and results in both oral and written forms
- compare outcomes with expectations and verify the suitability and reasonableness of a result.

Affective Objectives

It is highly desirable that students:

- develop an interest in mathematics, and acquire a positive attitude towards its use and power
- show a willingness to participate and persevere in the learning of mathematics
- develop confidence in their ability to use mathematics effectively
- appreciate the benefits of using technology in mathematics
- display responsibility for their organisation, presentation and learning of mathematics
- interact in a constructive and cooperative manner with peers and teachers and respond constructively to advice.

Recommended Preparation

A desirable preparation is a strong background in algebra.

Successful completion of Introductory Calculus provides a suitable preparation for the Year 12 subject Applicable Mathematics.

The recommended preparation for the Year 12 subject Calculus is the successful completion of both the Year 11 subjects, Introductory Calculus and Geometry and Trigonometry. However, it is possible to study Geometry and Trigonometry and Calculus concurrently, after the successful completion of Introductory Calculus.

A set of *Counselling Notes* outlining the subject options in the mathematics area for Year 11 and Year 12 students has been distributed to schools. Additional copies can be obtained from the Curriculum Council.

Teaching – Learning Program

The topics, or objectives within topics, can be taught in any order in keeping with the needs of teachers and students.

1. Powers and Polynomials (15 hours)

Algebraic expressions involving simple powers of a variable will have been studied extensively in lower school. As a prelude to the ideas of calculus, the emphasis here is on their behaviour as functions. This should be illustrated with graphs wherever possible.

1.1 Investigate linear functions and essential features of their graphs including the relationships between the slopes of parallel and perpendicular lines.

1.2 Investigate quadratic functions, including essential features of their graphs, as indicated in the following three forms of a quadratic function:

$$y = ax^2 + bx + c$$
$$y = a(x - b)(x - c)$$

$$y = a(x - b)^2 + c$$

- 1.3 Use the quadratic formula to calculate zeros of quadratic functions.
- 1.4 Revise the concept of x^n (n fixed) as a function of x , and draw its graph.
- 1.5 Explore graphs of polynomial functions using appropriate technology and sketch these graphs identifying important features.
- 1.6 Match a function with its graph.

Notes:

- Techniques contained in this section should be applied whenever possible when modelling realistic situations.
- When dealing with the power functions x^n , the emphasis will be on the special values $n = 1, 2, 3, -1, -2, \frac{1}{2}$ and $\frac{1}{3}$. However, students should know the general shapes of the graphs of x^n for other values of n .
- When matching functions to their graphs (1.6) only functions that have obvious roots or are obvious translations and reflections should be considered. For quadratic functions, students may also be expected to factorise or complete the square.

Some examples:

$$y = x^2 \quad y = (x + 2)^2 \quad y = x^2 + 3x + 2 \quad y = x^2 - 2x - 2 \\ = (x + 1)(x + 2) \quad = (x - 1)^2 - 3$$

$$y = x^3 \quad y = (3 - x)^3 \quad y = x^3 + 3 \quad y = (x + 2)(x + 1)(x + 3)$$

$$y = x^4 \quad y = (x - 2)^4 \quad y = x^4 - 2 \quad y = x(x - 1)(x + 1)(x + 2)$$

$$y = \sqrt{x} \quad y = \sqrt{x + 2} \quad y = -\sqrt{x} + 2$$

$$y = \frac{1}{x} \quad y = \frac{1}{x + 2} \quad y = -\frac{1}{x} + 2$$

- The graphs of functions described above should include features such as turning points, the location of zeros, and behaviour as $x \rightarrow \pm\infty$.

2. Exponentials and Logarithms

(20 Hours)

The index laws are revised. The definition of a^x is extended in an intuitive manner to include non-integer values of x , and the basic properties of the exponential functions a^x are studied. Logarithms are defined as inverses of exponentials. Limit concepts are introduced, first via geometric series, and then via an important limit associated with the number e . This leads to the study of the natural exponential and logarithm functions, the importance of which will become clearer later in the subject.

- 2.1 Develop the index laws

$$a^{x+y} = a^x a^y \text{ and } (ab)^x = a^x b^x$$

for positive a and b , and arbitrary real x and y .

- 2.2 Establish and use the properties of exponential functions $y = Ca^{kx}$ ($a > 0$) and draw their graphs.
- 2.3 Develop the concepts of geometric growth and decay, using exponential functions as models.
- 2.4 Establish and use the formula for the sum of a finite geometric series.

- 2.5 Investigate the limiting behaviour of r^n as $n \rightarrow \infty$, (r fixed).

- 2.6 Establish and use the formula for the sum of a convergent infinite geometric series.

- 2.7 Investigate the limiting behaviour

$$\left(1 + \frac{a}{n}\right)^n \text{ as } n \rightarrow \infty, (a \text{ fixed}).$$

- 2.8 Define e as the limit of $\left(1 + \frac{1}{n}\right)^n$ as $n \rightarrow \infty$.

- 2.9 Define logarithms as the inverses of exponentials, and use the inverse relationship:

$$y = a^x \leftrightarrow x = \log_a y$$

- 2.10 Establish and use properties of the logarithm functions $\log_a x$ ($a > 0$), and draw their graphs.

- 2.11 Define the natural logarithm $\log_e x$.

Notes:

- The order of topics suggested here offers a quick and informal introduction to the number e and the associated natural exponentials and logarithms. However for some classes it may be more appropriate if the calculus of polynomials precedes the study of exponentials and logarithms. This would require some rearrangement. The treatment of some or all of this section would be postponed until the end of section 4.
- The definition of a^x is well understood for integer exponents x . The extension to other real values of x can be made plausible by appealing to continuity and the desirability of preserving the index laws. Limits are introduced not by formal definition but by means of examples of sufficient cogency to make the concept seem worthwhile and useful.

Artificial examples like $x^2 + 3 \rightarrow 7$ as $x \rightarrow 2$ should be avoided, as should sterile over-formal treatments. On the other hand, the limit investigated in 2.7 provides an excellent introduction to the concept. The result is non-trivial and important, it can be easily motivated using the notion of continuous compounding of interest, and it allows for easy numerical experimentation.

3. Differentiation

(20 hours)

The calculus studied here is the calculus of polynomials and simple rational functions, logarithms and exponentials. The derivatives of powers x^n are made plausible algebraically. The formula for the derivative of $\ln x$ is obtained starting from the approximation $\ln(1+x) \approx x$ for small x , and e^x is differentiated by treating it as an inverse.

For a first meeting with calculus an intuitive approach is most appropriate. This should be supported wherever possible with the use of zoom facility of a graphics calculator, numerical experimentation and algebraic manipulation. The use of the incrementals

δx and δy and the $\frac{dy}{dx}$ notation is encouraged.

- 3.1 Develop the concept of average rate of change

of a function, and relate it to the slope of a chord on its graph.

3.2 Develop the concept of the derivative as instantaneous rate of change, and relate it to the slope of a tangent on its graph.

3.3 Use the various notations for the derivative:

$$y', f', f'(x), \frac{dy}{dx}, \frac{df}{dx} \text{ and } \frac{df(x)}{dx}.$$

3.4 Differentiate x^n for any real number n .

3.5 Investigate the limiting behaviour of

$$\frac{\ln(1+h)}{h} \text{ as } h \rightarrow 0.$$

3.6 Differentiate $\ln x$.

3.7 Use the sum, product and quotient formulas to differentiate polynomials and other simple algebraic combinations.

3.8 Use the chain rule to differentiate simple composite functions.

3.9 Differentiate inverse relationships, and use the

$$\text{formula } \frac{dx}{dy} \times \frac{dy}{dx} = 1.$$

3.10 Differentiate e^x .

3.11 Develop the concept of the derivative as a function.

3.12 Develop the concept of and find the second derivative.

Notes:

- There are many ways in which the differentiation formulas can be made plausible. At the most rudimentary level they can all be checked by numerical calculation of difference quotients. For some of the most elementary functions, such as x and x^2 , derivatives can be established with some simple algebra. Derivatives of higher powers x^n can be found using the product rule, the binomial theorem, or the expansion

$$x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})$$

- The result $\ln(1+x) \approx x$ for small x is a logarithmic formulation of the limit for e studied in 2.8.

- Results such as the continuity of polynomials and

$$\lim_{x \rightarrow a} (f(x) + g(x)) = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$$

which may be necessary for establishing formulas such as

$$\frac{d(x^n)}{dx} = nx^{n-1} \text{ and } \frac{d(y_1 + y_2)}{dx} = \frac{dy_1}{dx} + \frac{dy_2}{dx}$$

should be used implicitly, without direct reference.

- It is not intended in 3.8 and 3.9 that there should be any formal discussion of the underlying notions of composite and inverse functions. To avoid this it is probably best to present the chain rule and the rule for differentiating inverses in the Leibniz notation:

$$\frac{dz}{dx} = \frac{dz}{dy} \times \frac{dy}{dx} \text{ and } \frac{dx}{dy} = \left[\frac{dy}{dx} \right]^{-1}$$

In 3.9 'differentiating inverse relationships' means reversing the traditional roles of x and y , and differentiating an equation of the form $x = f(y)$ with respect to y . With this approach, and the formula relating $\frac{dy}{dx}$ to $\frac{dx}{dy}$ it is possible to obtain

derivatives of the exponential function e^x and fractional powers such as $x^{\frac{1}{2}}$.

- The functions $\ln f(x)$ and $e^{f(x)}$ can be differentiated by the use of the chain rule.

4. Applications of the Derivative

(25 hours)

Differentiation formulas should be applied as soon as they are obtained, and in a wide variety of settings.

- Determine the slope of a curve at a point.
- Establish and use the equation of a tangent to a curve.
- Develop the concept of a function of time, and determine and use time derivatives such as velocity and acceleration.
- Use 'marginal rate of change' synonymously with 'instantaneous rate of change', and relate it to the derivative.
- Find local extreme points of functions and determine their nature using the sign test or the second derivative test.
- Relate features of the graph of a function to properties of its first and second derivatives.
- Solve optimisation problems by finding global extreme points.
- Identify and solve problems involving geometric growth and decay.

Notes:

- Students should be able to use exponential functions Ae^{kx} as solutions of the differential equation $y' = ky$, and hence solve simple problems involving geometric growth and decay.
- Students should be exposed to optimisation problems from a wide variety of sources.
- The graphical features referred to in 4.6 are stationary points and inflection points, and intervals in which a graph is increasing or decreasing, convex or concave. Students should be able to recognise these on a graph and draw the right conclusions about derivatives. Conversely, they should be able to use information about derivatives to sketch graphs.

5. Integration (15 hours)

Integration is presented both as a continuous summation process for finding area under the curve and as the inverse of differentiation. The area approach has historical precedence and is the basis of approximate numerical techniques. However, anti-differentiation may be easier for students in their first meeting with calculus. The link between these two aspects of integration is the fundamental theorem of calculus. With this theorem area problems become

(generally easier) anti-differentiation problems.

5.1 Develop the concept of the indefinite integral as an anti-derivative.

5.2 Integrate x^n ($n \neq -1$).

5.3 Integrate x^{-1} .

5.4 Integrate e^x .

5.5 Integrate $(ax+b)^n$ and e^{ax+b} ($= Ae^{kx}$) using linear changes of variable.

5.6 Integrate linear combinations of powers $(ax+b)^n$ and exponentials e^{ax+b} .

5.7 Solve the differential equation $y' = f(x)$ by integration, and evaluate the constant of integration using $y(a) = b$.

5.8 Develop the concept of the definite integral as the signed area under a graph.

5.9 Estimate integrals of x and x^2 using approximating sums of the areas of rectangles.

5.10 Use the 'integral of a derivative' formula:

$$\int_a^b f'(x) dx = f(b) - f(a)$$

to calculate definite integrals.

Notes:

- In 5.2, 5.3, 5.4, 5.5 and 5.6 'integrate' means 'find the indefinite integral of'.
- In 5.5 there should be no need to formally discuss change of variable techniques for integration. It is sufficient to present the results and to justify them by differentiation.
- The function f in 5.7 is to be no more complicated than those in 5.6.
- The traditional approach to the version of the fundamental theorem given in 5.10 is to study the behaviour of

$$F(x) = \int_a^x f(t) dt$$

and to show that F is an anti-derivative of f . However, at this level a more convincing argument might be given using incrementals:

$$y(b) - y(a) = \Delta y = \sum \delta y_i$$

$$\approx \sum y'(x_i) \delta x_i \approx \int_a^b y'(x) dx$$

6. Applications of the Integral (15 hours)

With the integrals of powers and exponentials available, it is possible to demonstrate the power of the integral calculus in a variety of settings which are relevant to students of this level.

6.1 Evaluate total changes from given rates of change.

6.2 Determine areas under and between curves.

Notes:

- Section 6.1 includes not just traditional physical examples such as finding displacement from velocity and velocity from acceleration, but also examples from economics such as finding total

revenue from marginal revenue, and from other areas of science.

- In problems requiring the area between two curves to be calculated, the points of intersection should be either given or easily obtained.

Time Allocation

The subject has been designed to be completed through a structured education program of approximately 120 hours in any suitable contexts and series of learning experiences. Typically the subject will be studied over the period of one school year. For administrative reasons schools wishing to vary this delivery pattern (e.g. over a shorter period or over a longer period up to two school years) are required to notify the Chief Executive Officer of the Curriculum Council.

Subject Completion

Students must complete the school's structured educational and assessment program for a subject in order to be eligible to receive a grade unless there are exceptional and justifiable circumstances. In situations where the school considers that insufficient information has been gathered to justify the award of a grade for the subject, a result of U (for unfinished) should be allocated. The Curriculum Council offers the flexibility for the U to be converted to a grade after the final grades have been submitted. Further details on assessment and grading are provided in Volume I of the Syllabus Manuals.

Assessment Structure

Assessment structures are an integral part of all Accredited Subjects.

The structure specifies:

- the components and learning outcomes to be included in assessment
- weightings to be applied to these components
- the types of assessment considered appropriate for the subject.

Table 1

Syllabus Content	Weighting percentage
Power and polynomials	10-15
Exponentials and logarithms	15-20
Differentiation	15-20
Applications of the derivative	20-25
Integration	10-15
Applications of the integral	10-15

Table 2

Learning Outcomes	Weighting percentage
Lower order cognitive objectives *	45-55
Higher order cognitive objectives **	45-55

- * which include recall of skills, acquisition of concepts and routine use of mathematical knowledge.
- ** which describe processes such as generalisation, justification, deduction and the application of mathematical techniques and problem-solving strategies in non-routine ways.

Table 3

Types of Assessment	Weighting percentage
Extended pieces of work	25-50
Other forms of assessment	50-75

The assessment program must provide students with the opportunity to demonstrate achievement of the requirements of the subject.

AND

Students must complete the requirements of the subject.

Supporting Notes

1. In the assessment program developed by schools, due attention must be given to the full range of educational objectives. It is recommended that, in order to assess the achievement of the higher order cognitive objectives, **at least three (3)** extended pieces of work be included in the assessment program.
2. In many assessment tasks it is appropriate and desirable for students to work cooperatively and obtain help from various sources. However, assessment in mathematics should focus on the level of knowledge and understanding a student has attained and not the manner in which the learning has taken place. In all cases where students can collaborate or obtain outside help for assessable work, schools must ensure that marks awarded are measures of the student's own knowledge and understanding. Accordingly, validation is regarded as an integral part of the assessment procedure.
3. Student performance in the subjects will be graded on a scale of A, B, C, D and E according to the degree to which students have achieved the stated objectives of the subject.
A set of grade-related descriptors, which indicate the standard of student achievement required for each grade, is available for these subjects and should be used when grading student performance.

Notes on Table 3

Extended Pieces of Work: These are assessment tasks which allow students the opportunity to demonstrate higher order cognitive skills such as verification, justification, interpretation and deduction. They may involve an in-class and/or out of class component and may be completed over an extended period of time. The intent is to set tasks that can be completed free from the pressures of time. Careful consideration of the time allocated to complete such tasks is therefore essential.

Other Forms of Assessment: There are other forms of assessment that may be used to determine the extent to which students have achieved the objectives of the course. It would be reasonable to expect that tests and exams would figure prominently but checklists, homework assignments which emphasise skill development and oral presentations for instance could also be legitimately used.

Notes:

- While out of class assessment tasks at times allow (in fact may require) students to utilise resources beyond those typically available while being assessed under supervised conditions, it is expected that work finally submitted for assessment should be both known to and understood by the students concerned.
- Schools are responsible for determining procedures which should be applied to eliminate the likelihood of cheating, plagiarism, collusion and the like.
- School-based initiatives to exercise some form of 'control' over out-of-class assessment tasks are considered desirable as a means of both protecting students and retaining the integrity of using school assessments for external purposes.
- The following are examples of 'controls' that may be legitimately used:
 - professional judgement
 - in-class supervised tasks with open access to notes and other resources
 - achievement on 'at home' tasks measured by a brief in-class test on key concepts
 - research done out of class with reports completed in class under supervised conditions
 - teacher/student interview
 - part in-class and part out of class components.

Grade-Related Descriptors

Grade-Related Descriptors describe the student performance standards that are used to award grades in this subject. Schools delivering this subject have been provided with a copy of the document. Additional copies may be purchased from the Curriculum Council.