Solutions – Sem 1 2007 Exam

1. [7 marks]

(a) What is the physical significance of the slope of the graphs? [1]

Speed – the steeper the slope, the greater the speed

(b) Calculate the gradient of each graph. [2]

A: ***60 km/hr***

B: ***40 km/hr***

(c) Write an equation to give the distance travelled for car B as a function of time.

B(t) = 40t + 80

(d) Another car, car C, starts at a distance 200 km and travels at 50 km/hr in the opposite direction to car A. Write the equation to give distance travelled for car C as a function of time.

C(t) = -50t + 200 [2]

2. [6 marks]

Given the function $3x - 2y = 6$ state:

(a) the coordinates of the x intercept ***(2, 0)*** [1]

(b) the coordinates of the y intercept ***(0, -3)*** [1]

(c) the gradient ***3/2*** [1]

(d) Given a second function $2x + 3y = 12$, state the relationship between the two functions. Justify your answer. [3]

y = 3/2x – 3 and y = -2/3x + 4

The lines are perpendicular because product of gradients is -1

3. [7 marks]

Given $f(x) = \dfrac{e^{2x}}{(x+1)}$ and $g(x) = x^2(x-2)$ then:

(a) Determine $f(-2)$. $-\dfrac{1}{e^4}$ ***≅-0.0183*** [2]

(b) Determine a, if $g(a) = -1$. [2]

$a = -1.618$ *or* 1 *or* 1.618

(c) Determine p, if $g''(p) = -3$ [3]

$p = \frac{1}{6}$

4. [8 marks]

(a) What length of road is supported from above? [3]

Points of intersection at (47.53, 40) and (252.47, 40)

Length of road is 204.94 m

(b) What length of road is supported from below? [3]

Roots are 0 and 300.

Length = 47.53 + (300 – 252.47)

≈ 95.06 m

(c) Find the length of the central support wire? [2]

T.P. = (150, 75)

∴ length = 35 m

5. [7 marks]

Given $f(x) = x^3 - 3x^2$, when $x = 6$ and $h = 0.1$, evaluate

(a) $f(x)$ [1]

when x = 6; f(6) = 108

(b) $f(x+h)$ [1]

when x = 6 and h = 0.1; f(6.1) = 115.351

(c) $f(x+h) - f(x)$ [1]

115.351 – 108 = 7.351

(d) $\dfrac{f(x+h) - f(x)}{h}$ [1]

7.351 ÷ 0.1 = 73.51

(e) Explain how your answer to (d) relates to $f'(6)$ [1]

Finding the approximate gradient of the tangent to f(x) at x = 6 from first principles or by limiting chord process

(f) Evaluate $f'(6)$ EXACTLY [2]

f'(x) = 3x² – 6x

f'(6) = 72

6. [7 marks]

Simplify each of the following

(a) $\dfrac{30a^2b^3}{6b^2} \times \dfrac{(2b^2)^3}{75a^3b^3}$ [3]

$= \dfrac{8b^4}{15a}$

(b) $\left(\dfrac{3c^2}{m^{-1}}\right)^2 \times \sqrt[3]{\dfrac{m^{-6}}{27c^9}}$ [4]

$= 3c$

7. [11 marks]
Solve, to two decimal places where appropriate, the following equations.
Working is expected.

(a) $\log 3 + \log x = \log 5 + \log (x-2)$ [3]

$x = 5$

(b) $\log_9(x-5) + \log_9(x+3) = 1$ [3]

$x = 6$ *or* ($x = -4$ *gives log of a -ve)*

(c) $10^{-a} = 2$ [2]

$a = -\log 2$ *or* $\log\left(\frac{1}{2}\right)$

$\boxed{\cong -0.30}$

(d) $4 + 3\log_3(2x) = 9$ [3]

$x \approx 3.12$

8. [12 marks]

Graph (a) …….. *IV* …………………….. Graph (b) ………………………………….. *VIII*

Graph (c) ………………………………….. *VI* Graph (d) ………………………………….. *VII*

9. [7 marks]
Solve each equation exactly

(a) $27^y = 9$ [2]

$y = \frac{2}{3}$

(b) $5^x = 0.2$ [2]

$x = -1$

(c) $2^n + 2^{n+1} = 24$ [3]

$n = 3$

10. [21 marks]
Differentiate each of the following with respect to *x*:

(a) $y = x^3 - 4x^2$ [2]

$$\frac{dy}{dx} = 3x^2 - 8x$$

(b) $f(x) = \frac{-2}{x^4}$ $f'(x) = \frac{8}{x^5}$ [2]

(c) $g(x) = \frac{x^2(4-x)}{6}$ [3]

$g'(x) = \frac{4}{3}x - \frac{1}{2}x^2$

(d) $y = 4x^3(3x+1)^5$ Simplify your answer.[3]

$\frac{dy}{dx} = 12x^2(3x+1)^4(8x+1)$

(e) $F(x) = \frac{3x^3 + 2x}{4x^3 - 3}$ Do not simplify your answer. [3]

$$F'(x) = \frac{(9x^2 + 2)(4x^3 - 3) - 12x^2(3x^3 + 2x)}{(4x^3 - 3)^2}$$

(f) $y = \sqrt[3]{x^3 - 8}$ [4]

$y = (x^3 - 8)^{\frac{1}{3}}$

$\frac{dy}{dx} = \frac{x^2}{\sqrt[3]{(x^3 - 8)^2}}$

(g) $g(x) = \frac{1}{(6 - 12x^2)^2}$ [4]

$g'(x) = \frac{48x}{(6 - 12x^2)^3}$

11. 7 marks]

(a) How many hours after the drug is taken is the concentration of the drug in the bloodstream a maximum? [2]

GC: Maximum when t = 1.91
$\therefore$ ***after 1.91 hours***

(b) What is the maximum concentration of the drug in the bloodstream? [2]

$\approx$***0.274 mg***

(c) If another oral dose is given when the concentration of the drug first falls below 0.12 mg, when (to the nearest hour) should the second dose be given? [3]

When y = 0.12; t = 3.978 $\therefore$ ***after*** $\approx$***4 hours***

12. [4 marks]

What are the equations of the following two functions?

$y = \frac{k}{x - 2} + 2$

$y = \frac{1}{x - 2} + 2$

$y = \sqrt{x + 2}$

13. [5 marks]

Give the values of *a*, *b* and *c*.

a = ½ , b = 3, c = 2

14. [4 marks]

Evaluate $\log_m(n^3) + 2\log_m(mn)^2 - 7\log_m(n)$ = 4 [4]

15. [2 marks]

f(x) = a(x – b)(x – 2) where a and b are constants.

What is the equation of the line of symmetry? [2]

x-intercepts at (b, 0) and (2, 0)
LOS: x = (b + 2)/2

16. [9 marks]

(a) the distance she travels on the 6th day. [3]

GP: 100, 90, 81, a = 100, r = 0.9

$$T_n = ar^{n-1}$$

$$T_6 = 100 \times 0.9^5$$

$$= 59.049\ km$$

(b) how far she is from Perth at the end of the 8th day. [3]

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_8 = \frac{100(1 - 0.9^8)}{1 - 0.9}$$

$$= 569.53\ km$$

(c) whether she reaches Exmouth. (Show reasoning.) [3]

$$S_\infty = \frac{a}{1 - r}$$

$$= \frac{100}{1 - 0.9}$$

$$= 1000$$

She does not reach Exmouth

17. [7 marks]
(a) If the half life of Radium **226** is 1590 years, determine the exact value of k. [3]

$$0.5 = e^{-k}(1590)$$

$$\mathbf{k} = \frac{\ln 0.5}{-1590}$$

$$(\mathbf{k} = 0.000436)$$

(b) If the original mass found was 100mg, how much will remain after 1000 years? Give your answer to the nearest mg. [2]

$$M = 100e^{-0.000436 \times 1000}$$

$$= 64.661...$$

$$\approx 65\ mg$$

(c) When will the mass be reduced to 30mg?
Give your answer to the nearest year. ≈ 2762 *yea* [2]

18. [8 marks]
(a) What is the % increase in fish each year? ***1.2 %*** [1]
(b) What is the fish population in 5 years? [1]

when t = 5; n(5) ≈ 12.742 million

(c) After how many years will the number of fish reach 30 million? [2]

when n(t) = 30 million; t = 76.358. ∴ after 76.358 years or during the 77th year.

(d) Another population of fish can be modelled by: $m(t) = 8\,e^{0.015t}$
Show use of logarithmic rules to determine when the second species of fish will outnumber the first species. [4]

$t \approx 135.155$

After 135.155 years or during the 136th year

19. [11 marks]

(a) The first three terms of a geometric sequence are $x^2 - 1$, $5x - 1$ and -3. Find all possible values of ***a***, the first term and ***r***, the common ratio. [6]

$$\frac{5x-1}{x^2-1} = -\frac{3}{5x-1}$$

$$25x^2 - 10x + 1 = -3x^2 + 3$$

$$28x^2 - 10x - 2 = 0$$

$$14x^2 - 5x - 1 = 0$$

$$(7x+1)(2x-1) = 0$$

$$x = -\frac{1}{7} \text{ or } x = \frac{1}{2}$$

GP1: $-\frac{48}{49};\ -\frac{12}{7};\ -3 \quad a = -\frac{48}{49};\ r = \frac{7}{4}$

GP2: $-\frac{3}{4};\ \frac{3}{2};\ -3 \quad a = -\frac{3}{4};\ r = -2$

(b) (i) Find the sum of :

-24 + 12 - 6 + 3 - . . . [2]

$$S_\infty = -\frac{24}{1 - -0.5}$$

$$= -16$$

(ii) Find the sum of :

$\text{m} - \frac{\text{m}}{2} + \frac{\text{m}}{4} \ldots$ (Express your answer in terms of m) [3]

$$S_\infty = \frac{m}{1 - -0.5}$$

$$= \frac{2m}{3}$$