



ST STEPHEN'S SCHOOL

SEMESTER ONE EXAMINATION

2006

SOLUTIONS

QUESTION 1 [2 marks]

Use the formula given below to work out the Fahrenheit equivalent of 210° Celsius (Hint: C represents Celsius, F represents Fahrenheit)

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(210) + 32$$

$$F = 410^{\circ} \text{ Fahrenheit}$$

QUESTION 2 [3 marks]

Given the three numbers 3.4×10^3 , 2.3×10^{-4} and 0.00013, multiply the largest of these numbers by the smallest and give your answers in scientific notation.

$$3.4 \times 10^3 = 3400 \quad \text{largest}$$

$$2.3 \times 10^{-4} = 0.00023$$

$$0.00013 \quad \text{smallest}$$

$$3.4 \times 10^3 \times 0.00013$$

$$= 0.442$$

QUESTION 3 [2, 2, 2, 2 marks]

For each of the following, write an equation using the given information and solve your equation algebraically:

- a) Think of a number multiply it by six, add eight to the answer and then subtract the original number. If the answer is forty eight, what was the original number?

$$\begin{aligned} 6x + 8 - x &= 48 \\ 5x &= 40 \\ x &= 8 \end{aligned}$$

- b) Think of a number, subtract six, multiply the result by twelve and then divide the result by five. The answer is six more than the original number. What was the original number?

$$\frac{12(x-6)}{5} = x+6$$

$$\begin{aligned} 12(x-6) &= 5x+30 \\ 12x-72 &= 5x+30 \\ 7x &= 102 \end{aligned}$$

$$x = \frac{102}{7} = 14 \frac{4}{7}$$

- c) Think of a number and then subtract it from eight. Multiply the result by five, add twice the original number with the result being negative forty one. What was the original number?

$$\begin{aligned} 5(8-x) + 2x &= -41 \\ 40 - 5x + 2x &= -41 \\ -3x &= -81 \\ x &= 27 \end{aligned}$$

- d) Three times five more than a number is reduced 6 times the original number giving a result of thirty six. What was the original number?

$$\begin{aligned} 3(x+5) - 6x &= 36 \\ 3x+15 - 6x &= 36 \\ -3x &= 21 \\ x &= -7 \end{aligned}$$

QUESTION 2 [2, 2, 2, 4 marks]

- a) Solve the equation:
- $6(5x + 3) - 3(3x - 4) = 13$

$$30x + 18 - 9x + 12 = 13$$

$$21x + 30 = 13$$

$$21x = -17$$

$$x = \frac{-17}{21}$$

- b) Solve the equation:
- $\frac{22}{f} = -5$

$$22 = -5f$$

$$f = \frac{22}{-5}$$

$$f = -4\frac{2}{5}$$

- c) Solve the equation:
- $2(2 - x) = 4 - 3(x + 1)$

$$4 - 2x = 4 - 3x - 3$$

$$4 - 2x = 1 - 3x$$

$$x = -3$$

- d) Solve for x and y where:
- $3x = 11 - 5y$
- and
- $x + y = 5$

$$3x = 11 - 5y$$

$$x + y = 5$$

$$x = 5 - y$$

using
substitution.

$$3(5 - y) = 11 - 5y$$

$$15 - 3y = 11 - 5y$$

$$2y = -4$$

$$y = -2$$

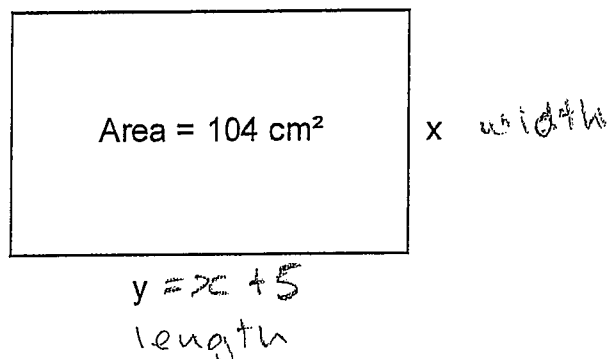
$$x = 5 - (-2)$$

$$x = 7$$

$$x = 7 \quad y = -2$$

QUESTION 3 [1, 2, 4 marks]

The length y of the rectangle below is 5 cm more than its width x . The area of the rectangle is 104 cm^2 .



a) Complete $y = x + \underline{5}$

b) Explain why $x(x + 5) = 104$.

Area of rectangle is length times width.
 length = $x + 5$ and width = x and Area = 104 cm^2
 $\therefore x(x + 5) = 104$

c) Find the dimensions of the rectangle. Show your method clearly.

$$x(x + 5) = 104$$

$$x^2 + 5x - 104 = 0$$

$$(x + 13)(x - 8) = 0$$

$$x = -13 \quad x = 8$$

width cannot be negative

so $x = 8$

$$y = x + 5$$

$$y = 8 + 5 = 13$$

$$x = 8 \text{ cm} \quad y = 13 \text{ cm}$$

Trial and Error method

x	$x + 5$	Area
5	10	50
6	11	66
7	12	84
8	13	104 ✓

$x = 8 \text{ cm}$
 $y = 13 \text{ cm}$

QUESTION 4 [4 marks]

An investor has \$60 000 to invest for one year. She decides to put some of it in a secure deposit account and the rest in a more risky investment. At the end of the year the secure deposit account pays interest of 10%, the more risky investment pays 15% and the investor receives a total of \$8 600 in interest from these two sources. How much of the \$60 000 had the investor put in the secure deposit?

Let x be amount invested in secure deposit.
So $(60000 - x)$ is invested in risky investment.

could also use
simultaneous
equations
Let x : secure invest.
 y : risky
 $x + y = 60000$
 $0.1x + 0.15y = 8600$
etc.

$$0.1x + 0.15(60000 - x) = 8600$$

$$0.1x + 9000 - 0.15x = 8600$$

$$400 = 0.05x$$

$$x = \frac{400}{0.05} = 8000$$

∴ \$8000 invested in secure deposit.

QUESTION 5 [4 marks]

My father gives me half as much pocket money as he gives my older sister, but one and a half times as much as he gives to my brother. He pays out \$6.60 in pocket money to the three of us each week. How much do I get each week?

Let x represent how much my brother gets

$$\frac{\text{me}}{1\frac{1}{2}x}$$

$$\frac{\text{brother}}{x}$$

$$\frac{\text{sister}}{3x}$$

$$1\frac{1}{2}x + x + 3x = 6.60$$

$$5\frac{1}{2}x = 6.6$$

$$\frac{11x}{2} = 6.6$$

$$11x = 13.2$$

$$x = 1.2$$

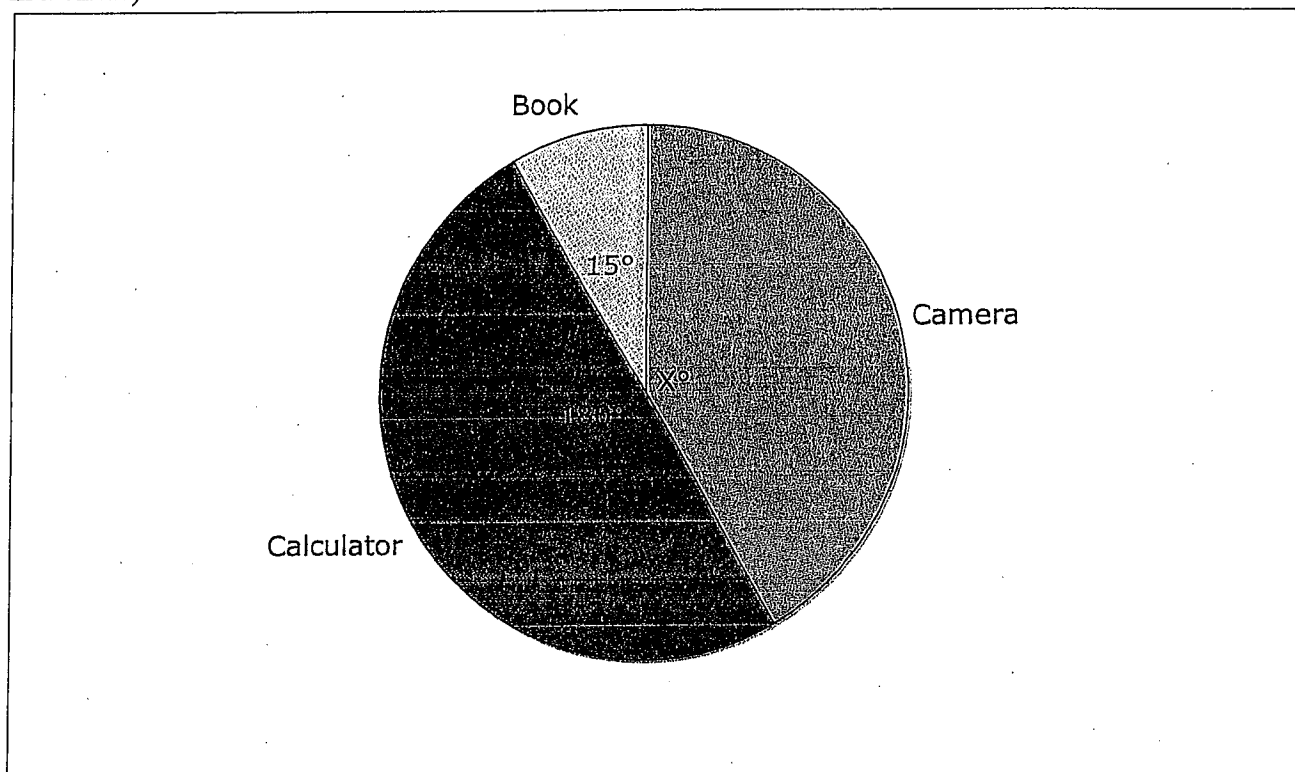
my brother gets \$1.20

So I get $1.5 \times 1.20 = 1.80$

I get \$1.80

QUESTION 6 [2, 1, 1, 1 marks]

Each child in a group of 120 was asked to choose a camera, a book or calculator as a present. Their choice are represented in the given pie chart (remember that there are 360° in a circle)



Use the diagram above to calculate

a) the value of X

$$165^\circ$$

b) the fraction of the group that chose a calculator.

$$\frac{180}{360} = \frac{1}{2}$$

c) the number of children who chose a camera.

$$\frac{165}{360} \times 120 = 55 \text{ children}$$

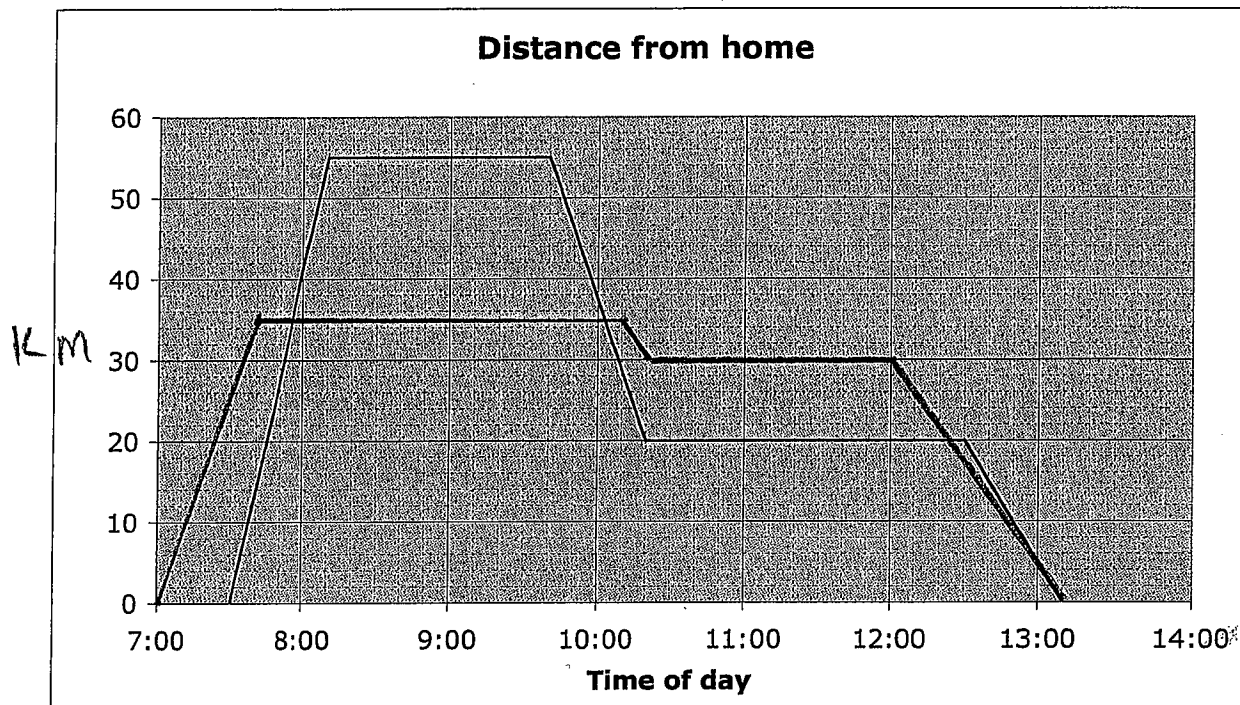
d) the percentage of children who chose a book.

$$\frac{15}{360} \times 100 = 4.1\bar{6}\%$$

$$\text{or } 4\frac{1}{6}\%$$

QUESTION 7 [1, 2, 1, 3, 3 marks]

Joe is a self employed contract gardener. On a particular Sunday he and his wife set out for the morning, Joe is off to work and his wife is going to visit some friends. The graph below shows the movements of Joe's wife for the day.



- a) How far from home was the first friend that Joe's wife visited?

55 km

- b) At what speed did she travel between the two friends house?

$$\text{Speed} = \frac{\text{distance}}{\text{time}} = \frac{55 - 20}{40} = 0.875 \text{ km/min} \\ = 52.5 \text{ km/hr}$$

- c) What time did she arrive home?

13:10
or 1-10 pm

(question 9 continued)

On this particular day Joe only has 2 jobs that he has to go. He leaves home at the same time as his wife and drives 35 km to his first job arriving 40 minutes later. This job is quite a long one and it takes him two and half hours to complete it. His second job for the day is 5km closer to home and takes him 10 minutes to travel to it. This job takes him 1 hour and 40 minutes to complete. He then returned directly home, arriving at the same time as his wife.

d) Use the graph on the previous page to show Joe's movement for the day.

See graph.

e) The speed limit between Joe's second job and his home is 50 km/hr. Did he have to speed to arrive home at the same time as his wife? Justify your answer mathematically.

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{30}{70} \text{ km/min}$$

$$= \frac{30}{70} \times 60 \text{ km/hr}$$

$$= 25.71 \text{ km/hr}$$

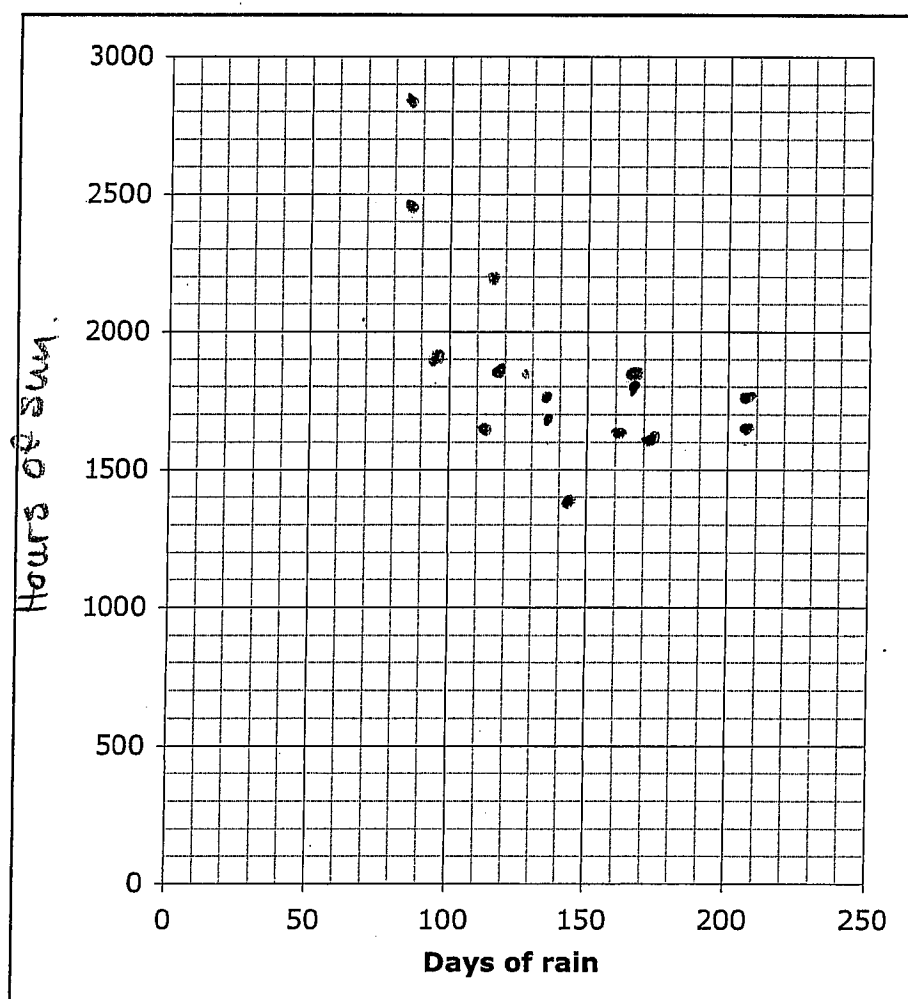
No, he did not have to speed. The speed limit is 50 km/hr and he went 25.71 km/hr on average.

QUESTION 8 [4 marks]

The average number of days of rain per year and the average number of hours of sunshine per year for a number of cities around the world is given in the table below.

City	Days of Rain	Hours of Sun
Amsterdam	206	1651
Brussels	206	1551
Copenhagen	172	1603
Dublin	142	1386
London	113	1514
Madrid	84	2843
Montreal	166	1852
Munich	128	1862
Oslo	161	1632
Paris	164	1840
Rome	69	2491
Tokyo	115	2019
Vienna	96	1891
Zurich	135	1694

Draw a scatter graph for the above data using the grid below.



QUESTION 9 [4, 1, 1, 2, 2 marks]

A total of 80 cars were observed in a car park in Paris. Of these, 50 had car radios, 60 had fog lights and 10 had neither.

- a) Complete the two-way table below using the above information

	Radio	No radio	Total
Fog lights	40	20	60
No fog lights	10	10	20
Total	50	30	80

- b) Use the two way table of results to determine

- i) how many cars had fog lights and a radio

40

- ii) the number of cars that had a radio but no fog lights

10

- iii) the percentage of cars that had fog lights only

$$\frac{20}{80} \times 100 = 25\%$$

- iv) what percentage of those who had radios also had fog lights

$$\frac{40}{50} \times 100 = 80\%$$

QUESTION 10 [5 marks]

A new student council was being elected which reflected the proportional representation for each year group at a school. The total number of students on the council was to be twelve.

At the time of the election the school population consisted of:

Year	Number of students
8	150
9	180
10	165
11	130
12	125
Total	750

Determine the number who should be elected to the student council to represent each year group.

<u>Year</u>	<u>Calculation</u>	<u>Rounded</u>	
8	$\frac{150}{750} \times 12 = 2.4$	≈ 2	rounding to nearest integer. Can't have part of a student!
9	$\frac{180}{750} \times 12 = 2.88$	≈ 3	
10	$\frac{165}{750} \times 12 = 2.64$	≈ 3	
11	$\frac{130}{750} \times 12 = 2.08$	≈ 2	
12	$\frac{125}{750} \times 12 = 2$	≈ 2	
Total		<u>12</u> ✓	<u>12</u> ✓

Answer: Gr 8 2 representatives
 Gr 9 3 "
 Gr 10 3 "
 Gr 11 2 "
 Gr 12 2 "

QUESTION 11 [1, 1, 4 marks]

John is organizing a guide camp. The camping grounds gave him the following prices so that he can work out how much the camp will cost:

	Child	Adult
Accommodation	\$4.00	\$5.00
Breakfast (8.00am)	\$2.00	\$2.00
Lunch (1.00pm)	\$4.00	\$5.00
Dinner (6.30 pm)	\$7.00	\$10.00
Supper (9.00pm)	\$1.00	\$1.50

Use the table to calculate the following

- a) The total cost per adult per whole day's stay.

$$5 + 2 + 5 + 10 + 1.50 = \$23.50$$

- b) The total cost per child per whole day's stay.

$$4 + 2 + 4 + 7 + 1 = \$18.00$$

- c) The total cost of the camp if there are 3 adults and 40 children attending and they are arriving at 3.00pm on Friday and departing at 4.00 pm Monday. Show all working.

Adults: $3 \times 23.50 \times 3 = 211.50$

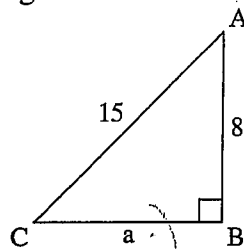
Children $40 \times 18 \times 3 = 2160$

Total = \$2371.50

$(\text{No people} \times \text{cost per day} \times \text{No days})$

QUESTION 12 [2, 2, 2 marks]

With reference to this triangle:



- a) Find length
- a

$$\begin{aligned}a^2 &= 15^2 - 8^2 \\a^2 &= 161 \\a &= \underline{12.69 \text{ units}}\end{aligned}$$

- b) Find the size of
- $\angle A$

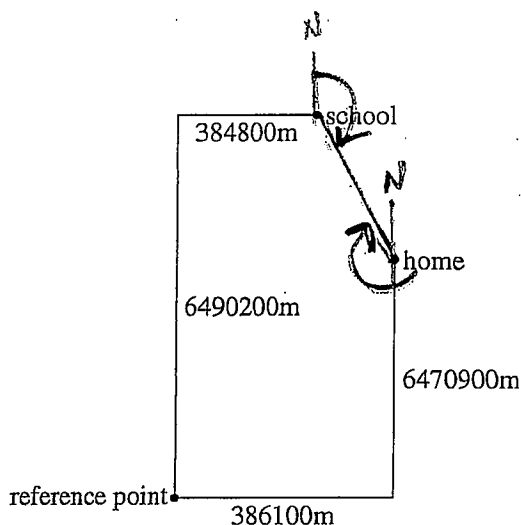
$$\begin{aligned}\angle A &= \cos^{-1}\left(\frac{8}{15}\right) \\&= \underline{57.77^\circ}\end{aligned}$$

- c) Find the area of
- $\triangle ABC$

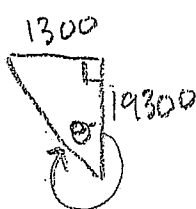
$$\begin{aligned}\text{Area Triangle} &= \frac{1}{2} \text{ base} \times \text{height} \\&= \frac{1}{2} \times 12.69 \times 8 \\&= \underline{50.76 \text{ units}^2}\end{aligned}$$

QUESTION 13 [4, 1, 2 marks]

Map grid coordinates give a distance in metres east and north from an arbitrary reference point. I want to set up a wireless networking connection between my home and the school and I need to know what direction to point the antennas. To the nearest 100m, the grid coordinates of my home are Easting 386100, Northing 6470900. Similarly, the grid coordinates of the school are Easting 384800, Northing 6490200.



- a) On what bearing should I point the antenna at my home?



$$\theta = \tan^{-1}\left(\frac{1300}{19300}\right) = 3.85^\circ$$

$$360 - 3.85 = 356.15$$

Bearing 356.15°

- b) On what bearing should I point the antenna at the school?



$$180 - 3.85 = 176.15$$

Bearing 176.15°

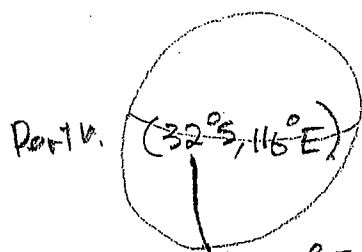
- c) How far apart will the two antennas be?

$$\begin{aligned} \text{distance} &= \sqrt{1300^2 + 19300^2} \\ &= \underline{19343.73\text{m}} \end{aligned}$$

QUESTION 15 [2, 2, 2 marks]

For this question, use 6350km as the radius of the Earth.

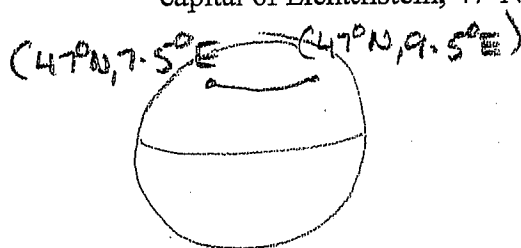
- a) How far is it from Perth (latitude 32° South, longitude 116° East) and the South pole, travelling along the 116° East meridian?



$$\begin{aligned} \text{Distance} &= \frac{\text{angle}}{360} \times 2\pi r & \text{angle} &= 90 - 32 = 58^\circ \\ &= \frac{58}{360} \times 2 \times \pi \times 6350 \\ &= \underline{6428.05 \text{ km}} \end{aligned}$$

South pole (90°S, 116°E)

- b) How far is it between Bern (the capital of Switzerland, 47° N, 7.5° E) and Vaduz (the capital of Lichtenstein, 47° N, 9.5° E) along their common line of latitude?



$$\begin{aligned} \text{Angle} &= 9.5 - 7.5 = 2^\circ \\ \text{Distance} &= \frac{2}{360} \times 2 \times \pi \times 6350 \times \cos 47^\circ \\ &= \underline{151.17 \text{ km}} \end{aligned}$$

- c) Explain the reason for the difference between the formula for distance along a common line of latitude and that for the distance along a common line of longitude.

Travelling along the line of longitude is a circle whose radius is that of the earth, whereas travelling along a line of latitude is a circle with a smaller radius (unless travelling along equator).

QUESTION 16 [1, 1, 2 marks]

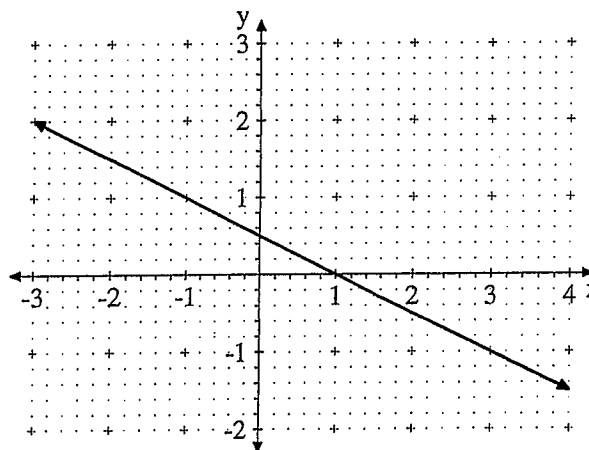
With reference to the line shown,

- a) What is the gradient of the line?

$$-\frac{1}{2}$$

- b) What is the y-intercept of the line?

$$(0, 0.5)$$



- c) Write the equation of the line.

$$y = -\frac{1}{2}x + 0.5$$

QUESTION 17 [2, 2 marks]

Use the graph shown to determine the solutions to the following simultaneous equations, and mark the point on the graph that corresponds to each solution.

a) $y = x - 10$

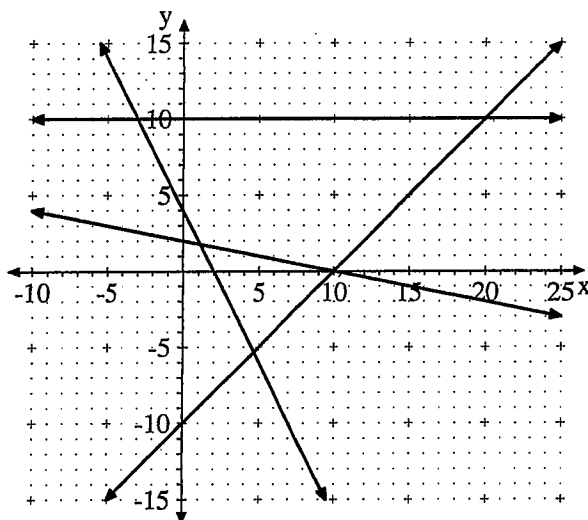
$y = 10$

$x = 20$ $y = 10$

b) $x - y = 10$

$y = 2 - 0.2x$

$x = 10$ $y = 0$

**QUESTION 18 [2, 3 marks]**

- a) For each of the following tables determine whether the relationship between x and y is linear.

i)

x	0	1	2	3	4	5
y	-10	-6	-2	2	6	10

linear

ii)

x	6	7	8	9	10	11
y	228	164	132	116	108	104

not linear

iii)

x	-2	-1	0	1	2	3
y	18.50	18.25	18.00	17.75	17.50	17.25

linear

iv)

x	1	2	3	4	5	6
y	-0.5	0	1.5	4.0	7.50	12.0

not linear

- b) Determine the equation of the relationship shown in the following table:

x	0	1	2	3	4	5
y	-12.5	-15.0	-17.5	-20.0	-22.5	-25.0

$c = -12.5$
 $m = -2.5$

$m = -2.5$

$c = -12.5$

$y = -2.5x - 12.5$

QUESTION 19 [4, 4, 2 marks]

- a) Find the equation of a line that passes through point (1,5) and has a gradient of $m=2$.

$y = mx + c$

$5 = 2 \times 1 + c$

$c = 3$

$y = 2x + 3$

- b) Find the equation of a line that passes through points (3, 3) and (-2, 2).

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 3}{-2 - 3} = \frac{1}{5}$

$y = mx + c$ $(3, 3)$ $m = \frac{1}{5}$

$3 = \frac{1}{5} \times 3 + c$

$c = 2\frac{2}{5}$

$y = \frac{1}{5}x + 2\frac{2}{5}$

OR $y = 0.2x + 2.4$

- c) Using your graphics calculator, or otherwise, find the coordinates of the point where the line in part a intersects the line in part b.

$y = 2x + 3$ ——— ①

$y = 0.2x + 2.4$ ——— ②

$10y = 2x + 24$ ——— ② $\times 10$

$-(y = 2x + 3)$ ——— ①

$9y = 21$

$y = \frac{21}{9} = 2\frac{1}{3}$

$2\frac{1}{3} = 2x + 3$

$2x = -\frac{2}{3}$

$x = -\frac{1}{3}$

Answer: $(-\frac{1}{3}, 2\frac{1}{3})$

Elimination method.

QUESTION 20 [2, 3, 5 marks]

For the following questions, point A has coordinates (3,11) and point B has coordinates (8,23).

- a) Find the distance between point A and point B.

$$\begin{aligned} \text{distance} &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(23 - 11)^2 + (8 - 3)^2} \\ &= 13 \text{ units} \end{aligned}$$

- b) Find the coordinates of point G, lying on AB, such that AG:GB = 4:3.

$$\begin{array}{c} \text{A} \quad \quad \quad \text{G} \quad \quad \quad \text{B} \\ (3, 11) \quad \quad \quad (8, 23) \\ \times 3 \quad \quad \quad \times 4 \\ \hline 7C = \frac{(23 \times 3) + (8 \times 4)}{7} = \frac{41}{7} \\ \hline \end{array} \quad \begin{array}{c} y = \frac{(11 \times 3) + (23 \times 4)}{7} \\ = \frac{125}{7} \\ \hline \end{array}$$

$\left(\frac{41}{7}, \frac{125}{7} \right)$

- c) Point J has coordinates (k, 17.0) and is 18.2 units away from point A. Find two possible values for k. (Note: point J does not lie on $\overline{AB}$)

$$\begin{aligned} \text{dist} &= \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} \\ 18.2 &= \sqrt{(k - 3)^2 + (17 - 11)^2} \\ 331.24 &= (k - 3)^2 + 36 \\ 295.24 &= (k - 3)^2 \\ k - 3 &= \pm \sqrt{295.24} \\ k - 3 &= 17.18 \text{ or } -17.18 \end{aligned}$$

Answer: $k = -14.18$ $k = 20.18$

QUESTION 21 [1, 3 marks]

For $f(x) = 8x - 11$ and $g(x) = -0.5x + 2$

- a) Determine $f(3)$

$$\begin{aligned} f(3) &= 8 \times 3 - 11 = 13 \\ f(3) &= 13 \end{aligned}$$

- b) Find the value of p where $f(p) - g(p) = 46.5$

$$\begin{aligned} 8p - 11 - (-0.5p + 2) &= 46.5 \\ 8p - 11 + 0.5p - 2 &= 46.5 \\ 8.5p &= 59.5 \\ p &= 7 \end{aligned}$$

QUESTION 22 [3, 3]

- a) What numbers (if any) must be **excluded** from the domain of x for the following functions?

i) $h(x) = \sqrt{x+9}$

$$x < -9$$

ii) $j(x) = \frac{6}{x-2.5}$

$$x = 2.5$$

iii) $k(x) = x^2 - 9$

none

- b) What numbers (if any) are impossible values for the following functions to output? (That is, what numbers are **excluded** from their range?)

i) $m(x) = \frac{2}{2x+3}$

$$m(x) = 0$$

ii) $p(x) = \sqrt{x-1}$

$$p(x) < 0$$

iii) $q(x) = x^2 + 2$

$$q(x) < 2$$

** END OF EXAM **

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$$m(x) = 0$$

ii) $p(x) = \sqrt{x-1}$

$$p(x) < 0$$

iii) $q(x) = x^2 + 2$

$$q(x) < 2$$

**** END OF EXAM ****