

ST STEPHEN'S SCHOOL CARRAMAR

INTRODUCTORY CALCULUS

TEST 3: INDICES, LOGARITHMS, SERIES – APRIL 2007

TIME: 75 minutes

MARKS: 72

NAME: _____

In order to receive maximum marks, show **all** working:

Question 1 [2, 2, 2 marks]

Simplify each of the following, expressing with positive indices:

a)
$$\frac{(a^2 b^{-1})^2}{8ab^2} \times \frac{(2a^3 b^2)^4}{4a^2 b}$$

b)
$$2^{2x+2y} \cdot 4^{2x+y} \cdot 16^{0.5x-y}$$

c)
$$\frac{5 \cdot 7^x + 45}{27 + 3 \cdot 7^x}$$

Question 2 [3, 3, 1 marks]

Solve correct to 3 decimal places by using any appropriate method:

a) $(3x - 1)^{\frac{2}{3}} = 4$

b) $5^{x-1} = 168$

c) Evaluate: $\log_4 32$

Question 3 [5, 3 marks]

Simplify each of the following, express with positive indices:

a) $\sqrt{\frac{4^{-1} x y^{-2} z^3}{4 x^3 y^{-4} z}} \div \left(\frac{2 x^{-2} y^0 z^{-1}}{3 x^{-3} y^2 z} \right)^{-2}$

b)
$$\frac{2^{x+1} + 2^{x-1}}{2^{x+2}}$$

Question 4 [1, 1, 2, 3 marks]

If $x = \log p$, $y = \log q$ and $z = \log r$, then determine each of the following in terms of x , y and/or z .

a) $\log p \cdot q$

b) $\log r^2$

c) $\log \frac{pr}{q}$

d) $\log \sqrt{qr^3}$

Question 5 [5 marks]

A medical supplier packs and sells first aid kits. The time taken to pack a kit depends on the experience of the packer. A first - time packer takes 12 minutes to pack a kit. Thereafter , the time for each kit is 4% less than the time taken for the previous kit.
How many hours would it take to pack

- (a) the tenth kit?

- (b) 10 kits?

- (c) How many kits could be packed in a 3-hour shift?

Question 6 [6, 3, 4 marks]

(a) Given a geometric sequence where the sixth term is -5.0625 and the second term is -1 then determine :

(i) a , the first term and r , the common ratio .

(ii) the sum to infinity.

b) Determine the value(s) of x such that $1 + x + x^2 + x^3 + \dots$ is equal to 2.5

c) Find the value(s) of x for which $x + 1$, $4x - 2$, $10x + 4$, form a geometric progression and then give the 1st 3 terms.

Question 7 [5 marks]

Solve for a and b , if $27^a = 9^b$ and $81^b = \frac{243}{3^a}$.

Question 8 [2, 3 marks]

Find y in terms of x :

a) $\log y = 3\log x - \log x^2$

b) $2\log y = \log (4x^2) + 2\log x$

Question 9 [1, 1, 2, 2, 3 marks]

Solve the following equations where possible expressing your answers as exact values.

(a) $\log_3 1 = x$

(b) $5^{x+1} = -3$

(c) $\log(x-3) = 2$

(d) $\log_3\left(\frac{2x+1}{x}\right) = -2$

(e) $\frac{5}{\sqrt[3]{x^2}} = 80$

Question 10 [2, 2, 3 marks]

A disease is spreading through a local community. The first day the disease was identified, 150 people were diagnosed as having it. The following day another 162 people were diagnosed as having the disease. On the third day of this epidemic a further 175 people were diagnosed positive and another 189 on the fourth day. Assume the trend continues.

- (a) Find the number of people diagnosed on the 10th day.

- (b) How many people were diagnosed positive after 12 days?

- (c) If the population of this small community is 12000, how many days will pass before everybody is diagnosed as positive?