YEAR 10 MATHEMATICS DEVELOPMENT EXAM REVISION SEMESTER 2

Chapter 1: Number

1. Simplify the following ratios:
   a. 1.25 : 5
   b. 12: 18 : 24
   c. $2\frac{1}{4} : 5\frac{3}{4}$
   d. 6 m : 200 cm
   e. 1200 s : 12 min
   f. 200 mm : 4 cm
   g. 2.5 L : 25 mL

2. Sally makes tomato sauce by mixing crushed tomatoes, salt and sugar in the ratio of 25 : 1 : 4. In 120 cups of sauce how much sugar is there?

3. List the prime factors of 630, 924 and 2730 and use them to find their highest common factor.

4. Water is leaking out of a tap at a rate of 500 mL/h:
   a. How much will leak out in 1 day?
   b. How much would leak out in 1 year? Give your answer in kilolitres.
   c. If water costs \$20 for the first kilolitre and then \$0.68 for every kilolitre thereafter, how much would the water leak cost for a year?

5. Calculate each of the following, using scientific notation correct to four significant figures:
   a. $(3.2 \times 10^{-3})^3 \times \pi^{-4}$
   b. $\dfrac{\sqrt{3.098 \times 10^{-12}}}{18.093^{\frac{2}{3}}}$

6. Find the distance that a car moving at 110 km/h would travel in:
   a. 2 hours
   b. 30 minutes
   c. 45 seconds

7. Calculate the following, expressing answers using scientific notation expressed to three significant figures.

   a. $(5.48 \times 10^2) \times (7.287 \times 10^5)$

   b. $\dfrac{(1.025 \times 10^{-2} + 23.023)^3}{(8.62 \times 10^3 - 9.26 \times 10^5)^{-1}}$

8. Find the density of granite in grams per cubic centimeter if $230\ \boldsymbol{cm}^3$ has a mass of 2.047kg.

9. Find the mass of nylon if the density is 1.14 g/cm^3 and the volume is 52 cm^3

Chapter 2: Business Maths

10. Express each of the following as a percentage:

a. $\frac{2}{5}$

b. $\frac{3}{7}$

c. 0.12

d. 2.67

11. Find the percentage change for the following share prices:

a. Aussie Pets: \$1.20 to \$0.85

b. Flemington Bank: \$9.34 to \$12.22

c. Bikea: \$0.76 to \$1.24

12. The price of cricket bats is marked up by 25% at the local sport shop. If the shop buys them for \$18, calculate:

   a. the selling price

   b. the profit made when each bat is sold

13. Francesca, who sells real estate, is paid a monthly amount of \$500 plus 1.5% of sales up to \$120 000 and 0.6% of sales above \$120 000. Find her salary for the month after she sold properties worth \$189 000, \$265 000 and \$345 000

14. Find the simple interest paid when \$20 000 is invested at the rate of 7.2% for three and a half years.

15. How much money would I need to invest at a simple interest rate of 7% over $5\frac{1}{2}$ years, in order to make a total of $8470 in interest?

16. Find the difference in interest payable when $128 000 is invested at the rate of 8% per annum for 2 years being compounded 6-monthly compared with being compounded monthly.

17. $42 000 is invested at the annual rate of 4.8% compounding monthly. Find the value of the investment after 4 months and so find the annual simple interest rate that would be required to produce the same result.

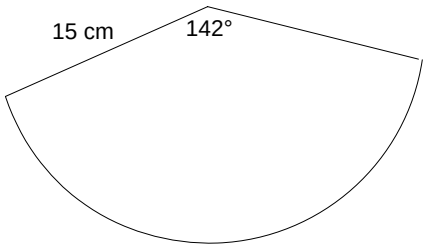
18. Honest Ernie the moneylender lends William $12 000 at the rate of 12% per annum compounding daily. Find the amount of interest that needs to be paid in a non-leap year if the loan is paid off after:
    a. 100 days
    b. 250 days
    c. 320 days

Chapter 3: Measurement

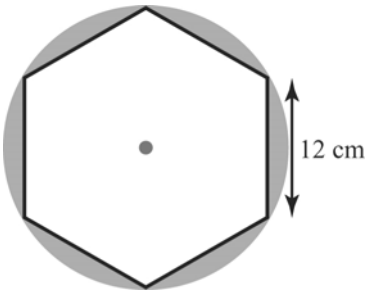
19. Find the perimeters of the following to the nearest millimetre:
    a. circle, diameter 2.45 m
    b. rhombus, edge length 12.067 cm
    c. right-angled triangle with adjacent sides of length 14.2 cm and 8.02 cm

20. Find the lengths of two possible isosceles triangles with one side 9 cm shorter than one of the other sides, so that its perimeter is 24.24 cm.

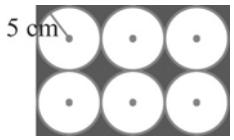
21. Find the area of this sector



22. Find the shaded area in the diagram below expressed:
    a. in exact form
    b. correct to two decimal places

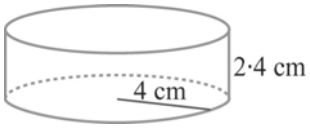


23. Find the shaded area, expressed to two decimal places:

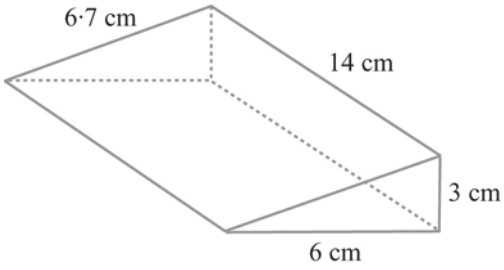


24. Calculate the total surface areas and volumes of the following correct to two decimal places where necessary:

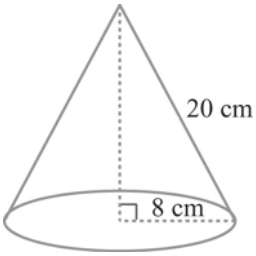
a.



b.

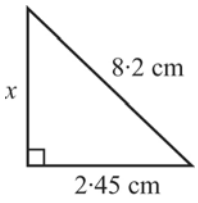


c.

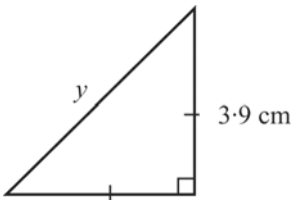


25. Find the value of the unknown sides expressed correct to two decimal places:

a.

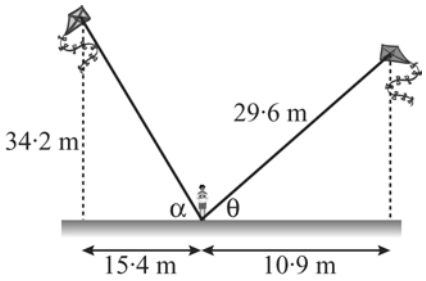


b.

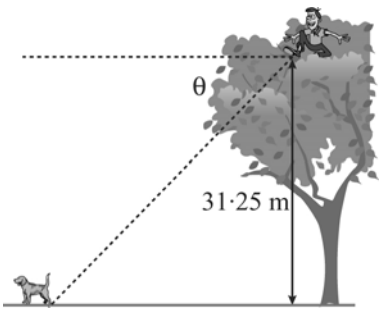


Chapter 5: Trigonometry

26. Two stunt kites are flown as shown below. Find the angles α and θ that the strings make with the ground expressed correct to two decimal places.



27. William has climbed a tree. When he is perched at the top, he is 31.25 metres above the ground. His dog Jo-Jo runs away from the base of the tree at 8 m/s. Find the angle of depression θ from Jo-Jo to William after:



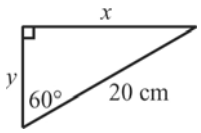
a. 2 s

b. 7.5 s

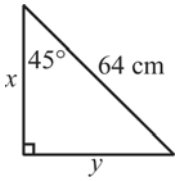
c. half a minute

28. Find the lengths of the unknown sides expressed in exact form:

a)



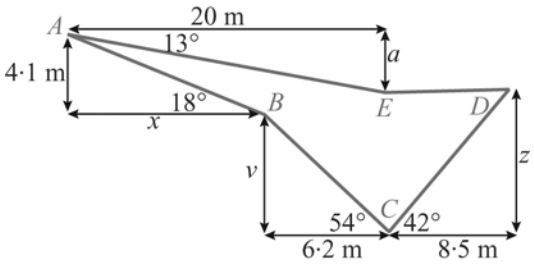
b)



29. An exercise circuit *ABCDEA* is shown below with point *E* vertically above *C*. Find, expressed to the nearest centimetre:

a. the unknown lengths

b. the total length of the course



30. Give the true bearings of the following compass bearings:

a. N42°E

b. S32.2°W

Give the compass bearings of the following true bearings:

c. 025.6°T

d. 165°T

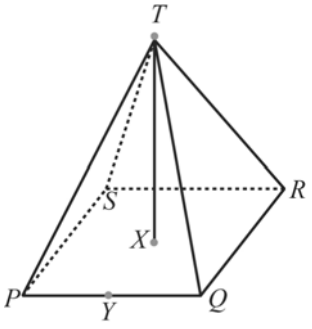
31. *PQRST* is a square-based pyramid with a base length of 60 cm and a height of 80 cm. The point *X* is the centre of the base and *Y* is the midpoint of *PQ*.

a. Find the length of the line *XY*.

b. Find the angle that the face *PQT* makes with the base expressed to the nearest tenth of a degree.

c. Find the length of a sloping edge correct to the nearest millimetre.

d. Find the angle that the sloping edge makes with the base expressed in degrees and minutes.



Chapter 6: Algebra Techniques

32. Expand the brackets and simplify each of the following:

a. $2x(x-1)-3(x+1)$ b. $(3x-1)^2$ c. $(x+1)(x-2)(x+3)$

33. Factorise the following:

a. $-5ab^2-10a^2b$ b. $2x(x+1)-3(x+1)$ c. $5x(1-x)+x-1$
d. $6xy+3a+6ax+3y$ e. $48b^3-75a^2b$ f. $(2x-3)^2-9x^2$
g. $(x-2)^2-3$ h. $16x^4-81y^4$ i. $(2x-1)^2-(3-x)^2$
j. x^2-x-20 k. $14+31x-10x^2$ l. $6x^2+23x+20$
m. $4x^2+2x-20$

34. Simplify the following:

a. $\frac{x}{4}-\frac{x}{3}$ b. $\frac{x+3}{5}+\frac{x}{2}$ c. $\frac{1}{x-2}+\frac{3}{x}$ d. $\frac{3}{x+4}+\frac{7}{x-2}$

35. Simplify the following:

. a. $\frac{18x-3y}{6x-36y}$ b. $\frac{x^2-9}{x^2-2x-3}$ c. $\frac{3x^2+2x-5}{2x^2+x-3}$

d. $\frac{x^2+2x-3}{x^2+5x+6}\times\frac{x^2+3x+2}{x^2+3x-4}$ e. $\frac{6x^2-11x-2}{x^2-1}\div\frac{6x^2-5x-1}{x^2+3x+2}$

36. Solve the following equations:

a. $-\frac{3}{4}x=21$ b. $\frac{x}{3}-11=-12$ c. $\frac{2x-4}{5}=13$

37. Solve the following equations:

a. $-2(x-4)=22$ b. $\frac{3(x+2)}{5}-6=12$

38. Solve the following equations:

a. $3x-6=7x+6$ b. $15(x-2)=2(4x-1)$ c. $\frac{3x}{2}+\frac{x}{3}=11$

d. $\frac{3x-1}{5}-\frac{x-2}{2}=2$

39. Solve the following equations

a. $x+4>-5$ b. $\frac{2x}{7}\le -6$ c. $3(x-4)<15$ d. $\frac{3x-1}{5}\ge -5$

e. $\frac{x-3}{2}+5<-1$

40. For each of the following, write an equation and solve it to find the unknown quantity:

a. When four kilograms of sand is added to three-fifths of some cement from a full bag of cement the total weight is 16 kg. Find the weight of cement in the full bag.

b. When a number is reduced by five and the result is halved, the result is twelve.

c. When two-thirds of a number has four added to it the result is negative eight.

41. The formula that can be used to find the surface area A cm^2 of a solid cylinder with radius r cm and height h cm is $A = 2\pi r(r + h)$.

a. Find A when $r = 10$, $h = 15$.

b. Transpose to make r the subject. Hence, find r when $A = 120$, $h = 20$.

c. Transpose to make h the subject. Hence, find h when $A = 220$,$r = 15$.

42. Simplify each of the following expressions:

a. $\dfrac{16x^3}{3y} \times \dfrac{\left(-2y^3\right)^2}{5x^2y^3}$

b. $(ab^3c)^2 \times (3abc^3)^3$

c. $\dfrac{-30a^2b^3}{3a^3b} \times \dfrac{2a^5b^3}{-6ab}$

d. $\dfrac{\left(3a^3b^2\right)^2}{(2ab)^3} \div \dfrac{\left(-4a^2b^3\right)^2}{8a}$

43. Simplify, writing answers with positive powers

a. $\left(\dfrac{a^2b}{x^2}\right)^3 \times \left(\dfrac{5ab}{12xy^2}\right)^{-1}$

b. $\left(\dfrac{3a^{-1}b^2}{x^{-3}y}\right)^{-1} \div \left(\dfrac{4a^3b}{5xy^{-2}}\right)^2$

44. Simplify, writing answers with positive powers

a. $\left(25a^{-4}\right)^{\frac{3}{2}}$

b. $\left(-3125a^5b^2\right)^{\frac{2}{5}}$

c. $\left(\dfrac{64y^3}{27x^{-2}}\right)^{-\frac{1}{3}}$

d. $\dfrac{3a^2b^{\frac{1}{4}}}{5^{-1}b^2} \times \dfrac{2a^{\frac{1}{4}}b^2}{25a^{-1}b^{\frac{3}{2}}}$

e. $\dfrac{27a^{\frac{3}{4}}b^{-\frac{1}{2}}}{4x^{\frac{3}{4}}y} \div \dfrac{9ab^2}{12xy^{\frac{2}{5}}}$

f. $\dfrac{ab^{\frac{2}{3}}}{x^{\frac{3}{4}}} \div \left(\dfrac{3a^{\frac{1}{3}}b^{-1}}{-2x^{-\frac{3}{4}}}\right)^2$

Chapter 7: Linear Functions

45. Find the x- and y-intercepts for the following and use them to sketch the lines on the sets of axes provided:

a. $2x - 3y = -6$

b. $3x + y = -6$

c. $y = 1\dfrac{1}{4}x + 5$

d. $3y = 12 - 4x$

46. Find the equation of the line that passes through the following points:

a. (4, 12) and (2, 6)

b. (–2, 4) and (3, –6)

47. Rearrange the following to make y the subject and state the gradient and y-intercept:

a. $2x + 3y = -6$

b. $y - 4x = 20$

c. $\frac{x+3}{4} = 3y$

d. $\frac{2}{3}x + \frac{4}{5}y = 1$

48. Find the equation of the line that:

a. has a gradient of 2 and passes through the point (1, 5)

b. passes through the point (1, 12) and a gradient of –2

49. The length of a rectangle (y) is 6 cm longer than its width (x). The perimeter of the rectangle is 102 cm.

a. Form two equations.

b. Solve the equations using substitution and so state the length and width of the rectangle.

50. Solve each of the following pairs of simultaneous equations:

a. $3x + 4y = -5$
$x - y = 3$

b. $2x - 3y = 11$
$y = 2x - 1$

51. For the following two points (–2, 8) and (3, –4) find the:

a. midpoint between the two points

b. distance between the two points

52. State the translation applied to the graph of the function $f(x) = |x|$ to produce:

a. $f(x) = |x + 3|$

b. $f(x) = |x| - 4$

c. $f(x) = |x - 1| + 2$

53. Determine the equation of the lines that are

i) perpendicular to the given line and contain the given point

ii) parallel to the given line and contains the given point.

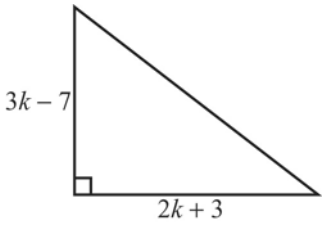
a. $y = 6x + 2$ $(9, 8)$

b. $y = -5x + 2$ $(1, 4)$

Chapter 8: Quadratic Functions

54. Write an algebraic expression for the area of each of the following:

a)



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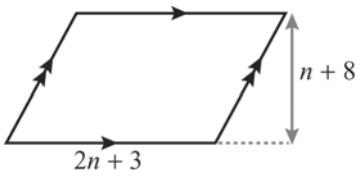


55. Solve each of the following equations:

a. $x^2 + 3x - 54 = 0$

b. $2x^2 - 17x + 35 = 0$

c. $4x^2 - 5x + 1 = 07x^2 = 16x + 15$

d. $\dfrac{(x-5)^2}{3} = 9$

e. $3(x + 3)^2 - 18 = 0$

f. $\dfrac{3(x-10)^2}{5} - 12 = 0$

56. Use the quadratic formula to solve the following equations, expressing the answers (i) in exact form and (ii) correct to two decimal places:

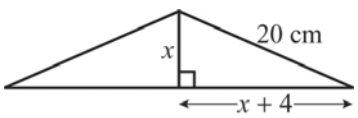
a. $2x^2 + 6x - 3 = 0$

b. $3x^2 - 4x - 6 = 0$

c. $-2x^2 + 7x - 5 = 0$

57. Find the value of x in the following:

a)



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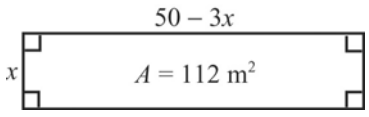


58. For each of the graphs listed, find, using a graphics calculator or otherwise:

a. $y = 2x^2 - 32$

b. $y = 2x^2 - x - 6$

i. the x- and y-intercepts

ii. the axis of symmetry

iii. the co-ordinates of the turning point

59. A series of parabolas have the general equation $y = (x - h)^2 + k$. Write the equation of each if the turning point is:

a. $(3, -2)$

b. $(-1, 0)$

c. $(-6, 9)$

d. $(\frac{1}{2}, 2\frac{1}{2})$

60. Starting with the parabola with equation $y = x^2$, state the transformations required to move it to the positions of the following:

a. $y = (x - 2)^2 - 4$

b. $y = (x + 1)^2 + 3$

c. $y = -(x - 1)^2 - 3$

d. $y = 2(x + 5)^2 + 2$

61. A parabola of the form $y = x^2 + bx + c$ has x-intercepts of -1, 3 and a y-intercept of -3.

a. State the equation of the parabola.

b. Find the co-ordinates of the turning point.

62. Solve each of the following:

a. $x^2 + 3x + 2 < 0$

b. $x^2 - x - 6 > 0$

c. $x^2 + 5x - 14 \geq 0$

d. $-x^2 + 7x - 12 \leq 0$

63. For the parabola $y = x^2 - 4x - 1$:

a. complete the square to express the equation in the form $y = (x - h)^2 + k$ and so state the co-ordinates of the turning point

b. find the x-intercepts expressed in exact form and correct to one decimal place

Chapter 9: Probability

64. Cards with numbers 1, 2, 3 and 5 are placed face down on a table. Two are chosen at random and are used to form a two-digit number.

Find the probability that the two-digit number is:

a. even
b. a multiple of 5
c. less than 20
d. greater than 20
e. at least 31

65. Twelve people participated in a survey to ascertain their favourite colour. Six people liked blue, five people liked red and one person liked neither red nor blue.

a. Draw a Venn diagram to show the information.
b. Find the probability that if a person was chosen at random they would prefer:
   i. blue only
   ii. red only
   iii. both red and blue

66. When students were surveyed as to which sport, tennis or squash, they enjoyed playing, the probability that a person enjoyed tennis was 0.2, that of squash was 0.5 and both 0.1.

a. Represent the information in a two-way table.
b. Find the probability that a person chosen at random would enjoy:
   i. squash but not tennis
   ii. tennis but not squash
   iii. neither sport
   iv. not squash
   v. not tennis

67. A coin is biased so that the chance of a head being tossed is three times that of a tail occurring. In a game the coin is tossed three times.

a. Show the outcomes on a tree diagram and write the probabilities along the branches.
b. Find the probability of tossing:

i) three tails in a row
ii) three heads in a row
iii) two heads and a tail in that order
iv) two heads and a tail
v) at least two heads
vi) at most one tail

68. A company has two machines that make basketballs. Machine A produces 95% acceptable basketballs while Machine B produces 90% basketballs that are acceptable and are able to be sold. Find the probability that Machine A produces basketballs that are:

a. acceptable

b. not acceptable

Find the probability that Machine B produces basketballs that are:

iv. acceptable

v. not acceptable

Machine A produces 40% of the basketballs manufactured at the factory. If a basketball is chosen at random find the probability that it was produced by:

vi. Machine A

vii. Machine B

A basketball that is chosen at random is found not to be acceptable. Find the probability that it was produced by:

viii. Machine A

ix. Machine B

69. Fourteen lollies are placed in a bag. Two are mints, three are jellybeans and the remainder are liquorice. Three lollies are selected at random without replacement. Find the probability that:

a. the lollies are the same

b. the lollies are different

Chapter 10: Statistics

70. Telephone calls made to a hardware store are timed and the times recorded in minutes are as follows:

2.4, 14.2, 1.4, 3.7, 3.4, 6.2, 11.3,
1.7, 4.7, 6.2, 4.2, 0.5, 1.9, 0.2,
0.3, 4.2, 5.2, 1.4, 8.3, 1.8, 2.8,
1, 1.2, 0.9, 2.6, 2.5, 2.5, 0.4,
0.1, 2.6, 3.2, 0.2, 0.4, 0.7, 0.2,
2, 8, 3.2, 3.8, 2.1, 2.4, 3.1,
3.9, 3.2, 10.4, 1.4, 1.1, 5.8, 5.8,
3.9, 3.5, 6, 4.7, 4.4, 5.8, 7.7,
2.8, 1.6, 1.3, 3.4, 1.7, 1.4, 0.2,
0.6, 3.5, 3.8, 13.2, 7.8, 0.3, 0.8,
2.3, 1.7, 9.2, 2,7, 3.3, 7.9, 15.8,
1.9, 1.8, 1.9, 1.4, 9, 3.2, 2.4,
5, 8.4

a. Make a frequency table using intervals of 2 minutes.

b. Show the information as a histogram.

c. Describe what the histogram shows.

71. Two types of light globes are tested to judge their quality. The times in hours of continuous operation for a sample of 100-watt globes are as follows:

Type A

243, 326, 278, 289, 303, 271

Type B

275, 265, 320, 333, 290, 227

c. Find the mean $\bar{x}$ and standard deviation S_x for each type of light globe, and, using these measures, state which gives the more consistent performance.

Calculate $\bar{x} \pm S_x$ for each type of globe and state the number of globes of each type that fall within these limits.

72. Billy Bob the boaster is always showing off by making incredible claims. At the latest comedy festival, a machine is set up to measure the volume of applause after each of his acts.

The scores were recorded as follows:

24, 46, 50, 48, 39, 47, 42, 60

d. Find the five-number summary for the data and use it to plot a boxplot of Billy Bob's scores.

e. Find the interquartile range of the scores.

At a party Billy Bob was overheard to state that his best score for the festival was about 75. Is this reasonable in comparison with his other scores?

Chapter 11: Variation, Exponential and Reciprocal Functions

73. The voltage *v* that powers a personal CD player is directly proportional to the current *I* amps. A personal CD player with constant resistance uses a current of 0.25 amp when powered by a voltage of 9 volts.

a. Find the resistance of the CD player which is the constant of proportionality.

b. Find the voltage that is needed to operate the CD player that would use a current of 0.4 amp.

c. If a 4-volt battery operated the CD player, what current would be used?

74. The volume v cm^3 of gas in a cylinder at fixed temperature is inversely proportional to the pressure p expressed in kg/cm^3.

a. If a volume of 26 cm^3 of gas exerts a pressure of 2 kg/cm^3, find a rule that connects the volume of gas in the cylinder with its pressure.

b. Find the pressure if the volume is:

i. increased to 52 cm^3

ii. decreased to 18 cm^3

c. What volume is required for the pressure of the gas to be:

i. increased to 2.5 kg/cm^3?

ii. decreased to 1.8 kg/cm^3?

iii. halved?

iv. doubled?

75. Write the rules for the following.

a. y varies as the square of x and when $y = 1.96$, $x = 2.8$.

b. a varies inversely as the cube of b and when $a = 8$, $b = 4$.

76. For the function $y = \frac{1}{x}$, state the new equation after the following transformations and state the equations of the asymptotes.

a. translated 2 units right

b. translated 1 units left

c. translated 6 units up and 3 units left

Chapter 12: Trigonometric and Cubic Functions

77.

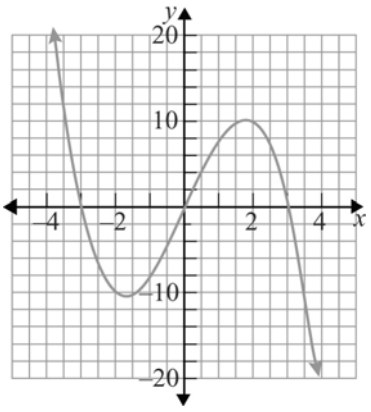
a. If the coefficient of x^3 is –1, state the equation of the cubic curve shown on the set of axes below.

b. State the equation of the cubic and sketch it on the set of axes after the following transformations:

i) translated 2 units right

ii) translated 1 units left

iii) reflected in the x-axis then translated 1 unit up



78. State the period and amplitude for the following functions:

a. $y = 6\cos 2x$

b. $y = \frac{1}{2}\sin 3x$

c. $y = -4\sin\left(\frac{x}{2}\right)$

d. $y = -2\tan 3x + 4$

79. State the equation of the function after $y = (x-1)(x+2)(x-4)$ is translated to the right by 2 units, down by 3 units, then reflected in the x-axis.

80. Write the equation of a sin function that has a period of 45^0 and an amplitude that is double that of $y = \frac{\sin x}{6}$

Answers

Chapter 1

1) a) 1:4 b) 2:3:4 c) 9:23 d) 3:1 e) 5:3 f) 5:1 g) 100:1

2) 16 cups

3) 630 = 2 x 3 x 3 x 5 x 7, 924 = 2 x 2 x 3 x 7 x 11 2730 = 2 x 3 x 5 x 7 x 13 Highest common factor: 2 x 3 x 7 = 42

4) a) 12 L b) 4·38 kL c) $22·30

5) a) $3{\cdot}364 \times 10^{-10}$ b) 2.554×10^{7}

6) a) 220 km b) 55 km c) 1375 m

7) a) $3{\cdot}99 \times 10^{8}$ b) -1.11×10^{10}

8) $8.9\ gm/cm^3$

9) 59.28 g

Chapter 2

10)

a 40%

b $42\frac{6}{7}\%$

c 12%

d 267%

11)

a 29·2% loss; $\left(\frac{0{\cdot}35}{1{\cdot}20} \times 100\right)\%$

b 30·8% gain; $\left(\frac{2{\cdot}88}{9{\cdot}34} \times 100\right)\%$

c 63·2% gain; $\left(\frac{0{\cdot}48}{0{\cdot}76} \times 100\right)\%$

12)

a $18 × 1·25 = $22·50.
The selling price is $22·50.

b Profit: $22·50 – $18 = $4·50

13)

$189 000 + $265 000 + $345 000 = $799 000
Francesca's monthly pay is $500 + 1·5% of $120 000 + 0·6% of $679 000 and totals $6374.

14)

$$\$20\,000 \times \frac{7{\cdot}2}{100} \times 3{\cdot}5 = \$5040$$

The simple interest paid is $5040.

15) $22000

16) $\$128\,000((1{\cdot}006)^{24} - (1{\cdot}04)^4) = \$387{\cdot}76$

17)

$42 000 increases to $42 676·04 through interest of $676·04 in 4 months. The equivalent simple interest rate x% is such that $42\,000 \times x = 676{\cdot}04$.
$x = 1{\cdot}6$% for four months and 4.8% per annum.

18)

An interest rate of 12% per annum converts to $\frac{12}{36\,500}$ % per day.

a $\$12\,000 \times \left(1 + \frac{12}{36\,500}\right)^{100} = \$12\,401{\cdot}01$

The interest is $401·01.

b $\$12\,000 \times \left(1 + \frac{12}{36\,500}\right)^{250} = \$13\,027{\cdot}79$

The interest is $1027·79.

c $\$12\,000 \times \left(1 + \frac{12}{36\,500}\right)^{320} = \$13\,331{\cdot}04$

The interest is $1331·04.

Chapter 3

19)

a 7697 mm b 483 mm c 385 mm

20)

If one side is x, the other sides are both $(x - 9)$.
$x + (x - 9) + (x - 9) = 24{\cdot}24$ and

$3x - 18 = 24{\cdot}24$, $x = \frac{42{\cdot}24}{3} = 14{\cdot}08$

The sides are 5·08 cm, 5·08 cm and 14·08 cm.

If one side is x, the other sides are $(x - 9)$ and x.
$x + (x - 9) + x = 24{\cdot}24$ and

$3x - 9 = 24{\cdot}24$, $x = \frac{33{\cdot}24}{3} = 11{\cdot}08$.

The sides are 11·08 cm, 2·08 cm and 11·08 cm.

21) 278.82 square cm

22)

The hexagon can be divided into 6 equilateral triangles of height $6\sqrt{3}$ cm and base length 12 cm. The area of each triangle in cm^2 is $\frac{1}{2} \times 12 \times 6\sqrt{3} = 36\sqrt{3}$.

a The shaded area is $(144\pi - 216\sqrt{3})$ cm^2.

b 78·27 cm^2.

23) $A = 20\times 30 - 6\pi\times 5^2 = 128{\cdot}76$
The area is 128·76 cm^2.

24) a) surface area = 160.85 sq cm volume = 120.64 cubic cm
b) surface area = 237.8 sq cm volume = 126 cubic cm
c) surface area = 703.72 sq cm volume = 1226.48 cubic cm

25)

a $x^2 + 2{\cdot}45^2 = 8{\cdot}12^2$
$x^2 = 67{\cdot}24 - 6{\cdot}0025$
$x = \sqrt{61{\cdot}2375} = 7{\cdot}83$
The length x is 7·83 cm.

b $3{\cdot}9^2 + 3{\cdot}9^2 = y^2$
$y^2 = 15{\cdot}21 + 15{\cdot}21$ and so $y = \sqrt{30{\cdot}42} = 5{\cdot}52$.
The length y is 5·52 cm.

Chapter 5

26) From the diagram $\tan\alpha = \frac{34{\cdot}2}{15{\cdot}4}$ and $\cos\theta = \frac{10{\cdot}9}{29{\cdot}6}$
so $\alpha = 65{\cdot}7583°$ and $\theta = 68{\cdot}3926°$.
$\alpha = 65°45'$ and $\theta = 68°24'$.

27) a After 2 s the angle of depression θ is given by
$\theta = \tan^{-1}\frac{31{\cdot}25}{2\times 8}$, θ is 62·9°.

b After 7·5 s, $\theta = \tan^{-1}\frac{31{\cdot}25}{7{\cdot}5\times 8}$, θ = 27·51°

c After 30 s, $\theta = \tan^{-1}\frac{31{\cdot}25}{30\times 8}$, θ = 7·42°

28)

a $x = 20\sin 60°$ and $y = 20\cos 60°$
$= 20\times\frac{\sqrt{3}}{2}$ $= 20\times\frac{1}{2}$
$x = 10\sqrt{3}$ cm and $y = 10$ cm.

b As the triangle is isosceles $x = y$.
$x = 64\cos 45° = 64\frac{1}{\sqrt{2}} = 32\sqrt{2}$.
x and y are each $32\sqrt{2}$ cm.

29) a $a = 20\tan 13° = 4{\cdot}62$ m
$v = 6{\cdot}2\tan 54° = 8{\cdot}53$ m
$x = \frac{4{\cdot}1}{\tan 18°} = 12{\cdot}62$ m
$z = 8{\cdot}5\tan 42° = 7{\cdot}65$ m
AB has length 13·27 m $\left(\frac{4{\cdot}1}{\sin 18°}\right)$.
BC has length 10·55 m $\left(\frac{6{\cdot}2}{\cos 54°}\right)$.
CD has length 11·44 m $\left(\frac{8{\cdot}5}{\cos 42°}\right)$.
DE has length 8·51 m
$DE = \frac{8{\cdot}5}{\cos\theta}$ where $\theta = \tan^{-1}\frac{(z-(41+v-a))}{8{\cdot}5}$
$= 2{\cdot}425°$
$DE = 8{\cdot}51$ m
AE has length 20·53 m $\left(\frac{20}{\cos 13°}\right)$.

30) a 042°T b 212·2°T
c N25·6°E d S15°E

31)

a 30 cm
b If α is the angle $\tan\alpha = \frac{80}{30}$: α = 69·4°
c $TQ = \sqrt{80^2 + (30\sqrt{2})^2}$ cm = 90·6 cm.
The length of a sloping edge is 90·6 cm.
d The edge makes the angle θ with the base such that $\theta = \tan^{-1}\frac{80}{30\sqrt{2}}$: θ = 62·06° or 62°4'.

Chapter 6

32) a) $2x^2 - 5x - 3$ b) $9x^2 - 6x + 1$ c) $x^3 + 2x^2 - 5x - 6$

33) a) $-5ab(b + 2a)$ b) $(2x - 3)(x + 1)$ c) $(5x - 1)(1 - x)$ d) $(6x + 3)(y + a)$
e) $3b(16b^2 - 25a^2)$ f) $(-x - 3)(5x - 3)$ g) $(X - 2 - \sqrt{3})(x - 2 + \sqrt{3})$
h) $(4x^2 - 9y^2)(4x^2 + 9y^2)$ i) $(3x - 4)(x + 2)$ j) $(x - 5)(x + 4)$ k) $-(5x + 2)(2x - 7)$
l) $(3x + 4)(2x + 5)$ m) $2(2x + 5)(x - 2)$

34) a $\frac{-x}{12}$ b $\frac{7x+6}{10}$
c $\frac{2(2x-3)}{x(x-2)}$ d $\frac{2(5x+11)}{(x+4)(x-2)}$

35) a $\frac{6x-y}{2(x-6y)}$

b $\frac{(x+3)(x-3)}{(x-3)(x+1)} = \frac{(x+3)}{(x+1)}$

c $\frac{(3x+5)(x-1)}{(x-1)(2x+3)} = \frac{(3x+5)}{(2x+3)}$

d $\frac{(x+3)(x-1)\times(x+2)(x+1)}{(x+2)(x+3)\times(x+4)(x-1)} = \frac{(x+1)}{(x+4)}$

e $\frac{(6x+1)(x-2)\times(x+2)(x+1)}{(x+1)(x-1)\times(6x+1)(x-1)}$
$= \frac{(x+2)(x-2)}{(x-1)(x-1)}$

36) a –28 b –3 c $34\frac{1}{2}$

37) a $x = -7$ b $x = 28$

38) a -3 b 4 c 6 d 12

39) a $x > -9$, $(-9, \infty)$
b $x \leq -21$, $(-\infty, -21)$
c $x < 9$, $(-\infty, 9)$
d $x \geq -8$, $(-8, \infty)$
e $x < -9$, $(-\infty, -9)$

40) a If x is the mass of a bag of cement
$4 + \frac{3}{5}x = 16$ and so $x = 20$.
The mass of a bag of cement is 20 kg.
b If x is the number $\frac{1}{2}(x - 5) = 12$ and $x = 29$.
The number is 29.
c $\frac{2}{3}x + 4 = -8$. The number is -18.

41) a As $A = 2\pi r(r + h)$, $A = 20\pi(10 + 15) = 500\pi$
The surface area is 500π cm^2 (≈ 1571 cm^2)
b $r(r + h) = \frac{A}{2\pi}$ is a part answer. A full answer can be obtained through the use of the difference of two squares method of factorising.
c $r(r + 20) = \frac{120}{2\pi} = \frac{60}{\pi}$:
$r^2 + 20r + 100 - 100 - \frac{60}{\pi} = 0$
$(r+10)^2 - \left(\sqrt{\frac{100\pi + 60}{\pi}}\right)^2 = 0$
$(r+10)^2 - 119{\cdot}1^2 = 0$
$(r + 10 + 119{\cdot}1)(r + 10 - 119{\cdot}1) = 0$
$r = -129{\cdot}1$ or $r = 109{\cdot}1$ both of which are impossible.
The area value of 120 is too small for a height of 20 cm.
d $h = \frac{A - 2\pi r^2}{2\pi r}$
e Clearly there is no solution for h as $2\pi 15^2$ is greater than 220 and h would be negative.

42) a $\frac{64xy^2}{15}$ b $27a^5b^9c^{11}$ c $\frac{10a^3b^4}{3}$
d $\frac{9a^6b^4}{8a^3b^3} \times \frac{8a}{16a^4b^6} = \frac{9a^7b^4}{16a^7b^9} = \frac{9}{16b^5}$

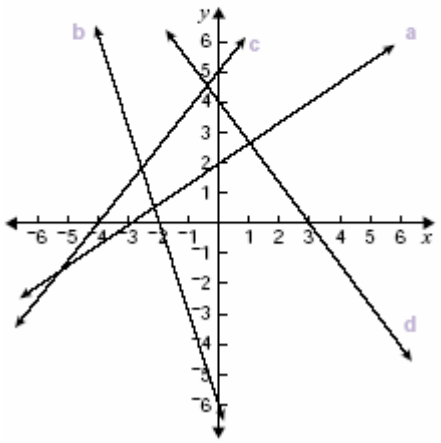
43) $\frac{a^6b^3}{x^6} \times \frac{12xy^2}{5ab} = \frac{12a^5b^2y^2}{5x^5}$
$\frac{x^{-3}y}{3a^{-1}b^2} \times \frac{25x^2y^{-4}}{16a^6b^2} = \frac{25}{48a^5b^4xy^3}$

44) a $\frac{125}{a^6}$ b $25a^2b^{\frac{4}{5}}$ c $\frac{3}{4x^{\frac{2}{3}}y}$
d $\frac{6a^{3\frac{1}{4}}}{5b^{1\frac{1}{4}}}$ e $\frac{9x^{\frac{1}{4}}}{a^{\frac{1}{4}}b^{2\frac{1}{2}}y^{\frac{3}{5}}}$ f $\frac{4a^{\frac{1}{3}}b^{2\frac{2}{3}}}{9x^{2\frac{1}{4}}}$

Chapter 7

45)

Question	x-intercept	y-intercept
a	-3	2
b	-2	-6
c	-4	5
d	3	4



46) a $y = 3x$ b $y = -2x$

47) a As $2x + 3y = -6$, $y = \frac{2}{3}x - 2$
The gradient is $\frac{2}{3}$ and y-intercept -2
b $y = 4x + 20$. The gradient is 4 and intercept 20.
c $y = \frac{1}{12}x + \frac{1}{4}$. The gradient is $\frac{1}{12}$ and intercept $\frac{1}{4}$.
d As $10x + 12y = 15$, $y = -\frac{5}{6}x + \frac{5}{4}$. The gradient is $-\frac{5}{6}$ and intercept is $\frac{5}{4}$.

48) a The linear equation has the form $y = mx + c$.
With gradient 2, $y = 2x + c$ and since the point (1, 5) lies on the line $5 = 2 \times 1 + c$, $c = 3$.
The equation is $y = 2x + 3$.
b As in part a, $y = -2x + c$ and $12 = -2 \times 1 + c$, $c = 14$. The equation is $y = -2x + 14$.

49) a The equations are $y = x + 6$ and $2x + 2y = 102$.

As $2x + 2y = 102$

$2x + 2(x + 6) = 102$

$4x + 12 = 102$

$4x = 90$

$x = 22\frac{1}{2}$ and $y = 28\frac{1}{2}$

The length is $28\frac{1}{2}$ cm and width $22\frac{1}{2}$ cm.

50)

a Substituting $x = y + 3$ in $3x + 4y = -5$ gives $7y = -14$, $y = -2$ and so $x = 1$.
The solution is $x = 1$, $y = -2$.

b Adding the two equations gives $-2y = 10$, $y = -5$ and so $x = -2$.
The solution is $x = -2$, $y = -5$.

51) a) midpoint is (1/2, 2) b) distance =13 units

52)

a $f(x) = |x|$ is translated left 3 units.

b $f(x) = |x|$ is translated down 4 units.

c $f(x) = |x|$ is translated up 2 units and right 1 unit.

53) a) i) perpendicular $y = -\frac{1}{6}x + 9\frac{1}{2}$

ii) parallel $y = 6x - 46$

b) i) perpendicular $y = \frac{1}{5}x + 3\frac{4}{5}$

ii) parallel $y = -5x + 9$

Chapter 8

54)

a The area A is $A = \frac{1}{2}(2k + 3)(3k - 7)$

b The area A is $A = (2n + 3)(n + 8)$

55)

As $(x + 9)(x - 6) = 0$, $x = -9$ or 6

As $(2x - 7)(x - 5) = 0$, $x = 3\frac{1}{2}$ or 5

As $(4x - 1)(x - 1) = 0$, $x = \frac{1}{4}$ or 1

$(x - 5)^2 - (3\sqrt{3})^2 = 0$

$(x - 5 + 3\sqrt{3})(x - 5 - 3\sqrt{3}) = 0$, $x = 5 \pm 3\sqrt{3}$

$(x + 3)^2 - \sqrt{6}^2 = 0$, $x = -3 \pm \sqrt{6}$

$(x - 10)^2 - (2\sqrt{5})^2 = 0$, $x = 10 \pm 2\sqrt{5}$

56)

a $x = \frac{-6 \pm \sqrt{6^2 - 4 \times 2 \times -3}}{4} = \frac{-6 \pm \sqrt{36 + 24}}{4}$

$x = \frac{-6 \pm \sqrt{60}}{4} = \frac{-6 \pm 2\sqrt{15}}{4} = \frac{-3 \pm \sqrt{15}}{2}$

i $x = \frac{-3 \pm \sqrt{15}}{2}$

ii $x = -3{\cdot}44, 0{\cdot}44$

b $x = \frac{4 \pm \sqrt{4^2 - 4 \times 3 \times -6}}{6} = \frac{4 \pm \sqrt{88}}{6} = \frac{4 \pm 2\sqrt{22}}{6}$

$x = \frac{2 \pm \sqrt{22}}{3}$

i $x = \frac{2 \pm \sqrt{22}}{3}$

ii $x = 2{\cdot}23, -0{\cdot}90$

c $x = \frac{-7 \pm \sqrt{7^2 - 4 \times -2 \times -5}}{-2 \times 2} = \frac{-7 \pm \sqrt{49 - 40}}{-4}$

$x = \frac{-7 \pm \sqrt{9}}{-4} = \frac{7 \pm 3}{4}$

i $x = 1, 2\frac{1}{2}$

ii $x = 1{\cdot}00, 2{\cdot}50$

57) 8)

a Applying Pythagoras's Theorem
$x^2 + (x + 4)^2 = 20^2$, $2x^2 + 8x + 16 - 400 = 0$
$2(x^2 + 4x - 192) = 0$, $(x + 16)(x - 12) = 0$
The only possible solution is that $x = 12$ cm.

b The area $A = 112 = x(50 - 3x)$
$-3x^2 + 50x - 112 = 0$, $3x^2 - 50x + 112 = 0$
$(3x - 8)(x - 14) = 0$, $x = 2\frac{2}{3}$ or 14.

The width of the rectangle is 2·66 m o4 14 m.

58)

a i The x-intercepts are 4, –4 and the y-intercept is –32.

ii The axis of symmetry is $x = 0$.

iii The turning point is the point (0,–32).

b i The x-intercepts are 2, $-1\frac{1}{2}$ and the y-intercept is –6.

ii The axis of symmetry is $x = \frac{1}{4}$.

iii The turning point is the point $(\frac{1}{4}, -6\frac{1}{8})$.

59)

a $y = (x - 3)^2 - 2$ b $y = (x + 1)^2$

c $y = (x + 6)^2 + 9$ d $y = (x - \frac{1}{2})^2 + 2\frac{1}{2}$

60)

a A translation 2 units right and 4 units down is required.

b A translation 1 unit left and 3 units up is needed.

c A reflection in the x-axis followed by a translation 1 unit right and 3 units down are needed.

d A dilation of 2 units from the x-axis and a translation 5 units left and 2 units up are needed.

61) a The equation of the parabola is $y = x^2 - 2x - 3$.

b The turning point is $(1, -4)$.

62) a $\{x : -2 < x < -1\}$ or $(-2, -1)$

b $\{x : x > 3\} \cup \{x : x < -2\}$ or $(-\infty, -2) \cup (3, \infty)$

c $\{x : x \leq -7\} \cup \{x : x \geq 2\}$ or $(-\infty, -7] \cup [2, \infty)$

d $\{x : x \leq 3\} \cup \{x : x \geq 4\}$ or $(-\infty, -3] \cup [4, \infty)$

63) a In turning point form the parabola is $y = (x-2)^2 - 5$ Turning point is $(2, -5)$

b The x-intercepts are $2 \pm \sqrt{5}$ or $-0{\cdot}2$ and $4{\cdot}2$ to one decimal place.

Chapter 9

64) The possible two-digit numbers are

12, 13, 15
21, 23, 25
31, 32, 35
51, 52, 53

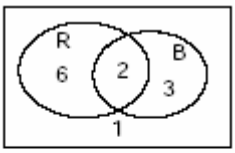
a Pr(even) $= \frac{3}{12} = \frac{1}{4}$

b Pr(multiple of 5) $= \frac{3}{12} = \frac{1}{4}$

c Pr(<20) $= \frac{1}{4}$ d Pr(>20) $= \frac{9}{12} = \frac{3}{4}$

e Pr(≥31) $= \frac{1}{2}$

65) a



b i Pr(Blue only) $= \frac{3}{12} = \frac{1}{4}$.

ii Pr(Red only) $= \frac{6}{12} = \frac{1}{2}$.

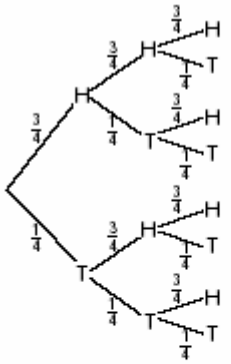
iii Pr(Red only) $= \frac{6}{12} = \frac{1}{2}$

66) a

	S	S′	
T	0·1	0·1	0·2
T′	0·4	0·4	0·8
	0·5	0·5	1·0

i Pr(S∩T′) = 0·4
ii Pr(T∩S′) = 0·1
iii Pr(S′∩T′) = 0·4
iv Pr(S′) = 0·5
v Pr(T′) = 0·8

67) a



b i Pr(TTT) $= \frac{1}{64}$ ii Pr(HHH) $= \frac{27}{64}$

iii Pr(HHT) $= \frac{9}{64}$ iv Pr(2Hs and 1T) $= \frac{27}{64}$

v Pr(At least 2 heads) $= \frac{27}{32}$

vi Pr(At most 1 tail) $= \frac{27}{32}$

68) a Pr = 0·95 b Pr = 0·05

c Pr = 0·90 d Pr = 0·10

e Pr = 0·40 f Pr = 0·60

g $\text{Pr} = \frac{0{\cdot}4 \times 0{\cdot}05}{0{\cdot}4 \times 0{\cdot}05 + 0{\cdot}6 \times 0{\cdot}1} = \frac{0{\cdot}02}{0{\cdot}02 + 0{\cdot}06}$

$= 0{\cdot}25$

h $\text{Pr} = \frac{0{\cdot}6 \times 0{\cdot}1}{0{\cdot}4 \times 0{\cdot}05 + 0{\cdot}6 \times 0{\cdot}1} = \frac{0{\cdot}06}{0{\cdot}02 + 0{\cdot}06}$

$= 0{\cdot}75$

69) a If the three lollies are to be the same they must be all jelly beans or all liquorice.

Pr(JJJ) $= \frac{3}{14} \times \frac{2}{13} \times \frac{1}{12} = \frac{1}{364}$ and

Pr(LLL) $= \frac{9}{14} \times \frac{8}{13} \times \frac{7}{12} = \frac{84}{364}$ so

Pr(Three same) $= \frac{1}{364} + \frac{84}{364} = \frac{85}{364}$

b Pr(Three different) $= 6 \times \frac{2}{14} \times \frac{3}{13} \times \frac{9}{12} = \frac{27}{182}$

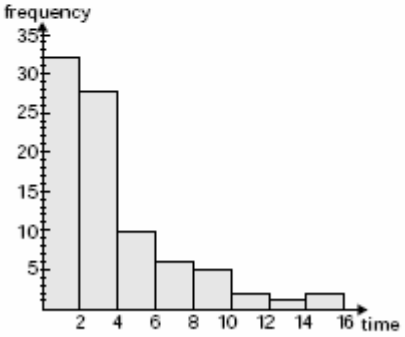
Chapter 10

70)

a The time-frequency details for calls to the hardware store are summarised in the table:

Time (min)	Frequency
0–	32
2–	28
4–	10
6–	6
8–	4
10–	2
12–	1
14–	2

b



c The display shows that most calls are short – in the range of 0–2 or 2–4 minutes.

71)

a For type A bulbs the mean operational time is 285 hours and the standard deviation of the measure is 28·4. For type B the figures are 285 and 38·5 respectively.
Both light bulbs have the same average life span but type A bulbs have a smaller standard deviation and so their operational life is more consistent than that of type B.

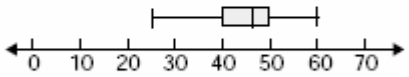
b For Type A $285 + 28{\cdot}4 = 313{\cdot}4$
$285 - 28{\cdot}4 = 256{\cdot}6$
Four globes fit into this range.
For Type B $285 + 38{\cdot}5 = 323{\cdot}5$
$285 - 38{\cdot}5 = 246{\cdot}5$
Four globes fit into this range.

72)

The scores in order are: 24, 39, 42, 46, 47, 48, 50 and 60.

i The lowest score is 24
ii The first quartile is 40·5
iii The median is 46·5
iv The third quartile is 49
v The highest score is 60

a



b The interquartile range is $49 - 40{\cdot}5 = 8{\cdot}5$

c The 75 value is an outlier as the upper limit is $60 + 1{\cdot}5 \times 8{\cdot}5 = 72{\cdot}75 \approx 73$. It is most likely that Billy Bob is showing off as the score is not consistent with the recorded scores.

Chapter 11

73)

a As $V \propto I$ and R is the constant of proportionality
$V = RI$, $R = \frac{9}{0{\cdot}25} = 36$.
The resistance is 36 ohms.

b $V = 36 \times 0{\cdot}4 = 14{\cdot}4$. 14·4 volts are needed.

c As $I = \frac{V}{R}$, $I = \frac{4}{36}$. The current used is $\frac{1}{9}$ amps.

74)

a As $v \propto \frac{1}{p}$, $v = k\frac{1}{p}$ and $k = 26 \times 2$.
The rule is given by $v = \frac{52}{p}$.

b i The pressure p is given by $p = \frac{52}{p} = 1$. The pressure is 1 kg/cm^3.

ii Here $p = \frac{52}{18} = 2\frac{8}{9}$ units.

c i With $p = 2.5$ units $v = \frac{52}{2{\cdot}5} = 20{\cdot}5$.
The volume is 20·5 cm^3.

ii The volume is $28\frac{8}{9}$ cm^3 $\left(v = \frac{52}{1{\cdot}8}\right)$.

iii The volume is 52 cm^3 $\left(v = \frac{52}{1}\right)$

iv The volume is 13 cm^3 $\left(v = \frac{52}{4}\right)$.

75)

a As $y \propto x^2$, $y = kx^2$ and $k = \frac{y}{x^2}$ so
$k = \frac{1{\cdot}96}{2{\cdot}8^2} = \frac{1}{4}$.
The rule is $y = \frac{x^2}{4}$.

b Given that $a \propto \frac{1}{b^3}$, $a = \frac{k}{b^3}$ so $k = ab^3 = 8 \times 64$.
The rule is $a = \frac{512}{b^3}$.

76) a) equation $y = \frac{1}{x-2}$, asymptotes $x = 2 \quad y = 0$

b) equation $\frac{1}{x+1}$, asymptotes $x = -1 \quad y = 0$

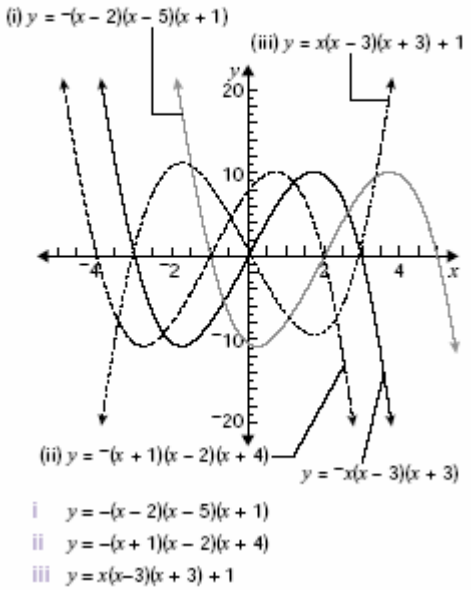
c) equation $\frac{1}{x+3} + 6$, asymptotes $x = -3 \quad y = 6$

Chapter 12

77)

a The equation of the cubic is $y = -x(x-3)(x+3)$

b



i $y = -(x-2)(x-5)(x+1)$

ii $y = -(x+1)(x-2)(x+4)$

iii $y = x(x-3)(x+3) + 1$

78) a) period=180, amp=6 b) period=60, amp = ½ c) period=720, amp=4 d) period=60, amplitude=undefined

79) $y = -[x(x-3)(x-6) - 3] = -x(x-3)(x-6) + 3$

80) $y = \frac{(\sin 8x)}{3}$