

2. [8 marks]

In a study of homes, researchers gathered the following information about the number of bedrooms in a person's home.

No. of bedrooms	1	2	3	4	5
No. of homes	0	3	28	50	4

→ list 1
→ list 2

- (a) Find the mean and standard deviation for the number of bedrooms per home (to 3dp) [2]

$$\bar{x} = 3.647 \quad \checkmark$$

$$s = 0.627 \quad \checkmark$$

- (b) Explain what would happen to the mean and standard deviation if:

- (i) a person with a 5 bedroom home was withdrawn from the data [2]

✓ $\bar{x}$ will decrease ✓

✓ s will decrease ✓

- (ii) a person with a 3 bedroom home was withdrawn from the data [2]

✓ $\bar{x}$ will increase ✓

✓ s will decrease ✓

- (iii) a new person with a 6 bedroom home is included in the original data. [2]

✓ $\bar{x}$ will increase ✓

✓ s will increase ✓

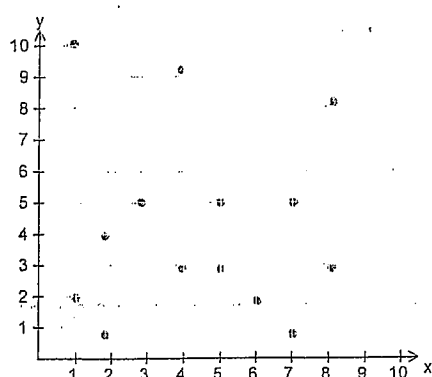
3. [5 marks]

In separate studies, each concerned with a relationship between two quantities, the following correlation coefficients were suggested:

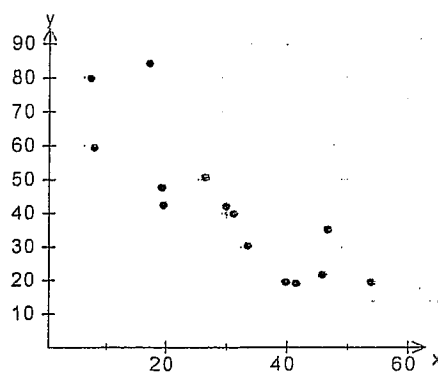
$$r = 0.45, r = -0.4, r = 0.09, r = 0.95, r = 1.23, r^2 = 1, r = -0.8$$

- (a) For each of the scatter diagrams below, state a corresponding correlation coefficient value from the list above.

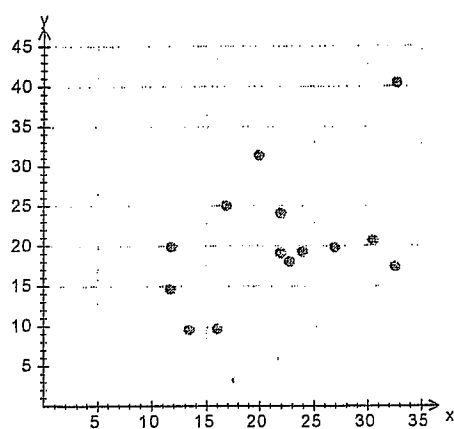
[3]



$$r = 0.09 \quad \checkmark$$



$$r = -0.8 \quad \checkmark$$



$$r = 0.45 \quad \checkmark$$

- (b) In each of the cases below, state the corresponding correlation value from the same list, used for part (a).

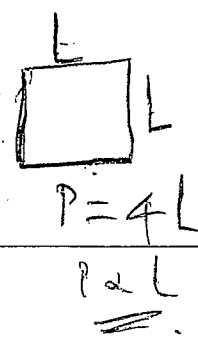
[2]

- (i) the age of an adult and the fitness of that adult.

$$r = -0.8 \quad \checkmark$$

- (ii) the perimeter of the square and the length of its side.

$$r = 1 \quad \checkmark$$



4. [13 marks]

(a) For the sequence 4, 7, 10, 13,.....

i) Write the recursive formula T_{n+1} [1]

$$T_{n+1} = T_n + 3 \quad \checkmark \quad T_1 = 4$$

ii) Write out the general rule T_n in terms of n [1]

$$T_n = 3n + 1 \quad \checkmark \quad T_n = 4 + (n-1)3$$

$$= 4 + 3n - 3$$

$$= 3n + 1$$

(b) For the sequence 256, -64, 16, -4, 1, -0.25,.....

i) Write the recursive formula T_{n+1} [1]

$$T_{n+1} = -\frac{T_n}{4} \quad \checkmark \quad T_1 = 256$$

ii) Write out the general rule T_n in terms of n [2]

$$T_n = 256 \times -0.25^{n-1} \quad \text{or} \quad -1024 \times -0.25^n$$

(c) Find T_2 , T_3 , T_4 for the recursive rule $T_n = (-1)^n(T_{n-1} - 2)$, $T_1 = -12$ [2]

$$T_2 = (-1)^2(-12 - 2) = -14$$

$$T_3 = (-1)^3(-14 - 2) = 16$$

$$T_4 = (-1)^4(16 - 2) = 14$$

(d) An arithmetic sequence has $T_{22} = 22$ and $T_{26} = 12$. Find the common difference d and first term a for this sequence. [3]

$$26 - 22 = 4 \quad 4d = -10 \quad \checkmark$$

$$d = -2.5 \quad \checkmark$$

$$T_1 = T_{22} - 21d \quad T_1 = 22 - (-52.5) \quad T_1 = 74.5 \quad \checkmark$$

(e) A geometric sequence with negative odd numbered terms has $T_{14} = 256$ and $T_{20} = 16384$. Find the ratio r and the first term a for this sequence. [3]

$$20 - 14 = 6 \rightarrow r^6 = \frac{16384}{256} \quad r^6 = 64$$

$$r = -2 \quad \checkmark \quad 256 = a(-2)^{13}$$

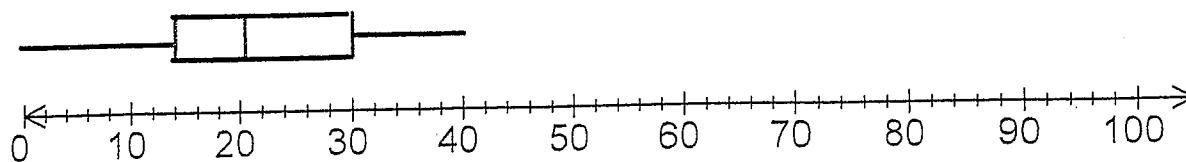
$$a = \frac{256}{(-2)^{13}} \quad a = -0.03125$$

$$T_{14} = 256$$

$$256r^6 = 16384$$

5. [6 marks]

A box plot is drawn below to show how students performed on a test.



- (a) Give the median and inter-quartile range of the scores. [2]

$$\text{Median} = 20 \quad \checkmark$$

$$\begin{aligned} \text{IQR} &= 30 - 14 \\ &= 16 \quad \checkmark \end{aligned}$$

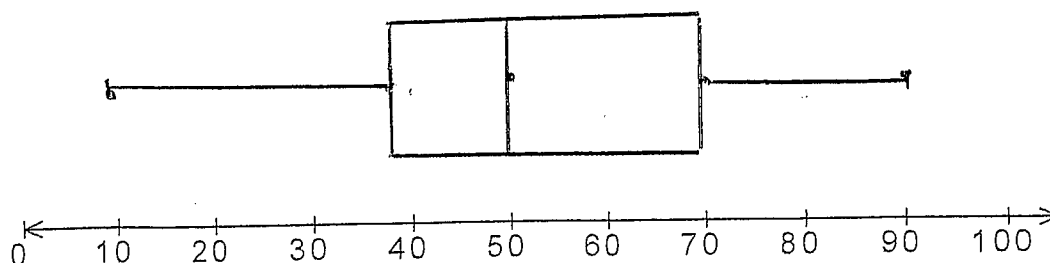
$\times 2$
 $+ 10$

- (b) The examiner decides the marks are too low and adjusts them by doubling each score then adding ten to each score. What will this do to the median and the inter-quartile range? [2]

Median will also double and add 10

IQR will double

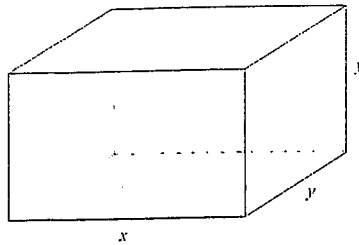
- (c) Draw the new box plot on the scale provided.



$$\begin{aligned} Q_3 &= 30 \times 2 + 10 = 70 \\ Q_1 &= 14 \times 2 + 10 = 38 \end{aligned}$$

6. [12 marks]

The drawing below is that of a steel frame for a box, with length x , width y and height y .



The steel frame is made from 16 metres of pipe.

(a) Complete:

$$4x + 8y = 16$$

[2]

The volume of the box can be calculated by using the formula;

$$V = (\text{length})(\text{width})(\text{height})$$

(b) Show algebraically that;

$$V = x \left(\frac{4-x}{2} \right)^2$$

$$4x + 8y = 16$$

$$8y = 16 - 4x$$

$$y = \frac{16 - 4x}{8}$$

$$y = \frac{4 - x}{2}$$

$$V = x \times \left(\frac{4-x}{2} \right) \times \left(\frac{4-x}{2} \right)$$

$$V = x \left(\frac{4-x}{2} \right)^2$$

[2]

(c) What range of values of x is sensible in this context? Explain.

[3]

$$0 < x \leq 4$$

Because x can't be negative and if it's greater than 4, it would make y negative.

(d) (i) Find the maximum volume that can be enclosed by the 16 metres of pipe. [2]

$$V = 2.37 \text{ m}^3 \quad \checkmark \quad (-1 \text{ no units or inaccurate})$$

(ii) State the dimensions of the frame for maximum volume.

[3]

length $x = 1.33 \text{ m} \quad \checkmark$

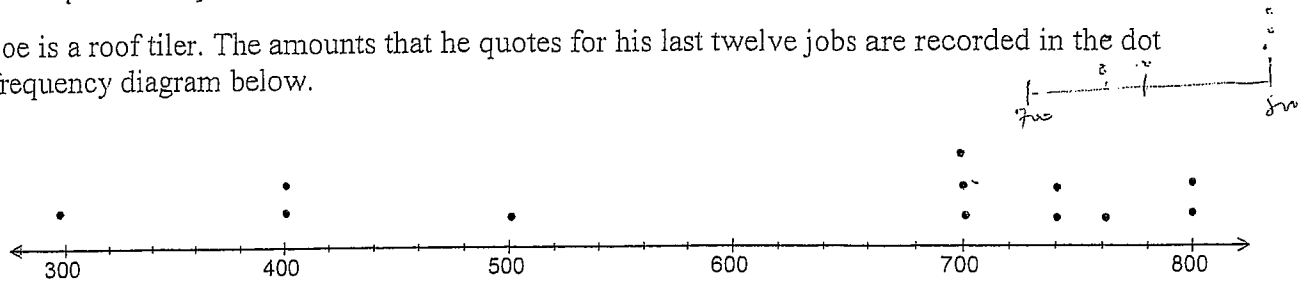
$$1.3$$

width $y = 1.34 \text{ m} \quad \checkmark \quad (1.335) \quad 1.35 \quad (1.4)$

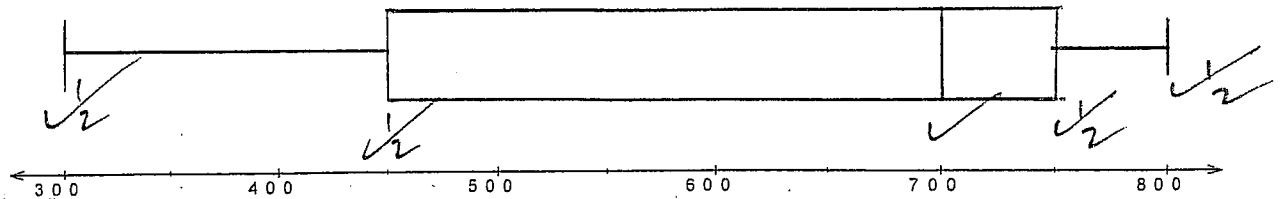
height $y = 1.34 \text{ m} \quad \checkmark \quad (1.335) \quad 1.35 \quad (1.4)$

7. [10 marks]

Joe is a roof tiler. The amounts that he quotes for his last twelve jobs are recorded in the dot frequency diagram below.



- (a) Draw a median box and whisker plot to represent the above data. [3]



- (b) State the median. [1]

700 ✓

- (c) Based on the shape of the distribution, is Joe's next job likely to be closer to \$300 or \$800? Explain. [2]

\$800 ✓

50% of distribution is between 700 and 800

- (d) The next job was actually an \$800 job.
If this job was included, what effect would this have on the:

- (i) range?

No effect ✓

- (ii) upper quartile?

Increase UQ to 780 ✓

[2] ↑
(430)

- (e) Joe needs to include GST of 10% on top of all original 12 quotes.
How will the addition of GST affect the:

- (i) median?

Increase by 10% ✓ (to 770)

- (ii) variance?

SD ↑ (SD ↑; $\times 1.1$, VAR ↑; $\times 1.21$)
Increase by 21% ✓

[2]

$SD = 169.6 \rightarrow SD^2 = 28767.16 \times 1.21$

8. [15 marks]

For 15 pupils in her Mathematics class, a teacher records the class mark and the Australian Problem Solving Competition mark to see if maths skills relate to problem solving ability.

Pupil No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Class Mark (%)	56	38	92	12	87	41	76	96	21	95	34	23	84	43	44
APSC (%)	43	27	66	8	75	17	55	56	3	89	15	4	69	35	88

- (a) Find the mean and standard deviation for the class marks

[2]

$$\text{Mean} = 56.13 \checkmark$$

$$\text{SD} = 28.47 \checkmark$$

- (b) If another class mark of 65 was included. Without calculating the new statistics, how would it affect the mean and standard deviation of the class marks?

[2]

Increase the mean $\checkmark$
 Decrease the standard deviation $\checkmark$

- (c) Identify the outlier (hint: use your Graphics Calculator and plot a scatter graph of APSC against Class Marks).

[1]

Person 15 (44, 88) $\checkmark$

- (d) Remove the outlier and find the equation of the line of regression to predict APSC results from the class marks.

[2]

$$y = 0.897x - 11.000 \checkmark$$

$$A\% = 0.897C\% - 11.000 \checkmark$$

- (e) Write down the correlation co-efficient for this line of regression. Comment on the statement. "If students study for class tests, they will do better in the APSC". [3]

$$r = 0.95 \checkmark$$

Although the correlation coefficient indicates a strong link between test results and APSC scores, it cannot be said that studying for one will help the other. $\checkmark$

- (f) A pupil who has a class mark of 56% was sick on the day of the APSC. Predict the pupils APSC mark. [2]

$$y = 0.897 \times 56 \checkmark - 11$$

$$y = 39\% \checkmark$$

- (g) Would you consider the prediction to be reliable? Give reasons for your answer. [3]

Given that $r = 0.95$ and 56% is inside the data range, $\therefore$ Interpolation, the result is reliable. $\checkmark$

10. [7 marks]

1993 1994 $\rightarrow$ 2000
 T_1 T_2 T_8

Each year starting in 1993, Mr Generous gives a donation to the Cancer Foundation. Each year he donates "d" dollars more than the previous year. This year, 2000, he donates \$207. In the year 2004 he will donate \$255.

(a) How much extra does he donate each year?

[3]

$$\begin{array}{l}
 (2000) T_8 \rightarrow \$207 \\
 (2004) T_{12} \rightarrow \$255
 \end{array}
 \left. \vphantom{\begin{array}{l} (2000) T_8 \rightarrow \$207 \\ (2004) T_{12} \rightarrow \$255 \end{array}} \right\}
 \begin{array}{l}
 4d = 255 - 207 \\
 4d = 48 \checkmark \\
 \underline{d = \$12} \checkmark
 \end{array}$$

(b) How much did he donate in 1993?

[2]

$$\begin{array}{l}
 T_8 = a + 7d \\
 207 = a + 7 \times 12 \checkmark \\
 \underline{a = \$123} \checkmark
 \end{array}$$

(c) Find a simplified expression for the amount (A) donated n years after 1993. [2]

$$\begin{array}{l}
 A = 123 + (n-1) \times 12 \checkmark \\
 = 123 + 12n - 12
 \end{array}$$

$$\therefore \underline{A = 12n + 111} \checkmark$$

11. [15 marks]

A lolly shop sells two *different* chocolate bars. A Scrunchy Bar contains 150 g of nuts and 50 g of sultanas. A Nicpic Bar contains 50 g of nuts and 100 g of sultanas. The manager estimates that the shop would need to sell a total of at least 40 packets each day to be profitable, but the shop would be unlikely to sell more than 60 packets in total. Hence, if x is the number of Scrunchy Bars sold each day and y is the number of Nicpic Bars sold each day two inequalities are formed,

(a) Write out the two inequalities in terms of x and y .

[2]

$$x + y \leq 60 \quad \checkmark$$

$$x + y \geq 40 \quad \checkmark$$

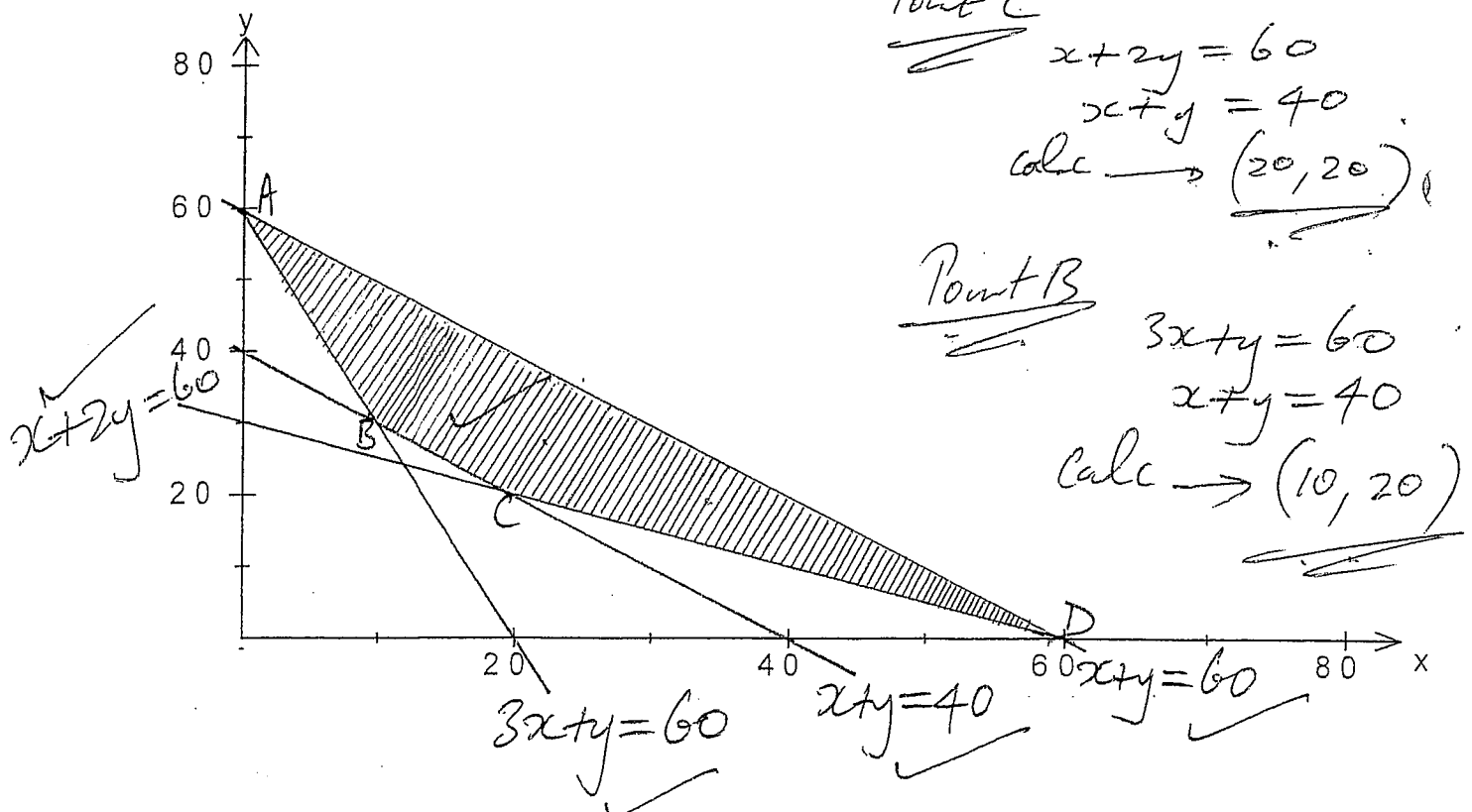
* Because of the commitments to suppliers, the shop must sell at least 3 kg of nuts and at least 3 kg of sultanas each day.

(b) Show that this piece of information generates two inequalities $3x + y \geq 60$ and $x + 2y \geq 60$ [2]

1500 →
 $150x + 50y \geq 3000$
 $\div 50$ gives $3x + y \geq 60 \quad \checkmark$

$50x + 100y \geq 3000$
 $\div 50$ gives $x + 2y \geq 60 \quad \checkmark$

(c) On the graph below, plot the inequalities, identify the feasible selling region and shade it in. [5]



(d) State the vertices of the feasible region

[2]

$$A \rightarrow (0, 60), \quad C \rightarrow (20, 20) \\ B \rightarrow (10, 30), \quad D \rightarrow (60, 0)$$

(e) If the price of a Scrunchy Bar is \$1.20 and of a Nicpic Bar is \$1.60, write down the objective function which gives the total revenue.

[1]

$$R = \$1.20x + \$1.60y$$

(f) What is the greatest possible revenue, and how many of each bar would need to be sold to achieve this? Show how you came to this conclusion.

[3]

	x	y	R
D	60	0	\$72
A	0	60	\$96 *
B	10	30	\$60
C	20	20	\$56

Greatest possible revenue = \$96 ✓

When number of Scrunchy Bars sold = 0
and number of Nicpic Bars sold = 60

12. [8 marks]

In a small class of just 3 students, Adam, Bree and Charlie are instructed by their teacher to work in groups of 1, 2 or 3. The list below shows the seven different groups that may be formed.

Individuals	A	B	C
Pairs	AC	BC	AB
Threes	ABC		

Your task is to determine the number of groups that may be formed if the same instruction was given to classes with:

- (a) 4 students A B C D ✓✓✓ [3]
- AB AC AD BC BD ~~BCD~~
- ABC ABD ACD BCD ABCD 15

- (b) 5 students A B C D E ✓✓✓ [3]
- AB AC AD AE BC BD BE CD CE DE
- ABC ABCDE 31

- (c) Place these values in the following table and then derive a formula in terms of n that will enable a quick calculation of the number of possible groups that could be formed from a class of any number of students.

Write the formula in the last line of the table.

[2]

Number of students in the class	Number of possible groups
1	1
2	3
3	7
4	15
5	31
n	$2^n - 1$ ✓✓

14. [8 marks]

My Grandmother use to make her own Jam. Every year she would make 20 jars but only use 6. When I first noticed this in Dec 1998, she had 102 jars on her shelf. I decided to study this mathematically.

(a) Complete the table.

[2]

Year Ending	n	Jam Jars on Shelf (J)
1998	1	102
1999	2	116
2000	3	130 ✓
2001	4	144 ✓

$\left. \begin{array}{l} +20 \\ -6 \end{array} \right\}$
 $\leftarrow$

(b) Show that the J values form an arithmetic sequence.

[1]

Each year add 20 & take 6

ie $+14$

$\therefore$ Common difference of 14 for AS ✓

(c) The number of jam jars on the shelf n years from 1998 can be re-written as $J = 14n + 88$. Using the general arithmetic sequence formula, show how this is done. [2]

$$a = 102, d = 14, T_n = J$$

$$J = a + (n-1)d \quad \checkmark$$

$$J = 102 + (n-1)14$$

$$= 102 + 14n - 14 \quad \checkmark$$

$$= 14n + 88$$

(d) At the end of what year will she have more than 282 jars left on the shelf? [3]

$$T_n = 282 \quad \checkmark$$

$$\therefore 282 = 14n + 88$$

$$14n = 194$$

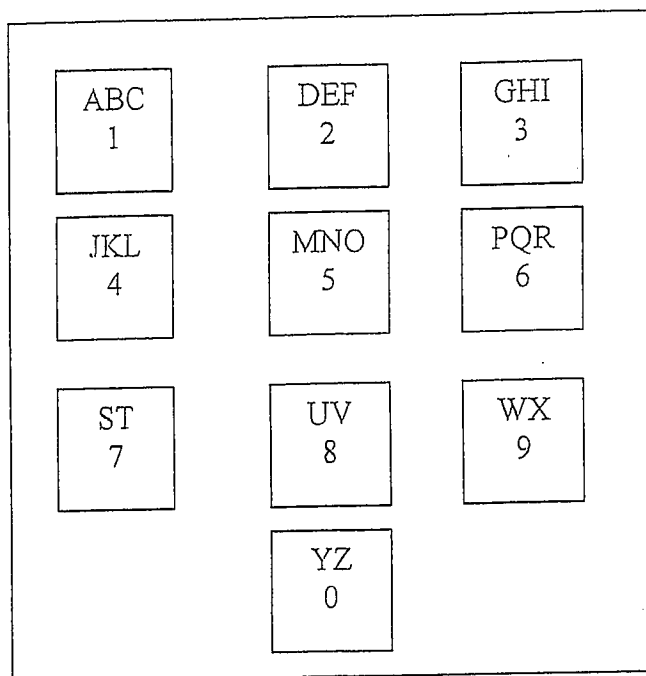
$$n = 13.85 \quad \checkmark$$

$$\approx 14$$

$\therefore$ At the end of 2012 ✓

17. [3 marks]

The keypad used to enter PIN numbers to unlock a safety deposit box has a display as shown below.



- (a) Any arrangement of letters is considered to be a 'word' e.g. CRWF is regarded as a "word". What is the PIN number that corresponds to the word CRWF? [1]

1692 ✓

- (b) How many "words" are there possible if the PIN number 8371 is used? [2]

$$2 \times 3 \times 2 \times 3 = 36$$

DISCRETE MATHEMATICS 2007

16. [10 marks]

a) An arithmetic progression has $T_6 = 17$ and $T_{10} = 29$.

i) Determine its first term and its common difference.

[2]

$$4d = 12 \quad d = 3 \quad \checkmark \quad a = 2 \quad \checkmark$$

ii) For this AP, evaluate $\frac{T_3 + S_2}{T_2}$ 2, 5, 8, ...

[2]

$$\frac{T_3 + S_2}{T_2} = \frac{8 + 7}{5} = 3 \quad \checkmark \checkmark$$

b) A second arithmetic progression has these terms:

300, 280, 260, ...

i) Write a recursive rule for this sequence.

[2]

$$T_{n+1} = T_n - 20 \quad \checkmark, \quad T_1 = 300 \quad \checkmark$$

ii) For this AP, evaluate $\frac{T_3 + S_2}{T_2}$

[2]

$$\frac{T_3 + S_2}{T_2} = \frac{260 + 580}{280} = 3 \quad \checkmark \checkmark$$

c) A Discrete Mathematics student conjectures that $\frac{T_3 + S_2}{T_2}$ is the same for any arithmetic progression. Is she correct? Justify your answer.

[2]

Yes $\checkmark$

$$\frac{(a+2d) + a + a+d}{a+d} = \frac{3a+3d}{a+d} = 3 \quad \checkmark$$

[or give more confirmatory examples]

DISCRETE MATHEMATICS 2007

17. [12 marks]

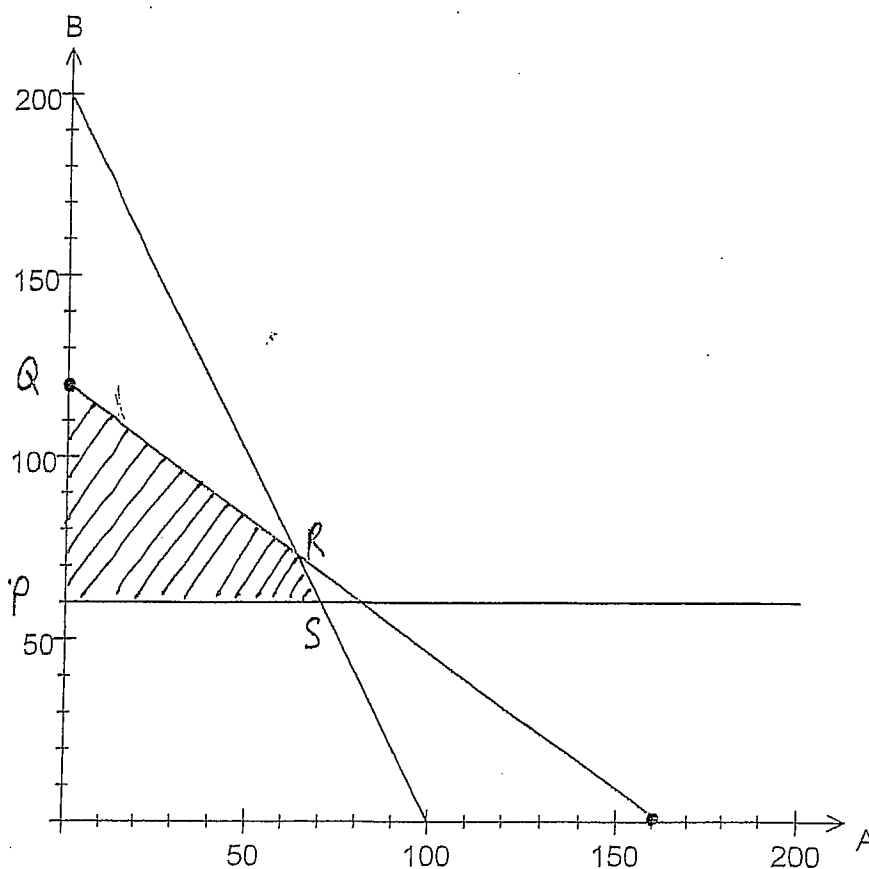
A pharmacist is making up two different medicines, Alphalox and Betalox. Both medicines have the same two ingredients, a liquid and a powder. The amounts required to make one bottle of each medicine are shown in the table below.

	Alphalox	Betalox
Liquid	100mL	50mL
Powder	30g	40g

The pharmacist has 10L of liquid and 4.8kg of powder available. He also knows that he will have to fill at least 60 prescriptions of Betalox.

Let A be the number of bottles of Alphalox and B the number of bottles of Betalox which the pharmacist prepares.

There are four constraints which apply to this situation. One of these is $A \geq 0$. The lines defining two of the other three have been drawn on the graph below.



DISCRETE MATHEMATICS 2007

- a) Write the equations of the two constraints (apart from $A \geq 0$) which have been drawn on the graph. [3]

$$100A + 50B \leq 10\,000 \quad \checkmark\checkmark$$

$$B \geq 60 \quad \checkmark$$

- b) Write the equation of the final constraint, and draw it on the graph. [3]

$$30A + 40B \leq 4800 \quad \checkmark\checkmark$$

drawn on graph $\checkmark$

- c) Shade the feasible region on the graph. [1]

see graph $\checkmark$

- d) If the profit on the each bottle of A is \$5.00 and the profit on each bottle of B is \$8.00, determine the maximum profit which can be obtained, and the amount of each product which must be produced to obtain this profit. [5]

Point	A	B	$5A + 8B$ $\checkmark$
P	0	60	480
Q	0	120	960
R	64	72 $\checkmark$	896
S	70	60 $\checkmark$	830

Maximum profit \$960 $\checkmark$

Produce 120 bottles of B

DISCRETE MATHEMATICS 2007

18. [12 marks]

The quarterly sales figures for a cafe are shown in the table below. All figures represent thousands of dollars.

Quarter ending:	Sales	4-point centred moving average	Residual
March 2005	171		
June 2005	156		
September 2005	143	A	-11
December 2005	152		
March 2006	159		
June 2006	C		
September 2006	B	138	-13
December 2006	130		
March 2007	143		
June 2007			
September 2007			D

a) Calculate the values A, B, and C.

$$A = \underline{154} \quad \checkmark$$

[1]

$$B = \underline{125} \quad \checkmark$$

[1]

$$C = \underline{146} \quad \checkmark \checkmark$$

[2]

b) The average residual for quarters ending in September is -11. Use this to find the value of D.

$$D = \underline{-9} \quad \checkmark \checkmark$$

(1 mark for Sep 2005 residual as in table)

[2]

DISCRETE MATHEMATICS 2007

- c) The model $\text{Prediction} = \text{Trend} + \text{Seasonal Component}$ can be used to forecast future sales figures.

The trend line can be modelled by the equation $\widehat{CMA} = 158.86 - 4.05t$, where t is the number of quarters, starting with $t = 1$ for the quarter which ends in March 2005.

- i) What is the value of t for the quarter ending in September 2008?

[1]

15 ✓

- ii) Predict the sales figures for the quarter ending in September 2008.

[3]

$$\text{Prediction} = 98.11 - 11 \approx 87.11$$

\$ 87 000 ✓

- d) In April 2007, just after the March sales figures were released, the cafe made a commercial which stated:

"Our business is booming. Sales increased by 10% last quarter."
Comment on this claim, using statistics to justify your comment.

[2]

This is false ✓

or Sales are decreasing overall
March is a peak, which disguises the trend ✓

QUESTION 10: [4, 1, 4, 2 = 11 marks]

Joes' Fish Restaurant shows the following sales.

Week	Day	t	Sales (100s \$)	5 Point Moving Average (y)	Residual
1	Tuesday	1	6.2	—	—
	Wednesday	2	7.0	—	—
	Thursday	3	9.3	9.2	0.1
	Friday	4	11.4	A	—
	Saturday	5	12.1	9.22	2.88
2	Tuesday	6	6.3	9.2	-2.9
	Wednesday	7	7.0	9.26	-2.26
	Thursday	8	9.2	9.4	B
	Friday	9	11.7	9.42	2.28
	Saturday	10	12.8	9.48	3.32
3	Tuesday	11	6.4	9.5	-3.1
	Wednesday	12	7.3	10.02	-2.72
	Thursday	13	C	10	—
	Friday	14	14.3	—	—
	Saturday	15	12.7	—	—

- (a) Calculate A, B and C.

$$A = \frac{7.0 + 9.3 + 11.4 + 12.1 + 6.3}{5}$$

$$A = 9.22 \checkmark$$

$$B = 9.2 - 9.4 = -0.2 \checkmark$$

$$\frac{12.7 + 14.3 + C + 7.3 + 6.4}{5} = 10 \checkmark$$

$$C = 9.3 \checkmark$$

- (b) Calculate the average seasonal residual for Wednesday.

$$\frac{-2.26 + -2.72}{2} = -2.49 \checkmark$$

* It is also known that the equation to determine the moving average from the time period is given as:

$$y = 0.08t + 8.81$$

- (c) Use this equation and your result from (b), to predict the sales, in whole dollars, for Wednesday of week 7. ~~✗~~

Wednesday of week 7, $t = 32$ ✓

$$\therefore y = 0.08(32) + 8.81$$

$$y = 11.37$$

$$\therefore \text{Predicted sales} = 11.37 + (-2.49) \\ = 8.88 \quad \checkmark$$

$$\therefore 8.88 \times 100 = \underline{\underline{\$888}} \quad \checkmark$$

- (d) State two factors that Joe would need to consider in judging reliability of the prediction made in (c). *calculations not required.* ~~✗~~

- | | |
|------------------------------|----------------|
| (i) Correlation Coefficient. | } ✓
Any two |
| (ii) Extrapolation. | |
| (iii) Seasonal components. | |

Question 2. [2, 3, 1, 3 = 9 marks]

The base perimeter of the tent shown is 6 metres, ie, $2W + 2L = 6$.

a) Show algebraically that $W = 3 - L$.

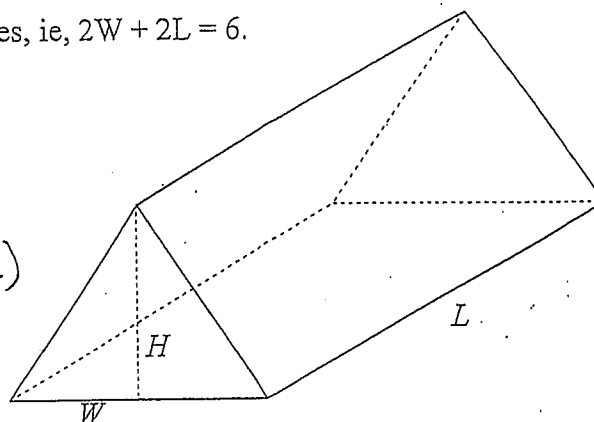
$$\text{Perimeter} = 6\text{m}$$

$$2W + 2L = 6 \quad (-2L)$$

$$2W = 6 - 2L \quad (\div 2)$$

$$W = 3 - L$$

hence proven



b) Given that $H = L$ and that the volume $V = \frac{1}{2} W \times H \times L$ complete the table below.

L	H	W	V
0	0	3.0	0
0.5	0.5	2.5	0.3125
1.0	1.0	2.0	1.0
1.5	1.5	1.5	1.6875
2.0	2.0	1.0	2.0
2.5	2.5	0.5	1.5625
3.0	3.0	0	0
	✓	✓	✓

c) What is the maximum volume of the tent?

$$2.0\text{m}^3 \checkmark$$

d) What are the dimensions which give the maximum volume?

$$L = 2.0\text{m} \checkmark$$

$$W = 1.0\text{m} \checkmark$$

$$H = 2.0\text{m} \checkmark$$

QUESTION 3. (12 marks)

Farmer John has 69 m of fencing with which to make a temporary rectangular enclosure to house orphan lambs.

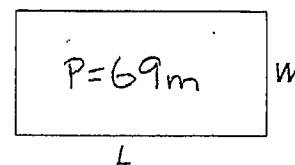
- (a) If W is the width of the enclosure, write an expression for the length in terms of W . [2]

$$P = 2L + 2W$$

$$69 = 2L + 2W \quad \checkmark$$

$$2L = 69 - 2W$$

$$L = \frac{(69 - 2W)}{2} = 34.5 - W \quad \checkmark$$



The table of values below considers values of width increasing by 5m.

Width, W (m)	Length, L (m)	Area (m^2)
0	34.5	0
5	29.5	147.5
10	24.5	245
15	19.5	292.5 *
20	14.5	290
25	9.5	237.5
30	4.5	135
16	18.5	296
17	17.5	297.5 *
18	16.5	297
19	15.5	294.5

- (b) Complete the table up to $w = 30$ and then use this table to determine the dimensions (length and width) of the enclosure that creates the greatest area, giving your answers for the width to the nearest 5m. [3]

$$L = 19.5\text{m} \quad \checkmark$$

$$W = 15\text{m} \quad \checkmark$$

to give a max. Area of 290m^2 .

- (c) Try other values of length and width, giving your answers for the width to the nearest 1m, so that the area is maximised. (Write these in the table above.) [2]

$$L = 17.5\text{m} \quad \checkmark$$

$$W = 17\text{m} \quad \checkmark$$

to give a max. Area of 297.5m^2 .

After John had worked for hours on this problem, his clever wife Tamara had the bright idea of using an existing stone wall as one of the longer sides of the enclosure. She explained that in this way, the 69 m of fencing could then be used to form the remaining three walls.

- (d) Clearly show the expressions for the dimensions.

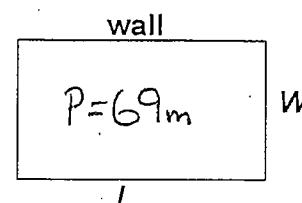
[2]

$$P = L + 2W$$

$$69 = L + 2W$$

$$L = 69 - 2W \quad \checkmark$$

$$W = \frac{69 - L}{2} = 34.5 - 0.5L \quad \checkmark$$



- (e) Fill in the table below and determine the dimensions of Tamara's new improved enclosure that again maximises the area, giving your answer to the nearest 1m.

Width, W (m)	Length, L (m)	Area (m ²)
15	39	585
16	37	592
17	35	595 *
18	33	594
19	31	589
20	29	580

$$L = 35m \quad \checkmark$$

$$W = 17m \quad \checkmark$$

to give a max Area of 595m².

[3]

5. [1, 2, 2, 2 = 7 marks]

A pile driver is a machine for hammering wooden poles into the ground. A team of workers is using a pile driver to drive wooden poles 4 metres long in to the ground to make the foundations for a bridge. The first hit of the pile driver drives the pole 50cm into the ground, the second hit drives the pole another 40 cm into the ground, the third hit drives the pole another 32 cm into the ground and successive distances form a geometric sequence.

- (a) How far into the ground will the pole go on the 6th hit?

1	2	3	4	5	6
50	40	32	25.6	20.48	16.38

- (b) After 8 hits how far will the pole have been driven into the ground?

$$S_8 = \frac{50(1 - 0.8)^8}{(1 - 0.8)} = 208.05$$

- (c) How far will the pole be driven into the ground if the pile driver is allowed to run until the pole has gone as far as possible into the ground?

$$S = \frac{50}{(1 - 0.8)} = 250\text{cm}$$

- (d) What percentage of the total distance that the pole can be driven into the ground is reached by the twentieth hit?

$$S_{20} = \frac{50(1 - 0.8^{20})}{(1 - 0.8)} = 247.118$$

$$\frac{247.118}{250} = 98.8\%$$