

# St Stephen's School

## YEAR 11 MAS 3AB

### PROGRAMME 2009

**Text:** A J Sadler. Unit 3A, Unit 3B

| Week       | Curriculum Council Objective and Content                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | Ref  | Assessment              |
|------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|-------------------------|
| 1.1        | <b>Unit 3A</b><br><b>Mathematical reasoning</b><br>5.3 investigate properties of the absolute value function (real)—analytically and graphically.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | Ch 1 |                         |
| 1.2<br>1.3 | <b>Trigonometry</b><br>2.3 establish and use the formula for the area of triangles<br>$\text{Area} = \frac{ab \sin C}{2}$<br>2.4 determine areas of sectors and segments in circles using exact and approximate values as appropriate<br>2.5 establish and use the sine and cosine rules to find distances and angles in triangles in two and three dimensional situations, including obtuse triangles and those triangles with two solutions (the ambiguous case)<br>2.6 establish the triangle inequality for the lengths of the sides of a triangle $ a + b  \leq  a  +  b $<br>2.7 solve practical problems; including angles of elevation and depression, surveying, bearings and navigation distances along circles of constant latitude or constant longitude on the surface of the Earth. | Ch 2 | I1 (5%)                 |
| 1.4<br>1.5 | <b>Vectors</b><br>1.1 distinguish between vector and scalar quantities<br>1.2 represent a directed line segment in the plane with magnitude and direction using vector displacement notation $\overrightarrow{AB}$ or <b>AB</b><br>1.3 develop the concept of equality of vectors, opposite vectors, unit vectors and the zero vector                                                                                                                                                                                                                                                                                                                                                                                                                                                             | Ch 3 | T1 (5%)<br><br>Ch 1 - 3 |
| 1.6        | <b>Vectors</b><br>1.4 represent a vector as an ordered pair $(a, b)$ , or as a row $[a \ b]$ , or as a column, matrix $\begin{pmatrix} a \\ b \end{pmatrix}$<br>1.5 represent vectors in the form $ai + bj$ , where <b>i</b> and <b>j</b> are the standard unit vectors<br>1.6 establish and use the formula $\ (a, b)\  = \sqrt{a^2 + b^2}$ for the magnitude or modulus of a vector as its length in the plane<br>1.7 determine the position vector $\overrightarrow{OP}$ , from the origin, of a point <i>P</i> in the Cartesian plane<br>1.8 use the parallelogram law or triangle law of vector addition and                                                                                                                                                                                 | Ch 4 |                         |

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|              | <p>establish the triangle inequality <math> a + b  \leq  a  +  b </math></p> <p>1.9 multiply a vector by a scalar and subdivide line segments internally</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |      |                      |
| 1.7<br>1.8   | <p><b>Trigonometry</b></p> <p>2.1 establish the relationship between radian measure and degree measure of angles and convert from one measure to the other</p> <p>2.2 determine arc lengths in circles, exactly and approximately</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Ch 5 |                      |
| 1.9<br>1.10  | <p><b>Exponentials and Logarithms</b></p> <p>3.1 develop and use the index laws for positive bases and rational exponents</p> <p>3.2 establish and use the properties of exponential functions <math>y = Ca^x</math> (<math>a &gt; 0</math>) and draw their graphs</p> <p>3.3 develop the inverse relationship between logarithmic and exponential functions: <math>x = \log_a y</math> and <math>y = a^x</math></p> <p>3.4 investigate and use the properties of the logarithmic functions <math>y = \log_a x</math> for <math>a &gt; 0</math>, and draw their graphs</p> <p>3.5 use the laws of logarithms</p> <p>3.6 solve growth and decay problems using exponential and logarithmic functions.</p> <p><b>Mathematical Reasoning</b></p> <p>5.2 establish the laws of logarithms</p>                                                                                                                                                                                   | Ch 6 |                      |
| 1.11         | <p><b>Vectors</b></p> <p>1.10 represent relative displacement and relative velocity as vectors.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         | Ch 7 | I2 (5%)              |
| 1.12<br>1.13 | <p><b>Functions</b></p> <p>4.1 develop the concept of function composition and obtain expressions for the composites of simple functions</p> <p>4.2 identify the domain and range of simple functions and their composites</p> <p>4.3 investigate the inverse of a function as a reflection in <math>y = x</math></p> <p>4.4 investigate relationships between domains and ranges of functions and their inverses</p> <p>4.5 solve, algebraically and geometrically, simple equations and inequalities involving absolute values of linear functions</p> <p>4.6 investigate the effects of varying <math>a</math>, <math>b</math>, <math>c</math> and <math>d</math> on the graph of <math>y = af(b(x - c)) + d</math> where <math>f(x)</math> is an exponential, logarithmic, power, reciprocal or absolute value function.</p> <p><b>Mathematical Reasoning</b></p> <p>5.1 identify and generalise number patterns for powers, exponential, and inverse relationships</p> | Ch 8 | T2 (10%)<br>Ch 4 - 7 |
| 1.14         | <p><b>Polar Coordinates</b></p> <p>7.1 develop the concept of polar coordinates <math>(r, \theta)</math> in the plane, where <math>r \geq 0</math></p> <p>7.2 use the relationship between Cartesian and polar coordinates in the plane and convert from one system to the other.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | Ch 9 |                      |

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| 1.13 | 3.2.1                  | calculate quantiles for normally distributed data with known mean and standard deviation                                                                                                                                                                                                                                                                                                                                                                                                                                          | Ch 10 |            |
| 1.14 | 3.2.2                  | use number of standard deviations from the mean (standard scores) to describe deviations from the mean in normally distributed data sets.                                                                                                                                                                                                                                                                                                                                                                                         |       |            |
| 1.15 | Review 3A              |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |            |
| 1.16 | Exams                  |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       | Exam Sem 1 |
| 1.17 |                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       | (20%)      |
| 1.18 | <b>Unit 3B</b>         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Ch 1  |            |
| 1.19 | <b>Complex Numbers</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |       |            |
| 1.20 | 6.1                    | develop the concept of the number $i$ as a solution of $x^2 = -1$                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |       |            |
|      | 6.2                    | investigate complex solutions of quadratic equations                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |            |
|      | 6.3                    | represent geometrically a complex number $z$ as a point in the complex plane                                                                                                                                                                                                                                                                                                                                                                                                                                                      |       |            |
|      | 6.4                    | represent the Cartesian form of $z$ as the sum of its real and imaginary parts: $z = a + bi$ , where $i^2 = -1$                                                                                                                                                                                                                                                                                                                                                                                                                   |       |            |
|      | 6.5                    | add, subtract, multiply and divide complex numbers in Cartesian form                                                                                                                                                                                                                                                                                                                                                                                                                                                              |       |            |
|      | 6.6                    | develop the concept of conjugates of complex numbers.                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |       |            |
| 2.1  | <b>Trigonometry</b>    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Ch 2  |            |
| 2.2  | 2.1                    | develop the concept of sine, cosine and tangent as functions, and establish and use the following properties:<br>Pythagorean: $\sin^2 x + \cos^2 x = 1$<br>parity:<br>$\cos(-x) = \cos x$ , $\sin(-x) = -\sin x$ , $\tan(-x) = -\tan x$<br><br>complementarity: $\sin x = \cos(\frac{\pi}{2} - x)$ ; $\cos x = \sin(\frac{\pi}{2} - x)$<br><br>periodicity: $\sin(x + 2\pi) = \sin x$ , $\cos(x + 2\pi) = \cos x$ ,<br>$\tan(x + \pi) = \tan x$<br>phase: $\sin x = \cos(x - \frac{\pi}{2})$ , $\cos x = \sin(x + \frac{\pi}{2})$ |       |            |
|      | 2.2                    | investigate the transformations of sine, cosine and tangent functions such as $y = a \sin b(x + c) + d$ and identify the effects of the constants $a$ , $b$ , $c$ and $d$ on amplitude, period, phase, and the locations of zeros and turning points (see 3AMAS 4.6)                                                                                                                                                                                                                                                              |       |            |
|      | 2.3                    | use appropriate technology to investigate and represent diagrammatically the roles of $a$ , $b$ , $c$ and $d$ in the linear scale changes studied in 2.2 above                                                                                                                                                                                                                                                                                                                                                                    |       |            |
| 2.3  | <b>Trigonometry</b>    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   | Ch 3  | T3 (10%)   |
| 2.4  | 2.4                    | use the addition and double angle formulas for sine, cosine and tangent:                                                                                                                                                                                                                                                                                                                                                                                                                                                          |       | Ch 1 - 3   |

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|              | $\sin(\theta \pm \varphi) = \sin \theta \cos \varphi \pm \cos \theta \sin \varphi$ $\cos(\theta \pm \varphi) = \cos \theta \cos \varphi \mp \sin \theta \sin \varphi$ $\tan(\theta \pm \varphi) = \frac{\tan \theta \pm \tan \varphi}{1 \mp \tan \theta \tan \varphi}$ $\sin 2\theta = 2 \sin \theta \cos \theta$ $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ $= 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$ $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$ <p>2.5 solve trigonometric equations of the form<br/> <math>\sin(ax) = k</math>, <math>\cos(ax) = k</math> and <math>\tan(ax) = k</math> for a given finite domain.</p> <p><b>Mathematical Reasoning</b></p> <p>5.2 establish the addition and double angle formulas for sine, cosine and tangent</p> <p>5.4 prove simple trigonometric identities by deduction, using the properties listed in 2.1 and 2.4 above</p> |      |         |
| 2.5          | <p><b>Vectors</b></p> <p>1.6 solve practical problems using vectors including. the study of bearings, forces and navigation problems involving apparent and true velocities.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Ch 4 |         |
| 2.6<br>2.7   | <p><b>Functions</b></p> <p>4.1 investigate the continuity and limiting behaviour of functions</p> <p>4.2 define the derivative of functions from first principles and apply to familiar functions (not trigonometric)</p> <p>4.3 investigate the differentiability of functions using limits</p> <p>4.4 draw and interpret graphs of gradient functions</p> <p><b>Mathematical Reasoning</b></p> <p>5.1 make conjectures regarding limiting patterns</p>                                                                                                                                                                                                                                                                                                                                                                                                                                      | Ch 5 |         |
| 2.8<br>2.9   | <p><b>Functions</b></p> <p>4.5 investigate piecewise-defined functions and continuity (including absolute value, the sign function <math>\text{sgn } [x]</math>, and the greatest integer function <math>\text{int } [x]</math>)</p> <p>4.6 apply the chain rule with appropriate notation to differentiate composite functions</p> <p>4.7 use the product and quotient rules to differentiate polynomial, exponential (base <math>e</math>) and natural logarithmic functions.</p> <p>4.8 develop the concept of the integral of a function as a limiting sum</p> <p><b>Mathematical Reasoning</b></p> <p>5.3 develop the chain rule for differentiating composite functions</p>                                                                                                                                                                                                             | Ch 6 | 13 (5%) |
| 2.10<br>2.11 | <p><b>Exponentials and logarithms</b></p> <p>3.1 investigate the limiting behaviour of<br/> <math>r^n</math> as <math>n \rightarrow \infty</math>, (<math> r  &lt; 1</math>)</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | Ch 7 |         |

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|------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|------------|
|      | <p>3.2 investigate the limiting behaviour of <math>\left(1 + \frac{a}{n}\right)^n</math> as <math>n \rightarrow \infty</math>, (<math>a</math> fixed)</p> <p>3.3 define <math>e</math> as the limit of <math>\left(1 + \frac{1}{n}\right)^n</math> as <math>n \rightarrow \infty</math></p> <p>3.4 investigate growth and decay problems of the form <math>y = a.e^x</math></p> <p>3.5 differentiate exponential and logarithmic functions including <math>e^{f(x)}</math> and <math>\ln[f(x)]</math>.</p>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |      |            |
| 2.12 | <p><b>Vectors</b></p> <p>1.1 develop the concept of the dot product of vectors in a plane, using projections, and the formula <math>\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2</math>, and establish the formula <math>\mathbf{a} \cdot \mathbf{b} =  \mathbf{a}  \mathbf{b} \cos\theta</math> where <math>\mathbf{a} = (a_1, a_2)</math> and <math>\mathbf{b} = (b_1, b_2)</math></p> <p>1.2 calculate the angle between two vectors and identify parallelism and perpendicularity</p> <p>1.3 establish and use the vector equation of a line in the plane in its various forms:<br/>one point and slope: <math>\mathbf{r} = \mathbf{r}_1 + \lambda \mathbf{l}</math><br/>two points:<br/><math>\mathbf{r} = \mathbf{r}_1 + \lambda(\mathbf{r}_2 - \mathbf{r}_1)</math><br/>normal: <math>\mathbf{r} \cdot \mathbf{n} = c</math></p> <p>1.4 establish and use the vector form of the equation of a circle in the plane: <math> \mathbf{r} - \mathbf{d}  = \rho</math></p> <p>1.5 solve practical problems using vector equations of lines and the dot product including tangency and shortest distance problems.</p> | Ch 8 | T4 (10%)   |
| 2.13 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |      | Ch 4 - 7   |
| 2.14 | Review 3A and 3B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |      |            |
| 2.15 | Exam 3A and 3B                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |      | Exam Sem 2 |
| 2.16 |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |      | (30%)      |

**Assessment Weightings**

| Type           | Number | Weightings %  | Total % |
|----------------|--------|---------------|---------|
| Investigations | 3      | 5, 5, 5       | 15      |
| Tests          | 4      | 5, 10, 10, 10 | 35      |
| Exams          | 2      | 20, 30        | 50      |

Full details of the Calculus syllabus is found at the Curriculum Council website at [www.curriculum.wa.edu.au](http://www.curriculum.wa.edu.au)