St Stephen's School Carramar
YEAR 12 CALCULUS
PROGRAMME 2008

Text: *Calculus, A J Sadler.*
Ref: *Calculus, O. T. Lee*

Week	Curriculum Council Objective and Content	Ref	Assess
1.1 1.2	2.7 Investigate graphs of absolute value functions. 2.8 Use the geometric interpretation of $\lvert x - y\rvert$ as the distance between x and y to establish the triangle inequality: $\lvert x + y\rvert \le \lvert x\rvert + \lvert y\rvert$. 2.9 Solve, algebraically and geometrically, simple equations and inequalities involving absolute values of linear functions.	Ch 1	
1.3 1.4	6.1 Define a complex number z as an ordered pair (a,b) of real numbers, and represent it as a point in the complex plane (Argand Diagram). 6.2 Add and multiply complex numbers according to the rules: $(a,b) + (c,d) = (a + c, b + d)$, and $(a,b) \times (c,d) = (ac - bd, ad + bc)$. 6.3 Investigate the correspondences $(1,0) \leftrightarrow 1$ and $(0,1)$ corresponds to i where $i^2 = -1$ 6.4 Express a complex number z in Cartesian form as the sum of its real and imaginary parts: $z = a + bi$, where $i^2 = -1$ 6.11 Find the conjugate $\bar{z}$ of a complex number z, expressed in Cartesian or polar form, and locate it in the complex plane. 6.12 Establish, algebraically and geometrically, the conjugation properties: $z\bar{z} = \lvert z\rvert^2$, $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ and $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ 6.16 Find all complex roots of a quadratic equation with real coefficients by completing the square or by using the quadratic formula.	Ch 2	Epw 1 (5%) PM
1.5	4.1 Determine the gradient of a curve and the rate of change of a function. 3.1 Revise the sum, product and quotient rules for differentiation 3.2 Revise the chain rule for differentiating composite functions. 3.3 Revise higher order differentiation. 4.4 Solve optimisation problems by finding global extreme points. 4.6 Evaluate total changes from given rates of change. 4.9 Model rectilinear motion by the differential equations $x' = f(t)$ and $x'' = g(t)$, and obtain solutions.	Ch 3	Test 1 (5%) Ch 1,2,3 NM
1.6 1.7	2.1 Develop the concept of a function as a mapping from one set into another and identify the domain and range. 2.2 Use the remainder theorem and the factor theorem to find linear factors of polynomials. 2.3 Define a rational function as the ratio of polynomials, locate its zeros, poles, point discontinuities and turning points, determine its behaviour at $\pm\infty$, and sketch its graph. 2.4 Develop the concept of a piecewise-defined function, and investigate the limiting behaviour of special piecewise-defined functions such as the sign function sgn x, and the greatest integer function int x. 2.10 Define the derivative of a function at a point using limit notation. 2.11 Determine the derivative of x^n, where n is a positive integer, using limit notation. 2.13 Develop the concept of function composition and obtain expressions for the composites of simple functions.	Ch 4	

	2.14 Develop the concept of an inverse function and obtain expressions for the inverses of linear, power, quadratic and reciprocal functions on appropriate domains. 2.15 Investigate the relationship between the graph of a function and the existence of an inverse. 4.3 Find local extreme points and inflection points of functions determining their nature using a sign diagram for *f'(x)* or the second derivative test. Understand the relationships between the graphs of *f(x), f'(x) and f''(x).*		
1.8	2.5 Use the limit notation $\lim_{x\to a} f(x), \lim_{x\to a^-} f(x), \lim_{x\to a^+} f(x), \lim_{x\to\infty} f(x), \lim_{x\to-\infty} f(x)$ or limits of the function *f* at the number *a*, and at ∞ and -∞. 2.6 Investigate discontinuities of piecewise-defined functions. 2.12 Investigate the non-differentiability of simple piecewise-defined functions using limit notation.	Ch 5	
1.9 1.10	1.1 Establish the inequalities: $x \cos x < \sin x < x$ for $0 < x < \frac{\pi}{2}$ using areas in the unit circle. 1.2 Establish the limit $\frac{\sin x}{x} \to 1 \text{ as } x \to 0$ algebraically, graphically and numerically. 1.3 Differentiate cos *x*, sin *x* and tan *x.* 3.4 Differentiate functions defined implicitly.	Ch 6	Test 2 (5%) Ch 4.5.6 SC
1.11 1.12	4.2 Approximate a curve by a tangent and use the incremental formula $\delta y \approx \frac{dy}{dx}\delta x$. 4.5 Solve related rate problems. 4.7 Examine the process of approximating the roots of an equation using the Newton-Raphson algorithm.	Ch 7	Epw 2 (5%) PM
1.13 1.14	6.5 Investigate the properties of $\lvert z \rvert$, the absolute value of the complex number *z*. 6.6 Investigate the properties of arg *z*, the argument of the complex number *z*. 6.7 Establish the exponential properties of $\text{cis}\,\theta = \cos\theta + i\sin\theta$: $\text{cis}(\theta+\phi) = \text{cis}\,\theta\,\text{cis}\,\phi$ $\text{cis}\,0 = 1,$ and $\text{cis}(-\theta) = (\text{cis}\,\theta)^{-1}$ 6.8 Express a complex number z in polar form: $z = r\,\text{cis}\,\theta$, where $r = \lvert z \rvert$ and $\theta = \arg z$. 6.9 Multiply complex numbers expressed in polar form. 6.10 Establish the zero-divisor property of complex numbers: 6.13 Divide one complex number by another, using both Cartesian and polar forms. 6.14 Establish $\frac{\bar{z}}{\lvert z \rvert^2}$ as the reciprocal of a non-zero complex number z. 6.15 Describe regions in the complex plane (Argand plane) by means of simple systems of equalities and inequalities. 6.16 Find all complex roots of a quadratic equation with real coefficients by completing the square or by using the quadratic formula. 6.17 Investigate De Moivre's formula: $\text{cis}\,n\theta = (\text{cis}\,\theta)^n$. 6.18 Use De Moivre's formula to find expansions of cos $n\theta$ and sin $n\theta$ in terms of cos θ and sin θ (for small positive integers *n*). 6.19 Find, and locate in the complex plane, all n'th roots of a non-zero complex number (for small positive integers *n*)	Ch 8	Ass 1 (7%) SC
1.15	Revise		

1.16 1.17	Exams 1		Ex 1 (20%) SC
1.18 1.19	1.4 Integrate cos x, sin x and tan x. 3.5 Integrate functions using the `change of variable' or `substitution' technique. 3.6 Develop the concept of a function defined as an integral: $F(x)=\int_a^x f(t)dt$ 3.7 Examine the relationship between the two parts of the fundamental theorem of calculus: $\frac{d}{dx}\int_a^x f(t)dt = f(x)$ and $\int_a^b f'(x)dx = f(b)-f(a)$	Ch 9	
1.20 2.1	4.10 Model simple harmonic motion by the differential equation $y'' + k^2y = 0$, and obtain solutions. 4.11 Calculate the area under a curve and the area between curves. 4.12 Find volumes of solids of revolution.	Ch10	Epw 3 (5%) MT
2.2 2.3	3.8 Investigate the logarithmic properties of the function $\int_1^x \frac{1}{t}dt$ 3.9 Define the natural logarithm $\ln x$ by the integral in 3.8, and establish its basic properties. 3.10 Use the change of base formula to convert logarithms from one base to another. 3.11 Revise the inverse relationship between exponentials and logarithms. 4.8 Model geometric growth and decay problems by the differential equation $y' = ky$, and obtain solutions.	Ch11	Test 3 (5%) Ch 9, 10, 11 NM
2.4 2.5	5.1 Develop the concept of a vector function of a scalar variable. 5.2 Express an arbitrary (plane) vector function $\boldsymbol{f}(u)$ in Cartesian form as the sum of scalar component functions: $\boldsymbol{f}(u) = f_1(u)\boldsymbol{i} + f_2(u)\boldsymbol{j}$. 5.3 Differentiate a vector function by differentiating its Cartesian components: $\boldsymbol{f}'(u) = f'_1(u)\boldsymbol{i} + f'_2(u)\boldsymbol{j}$. 5.4 Integrate a vector function by integrating its Cartesian components: $\int \boldsymbol{f}(u)du = \int f_1(u)du\,\boldsymbol{i} + \int f_2(u)du\,\boldsymbol{j}$ 5.5 Define the velocity and acceleration of an object moving in the plane as the first and second time derivatives of displacement: $\boldsymbol{v}(t) = \boldsymbol{r}'(t)$, and $\boldsymbol{a}(t) = \boldsymbol{v}'(t)$ 5.6 Find displacements, velocities and accelerations by differentiation and integration. 5.7 Calculate the speed, $\lvert\boldsymbol{v}(t)\rvert$, of an object moving in the plane. 5.8 Examine and use the tangential properties of the velocity vector $\boldsymbol{v}(t)$ on the path of a moving object.	Ch12	
2.6 2.7	6.20 Express cis θ as a complex exponential $e^{i\theta}$ and investigate the formulas: $\frac{d}{d\theta}e^{i\theta} = ie^{i\theta}$ and $\int e^{i\theta}d\theta = -ie^{i\theta}+c$	Ch13	Test 4 Ch 11,12,13 (5%) MT
2.8 2.9 2.10	Revise: TEE papers		Ass 2 (8%) SC
2.11	Exams 2		Ex 2 (30%) MT

Type	Number	Weightings %	Total %
Assignments	2	7, 8	15
Epws	3	5, 5, 5	15
Tests	4	5, 5, 5, 5	20
Exams	2	20, 30	50

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Rationale

Calculus and other Year 12 TES subjects in Mathematics must fulfil a variety of needs. They should be both a satisfying culmination of the work of earlier years and an adequate preparation for tertiary studies. To do this, these subjects must present mathematics as an organised body of useful knowledge and provide students with the skills and confidence necessary to apply this knowledge in practical situations.

Calculus meets these demands by offering studies in a range of topics in modern mathematics. These topics have the potential for useful application and are within the capabilities of the more mathematically able students of this age group. Students graduating from secondary school with a knowledge of these areas will appreciate the power of mathematics to provide a systematic way of understanding and interpreting the world around them.

This subject is designed for students desiring a thorough knowledge of calculus and its applications. It will be invaluable for those proceeding to tertiary studies in the more mathematically or scientifically orientated courses.

Calculus extends the theory and techniques of differential and integral calculus first studied in Introductory Calculus and combines them with trigonometric and vector methods of Geometry and Trigonometry. Complex numbers are also introduced. The study of complex numbers unifies algebraic, trigonometric, calculus and vector concepts, and provides a satisfying conclusion to high school mathematics.

Educational Objectives

This subject seeks to present mathematics as an organised body of knowledge which will provide students with a sound basis for later work in mathematics and other subjects. In addition the subject attempts to develop the following skills and attitudes.

Cognitive Objectives

Students will:

- recall mathematical facts and traditional terminology
- acquire mathematical concepts
- understand mathematical relationships
- acquire manipulative and computational skills
- use mathematical facts, traditional terminology, concepts, relationships and skills in routine ways
- comprehend information in oral and written forms including graphical, diagrammatic and tabular presentations
- select and use appropriate forms for representing mathematical data and relationships
- recognise and extend patterns and make conjectures, predictions and inferences from information given in oral and written forms
- understand and use deductive reasoning and proof
- apply suitable mathematical techniques and problem-solving strategies to both routine and non-routine situations
- select and use different technologies appropriately.
- communicate mathematical ideas and results in both oral and written forms

- compare results with expectations and verify the suitability and reasonableness of a result

Affective Objectives

It is highly desirable that students:

- develop an interest in mathematics, and acquire a positive attitude towards its use and power
- show a willingness to participate and persevere in the learning of mathematics
- develop confidence in their ability to use mathematics effectively
- appreciate the benefits of using technology in mathematics
- display responsibility for their organisation, presentation and learning of mathematics
- interact in a constructive and cooperative manner with peers and teachers and respond constructively to advice.

Recommended Background

The recommended preparation for Calculus is the successful completion of both Introductory Calculus and Geometry and Trigonometry. However it is possible to study Geometry and Trigonometry, and Calculus concurrently after the successful completion of Introductory Calculus.

A set of *Counselling Notes* outlining the subject options in the Mathematics area for Year 11 and Year 12 students has been prepared and distributed to schools. Additional copies can be obtained from the Curriculum Council.

Teaching – Learning Program

The topics, or objectives within topics, can be taught in any order in keeping with the needs of teachers and students.

1. Calculus of Trigonometric Functions (6 hours)

The task of finding derivatives and integrals of elementary functions, which commenced in Introductory Calculus, is completed here with a study of the trigonometric functions. The starting point is the basic trigonometric approximation sin x ≈ x for small x.

1.1 Establish the inequalities:

$$x\cos x < \sin x < x \quad \text{for} \quad 0 < x < \tfrac{\pi}{2}$$

using areas in the unit circle.

1.2 Establish the limit

$$\frac{\sin x}{x} \to 1 \text{ as } x \to 0$$

algebraically, graphically and numerically.

1.3 Differentiate cos *x*, sin *x* and tan *x*.

1.4 Integrate cos *x*, sin *x* and tan *x*.

Notes:

- Integrating tan *x* could be left until integration by substitution is met.
- The reciprocal trigonometric functions sec *x*, cosec *x* and cot *x* are not required in this subject but can be considered.
- The power of graphics calculators and/or computer packages should be used in the investigation of the trigonometric limits.

2. Functions and Limits (20 hours)

A study of calculus begins with the basic concepts of functions and limits. These have been met before. The intention here is to examine them in greater detail. The approach is still informal and intuitive and the underlying ideas are illustrated wherever possible by graphs and sketches. The aim is to provide students with an intuitive understanding of the fundamental concepts without overwhelming them with technical detail.

This section reviews the concept of a function and associated terminology and examines composition and inverses. New classes of functions are introduced in order to illustrate different types of limiting behaviour of functions at a point and at ±∞, continuity and discontinuity, differentiability and non-differentiability.

2.1 Develop the concept of a function as a mapping from one set into another and identify the domain and range.

2.2 Use the remainder theorem and the factor theorem to find linear factors of polynomials.

2.3 Define a rational function as the ratio of polynomials, locate its zeros, poles, point discontinuities and turning points, determine its behaviour at ±∞, and sketch its graph.

2.4 Develop the concept of a piecewise-defined function, and investigate the limiting behaviour of special piecewise-defined functions such as the sign function sgn x, and the greatest integer function int x.

2.5 Use the limit notation

$\lim_{x\to a} f(x), \lim_{x\to a^-} f(x), \lim_{x\to a^+} f(x), \lim_{x\to\infty} f(x), \lim_{x\to-\infty} f(x)$

for limits of the function *f* at the number *a*, and at ∞ and -∞.

2.6 Investigate discontinuities of piecewise-defined functions.

2.7 Investigate graphs of absolute value functions.

2.8 Use the geometric interpretation of $|x - y|$ as the distance between *x* and *y* to establish the triangle inequality: $|x + y| \le |x| + |y|$.

2.9 Solve, algebraically and geometrically, simple equations and inequalities involving absolute values of linear functions.

2.10 Define the derivative of a function at a point using limit notation.

2.11 Determine the derivative of x^n, where *n* is a positive integer, using limit notation.

2.12 Investigate the non-differentiability of simple piecewise-defined functions using limit notation.

2.13 Develop the concept of function composition and obtain expressions for the composites of simple functions.

2.14 Develop the concept of an inverse function and obtain expressions for the inverses of linear, power, quadratic and reciprocal functions on appropriate domains.

2.15 Investigate the relationship between the graph of a function and the existence of its inverse.

2.16 Investigate the reflection properties of the graphs of a function and its inverse.

Notes:

- Students must understand the link between concepts in 2.3 and those in 2.5, particularly to inform the use of a graphics calculator. Effective use of a calculator requires students to predict the vertical and horizontal asymptotes and point discontinuities.
- Graphics calculators and /or computer packages should also be used to support the teaching/learning of other syllabus objectives, for example, 2.7, 2.10 and 2.16.
- The term 'pole' has been borrowed from complex variable theory. A pole of a rational function is simply a zero of its reciprocal.
- It is not intended that ε-δ techniques be part of this subject. Limits are still to be defined in an intuitive manner and illustrated wherever appropriate with graphs and numerical calculations using graphics calculators and/or computer packages.

The discussion of function composition could be done in conjunction with discussions of the chain rule for differentiation and integration by substitution. Similarly, the discussion of inverses could well be delayed until the review of exponentials and logarithms.

3. Theory and Techniques of Calculus

(20 hours)

This section reviews fundamental operations of calculus – differentiation and integration – and the relationship between them. New integration techniques are introduced and an alternative, slightly more formal approach to logarithms and exponentials is examined.

3.1 Revise the sum, product and quotient rules for differentiation.

3.2 Revise the chain rule for differentiating composite functions.

3.3 Revise higher order differentiation.

3.4 Differentiate functions defined implicitly.

3.5 Integrate functions using the `change of variable' or `substitution' technique.

3.6 Develop the concept of a function defined as an integral:

$$F(x) = \int_a^x f(t)dt$$

3.7 Examine the relationship between the two parts of the fundamental theorem of calculus:

$$\frac{d}{dx}\int_a^x f(t)dt = f(x) \text{ and}$$

$$\int_a^b f'(x)dx = f(b) - f(a)$$

3.8 Investigate the logarithmic properties of the function

$$\int_1^x \frac{1}{t}dt$$

3.9 Define the natural logarithm $\ln x$ by the integral in 3.8, and establish its basic properties.

3.10 Use the change of base formula to convert logarithms from one base to another.

3.11 Revise the inverse relationship between exponentials and logarithms.

Notes:

- In 3.5 students will be expected to find the appropriate substitution $u = f(x)$ in cases where the derivative term $f'(x)$ is plainly visible. In other cases the substitution will be provided.
- The review of exponentials and logarithms implied in 3.10 should include an investigation of the use of 'scientific notation' for numbers, and other logarithmic scales in physics, chemistry and seismology.
- The calculus of trigonometric functions is extended through sections 3.1–3.8 e.g. the substitution technique of integration will include integrals like

$$\int \sin 3x dx \text{ and } \int \sin^3 xdx$$

- Students should be able to calculate derivatives at a given value and definite integrals both with and without their graphics calculator.

4. Applications of Calculus (28 hours)

Much of this section is a revision of the applications of calculus studied in Introductory Calculus. However there are some new techniques and a wider range of functions to which they can be applied.

The applications should be chosen from as wide a variety of sources as possible. In this way students will gain an appreciation of the utility of calculus and confidence in their ability to use it.

4.1 Determine the gradient of a curve and the rate of change of a function.

4.2 Approximate a curve by a tangent and use the incremental formula $\delta y \approx \frac{dy}{dx}\delta x$.

4.3 Find local extreme points and inflection points of functions determining their nature using a sign diagram for $f'(x)$ or the second derivative test. Understand the relationships between the graphs of $f(x)$, $f'(x)$ and $f''(x)$.

4.4 Solve optimisation problems by finding global extreme points.

4.5 Solve related rate problems.

4.6 Evaluate total changes from given rates of change.

4.7 Examine the process of approximating the roots of an equation using the Newton-Raphson algorithm.

4.8 Model geometric growth and decay problems by the differential equation $y' = ky$, and obtain solutions.

4.9 Model rectilinear motion by the differential equations $x' = f(t)$ and $x'' = g(t)$, and obtain solutions.

4.10 Model simple harmonic motion by the differential equation $y'' + k^2y = 0$, and obtain solutions.

4.11 Calculate the area under a curve and the area between curves.

4.12 Find volumes of solids of revolution.

Notes:

- The incremental formula can be used to estimate propagated errors.
- In 4.7 the importance of the judicious choice of the initial approximation should be noted.
- Examples of geometric growth and decay might include radioactive decay, heating and cooling, and population models. Doubling times and half-lives should also be examined.
- Amplitude and period should be included in the discussion of simple harmonic motion.
- The 'slice' formula for volumes of solids of revolution: $V = \int \pi y^2\, dx$, is appropriate at this level.
- Use of a graphics calculator can assist problem solving for many of these syllabus objectives, for example 4.4, and 4.8 - 4.12. Particularly relevant is the global view a graph can allow of a situation, but this requires that students know how to choose appropriate domain and range for the viewing window.

5. Vector Calculus (10 hours)

This section is a brief introduction to the application of calculus to the study of vector-valued functions. It is a natural way of combining and reinforcing calculus techniques with vector methods studied in Geometry and Trigonometry.

5.1 Develop the concept of a vector function of a scalar variable.

5.2 Express an arbitrary (plane) vector function $\boldsymbol{f}(u)$ in Cartesian form as the sum of scalar component functions:

$$\boldsymbol{f}(u) = f_1(u)\boldsymbol{i} + f_2(u)\boldsymbol{j}.$$

5.3 Differentiate a vector function by differentiating its Cartesian components:

$$\boldsymbol{f}'(u) = f'_1(u)\boldsymbol{i} + f'_2(u)\boldsymbol{j}.$$

5.4 Integrate a vector function by integrating its Cartesian components:

$$\int \boldsymbol{f}(u)du = \int f_1(u)du\,\boldsymbol{i} + \int f_2(u)du\,\boldsymbol{j}$$

5.5 Define the velocity and acceleration of an object moving in the plane as the first and second time derivatives of displacement:

$$\boldsymbol{v}(t) = \boldsymbol{r}'(t), \text{ and } \boldsymbol{a}(t) = \boldsymbol{v}'(t)$$

5.6 Find displacements, velocities and accelerations by differentiation and integration.

5.7 Calculate the speed, $|\boldsymbol{v}(t)|$, of an object moving in the plane.

5.8 Examine and use the tangential properties of the velocity vector $\boldsymbol{v}(t)$ on the path of a moving object.

Notes:

- Consideration of the position vector of a moving object is possibly the simplest way of introducing the concept of a vector-valued function.
- Two-dimensional vector functions are sufficient to illustrate the basic concepts and techniques studied in this section. It is not necessary at this level to study three-dimensional vector functions, even though the concepts and techniques are essentially the same.
- Circular motion and elliptical motion in the plane will be covered in syllabus objectives 5.5-5.8.
- Projectile motion, in simple cases, will serve as an application of uniform acceleration. The emphasis is on it being a simple application of vector calculus and not dependent upon knowledge of physics.
- The graph and table of values associated with parametric equations can support teaching/learning and problem solving in this topic.

6. Complex Numbers (21 hours)

Complex numbers were introduced as a theoretical device in order to solve certain polynomial equations. However, it is now realised that they have uses far beyond solving equations. They have profoundly influenced the development of mathematics and have applications in many branches of science and engineering.

The study of complex numbers also enriches and unifies earlier studies in algebra, geometry, trigonometry and calculus. For these reason it forms a satisfying conclusion to high school mathematics.

6.1 Define a complex number z as an ordered pair (a,b) of real numbers, and represent it as a point in the complex plane (Argand Diagram).

6.2 Add and multiply complex numbers according to the rules: $(a,b) + (c,d) = (a + c, b + d)$,
and $(a,b) \times (c,d) = (ac - bd, ad + bc)$.

6.3 Investigate the correspondences $(1,0) \leftrightarrow 1$ and $(0,1)$ corresponds to i where $i^2 = -1$

6.4 Express a complex number z in Cartesian form as the sum of its real and imaginary parts:

$z = a + bi$, where $i^2 = -1$

6.5 Investigate the properties of $|z|$, the absolute value of the complex number z.

6.6 Investigate the properties of arg z, the argument of the complex number z.

6.7 Establish the exponential properties of $\text{cis}\,\theta = \cos\theta + i\sin\theta$:

$$\text{cis}(\theta+\phi) = \text{cis}\,\theta\,\text{cis}\,\phi$$

$$\text{cis}\,0 = 1, \quad \text{and}$$

$$\text{cis}(-\theta) = (\text{cis}\,\theta)^{-1}$$

6.8 Express a complex number z in polar form:

$z = r\,\text{cis}\,\theta$, where $r = |z|$ and $\theta = \arg z$.

6.9 Multiply complex numbers expressed in polar form.

6.10 Establish the zero-divisor property of complex numbers:

$z_1 z_2 = 0$ if and only if $z_1 = 0$ or $z_2 = 0$.

6.11 Find the conjugate $\bar{z}$ of a complex number z, expressed in Cartesian or polar form, and locate it in the complex plane.

6.12 Establish, algebraically and geometrically, the conjugation properties:

$$z\bar{z} = |z|^2, \quad \overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$$

and $\overline{z_1 z_2} = \bar{z}_1\,\bar{z}_2$

6.13 Divide one complex number by another, using both Cartesian and polar forms.

6.14 Establish $\dfrac{\bar{z}}{|z|^2}$ as the reciprocal of a non-zero complex number z.

6.15 Describe regions in the complex plane (Argand plane) by means of simple systems of equalities and inequalities.

6.16 Find all complex roots of a quadratic equation with real coefficients by completing the square or by using the quadratic formula.

6.17 Investigate De Moivre's formula:

$$\operatorname{cis} n\theta = (\operatorname{cis}\theta)^n.$$

6.18 Use De Moivre's formula to find expansions of $\cos n\theta$ and $\sin n\theta$ in terms of $\cos\ \theta$ and $\sin\ \theta$ (for small positive integers n).

6.19 Find, and locate in the complex plane, all n'th roots of a non-zero complex number (for small positive integers n)

6.20 Express $\operatorname{cis}\theta$ as a complex exponential $e^{i\theta}$ and investigate the formulas:

$$\frac{d}{d\theta}e^{i\theta} = ie^{i\theta} \text{ and } \int e^{i\theta}d\theta = -ie^{i\theta} + c$$

Notes:

- Since addition of complex numbers is just component wise addition, it is a form of vector addition, and hence can be described geometrically as a parallelogram rule.
- The zero-divisor property is just one of the field axioms satisfied by the set of complex numbers and by the set of real numbers. Others include commutativity and associativity of addition and multiplication, and distributivity. It is not appropriate at this level to dwell on these basic number properties. However, they should not be ignored entirely. We choose to concentrate on the zero-divisor property because it is not entirely trivial, it can be easily demonstrated using polar forms, and it will be used implicitly later when solving equations.

The study and applications of the polar forms of complex numbers will reinforce earlier work in trigonometry, exponential functions, and calculus.

Time Allocation

The subject has been designed to be completed through a structured education program of approximately 110 hours in any suitable contexts and series of learning experiences. Typically the subject will be studied over the period of one school year. For administrative reasons schools wishing to vary this delivery pattern (e.g. over a shorter period or over a longer period up to two school years) are required to notify the Chief Executive Officer of the Curriculum Council.

Subject Completion

Students must complete the school's structured educational and assessment program for a subject in order to be eligible to receive a grade unless there are exceptional and justifiable circumstances. In situations where the school considers that insufficient information has been gathered to justify the award of a grade for the subject, a result of U (for unfinished) should be allocated. The Curriculum Council offers the flexibility for the U to be converted to a grade after the final grades have been submitted. Further details on assessment and grading are provided in Volume I of the Syllabus Manuals.

Examination Details

The examination will consist of one written paper of three hours duration.

The written paper will consist of a mixture of short routine questions and more challenging non-routine questions reflecting the educational objectives of the syllabus. There will be no choice of questions.

Resources:

- Notes on two unfolded A4 sheets of paper
- Curriculum Council *Mathematical Formulae and Statistical Tables* book
- Drawing instruments and templates

Calculators satisfying the conditions set by the Curriculum Council for this subject, which are listed on the Curriculum Council website:

http://www.curriculum.wa.edu.au/files/doc/139775_2.doc

Note: Personal copies of the tables book should not contain any handwritten or typewritten notes, symbols, signs, formulas or any other marks (including underlining and highlighting), except the name and address of a candidate, and may be inspected during the examination.

Assessment Structure

Assessment structures are an integral part of all Accredited Subjects.

The structure specifies:

1. the components and learning outcomes to be included in assessment
2. weightings to be applied to these components
3. the types of assessment considered appropriate for the subject.

Table 1

Syllabus Content	**Weighting percentage**
Calculus of trigonometric functions	5-10
Functions and limits	15-20
Theory and techniques of calculus	15-20
Applications of calculus	25-30
Vector calculus	5-10
Complex numbers	20-25

Table 2

Learning Outcomes)	**Weighting percentage**
	(relative significance of learning outcomes)
Lower order cognitive objectives *	45-55
Higher order cognitive objectives **	45-55

* which include recall of skills, acquisition of concepts and routine use of mathematical knowledge.

** which describe processes such as generalisation, justification, deduction and the application of mathematical techniques and problem solving strategies in non-routine ways.

Table 3

Types of Assessment	**Weighting percentage**
Extended pieces of work	15-30
Other forms of assessment	0-20
Tests and Examinations	50-75

The assessment program must provide students with the opportunity to demonstrate achievement of the requirements of the subject.

AND

Students must complete the requirements of the subject.

Supporting Notes

1. In the assessment program developed by schools, due attention must be given to the full range of educational objectives. The Syllabus Committee recommends that, in order to assess the achievement of the higher order cognitive objectives, **at least three (3)** extended pieces of work be included in the assessment program.
2. In many assessment tasks it is appropriate and desirable for students to work cooperatively and obtain help from various sources. However, assessment in mathematics should focus on the level of knowledge and understanding a student has attained and not the manner in which the learning has taken place. In all cases where students can collaborate or obtain outside help for assessable

work, schools must ensure that marks awarded are measures of the student's own knowledge and understanding. Accordingly, validation is regarded as an integral part of the assessment procedure.

3. Student performance in the subjects will be graded on a scale of A, B, C, D and E according to the degree to which students have achieved the stated objectives of the subject.

   A set of grade-related descriptors, which indicate the standard of student achievement required for each grade, is available for these subjects and should be used when grading student performance.

Notes on Table 3

Extended Pieces of Work: These are assessment tasks which allow students the opportunity to demonstrate higher order cognitive skills such as verification, justification, generalisation, deduction, proof, interpretation and application. Only tasks which meet these requirements can be included as EPWs. They may involve an in-class and/or out of class component and may be completed over an extended period of time. The intent is to set tasks that can be completed free from the pressures of time. Careful consideration of the time allocated to complete such tasks is therefore essential. Projects which involve higher order skills may be considered as EPWs.

Other Forms of Assessment: There are other forms of assessment that may be used to determine the extent to which students have achieved the objectives of the course. Assignments, projects, checklists, homework, teacher observation, and oral presentations should be included in this category.

Notes:

- While out of class assessment tasks at times allow (in fact may require) students to utilise resources beyond those typically available while being assessed under supervised conditions, it is expected that work finally submitted for assessment should be both known to and understood by the students concerned.
- Schools are responsible for determining procedures which should be applied to eliminate the likelihood of cheating, plagiarism, collusion and the like.
- School-based initiatives to exercise some form of ‘control’ over out-of-class assessment tasks are considered desirable as a means of both protecting students and retaining the integrity of using school
  assessments for external purposes.
- The following are examples of ‘controls’ that may be legitimately used:
  - professional judgement
  - in-class supervised tasks with open access to notes and other resources
  - achievement on ‘at home’ tasks measured by a brief in-class test on key concepts
  - research done out of class with reports completed in class under supervised conditions
  - teacher/student interview
  - a combination of in-class and out-of-class components.

Grade-Related Descriptors

Grade-Related Descriptors describe the student performance standards that are used to award grades in this subject. Schools delivering this subject have been provided with a copy of the document. Additional copies may be purchased from the Curriculum Council.

Mathematics Learning Area
St Stephen's School Carramar
January 2008

Extended Pieces of Work (EPW) Assessment Policy.
Year 11 and 12 TEE Mathematics Courses.

To the student.

An extended piece of work is any assignment, problem solving activity, project or investigation that you are required to complete out of class. After you hand up the completed EPW to your mathematics teacher he/she will give you a validation or test to verify that the EPW is your work. Consequently an EPW consists of two parts. A "take home" component and the "in class" assessment or validation.

The out of class or take home component will be given a weighting of no more than 20% of the total EPW weighting. On the same day that the out of class component is due you will be required to sit the in class validation. When your validation has been marked, your result is added to the mark that you received for the out of class component. The mark allocated to the out of class component is predominantly an effort mark. If you complete this component in a neat, clear and concise format then you can quite easily obtain a perfect mark. While the validation aims to check that the EPW is your own work, it also tests your level of understanding of the content and processes pertained within the EPW. The validation is likely to extend you beyond the mathematical requirements of the out of class component.

For Example: Betty received a mark of 4 out of 5 for the take home part of her investigation. On the day that this was due she sat the validation and she received a mark of 16 out of 20. So her final mark for this investigation is 4 + 16 = 20 out of a possible 25.

Your mathematics teacher may decide to give you an EPW at a designated time that does not contain a take home component. In this case the assessment is 100% weighted towards the in class component and marked similarly to a test.

If you hand up the out of class component late without an acceptable reason marks will be *deducted* at the rate stated in the St Stephen's Assessment Policy contained within your Student Work record. If you do not satisfactorily complete the out of class component you will receive a zero score for the EPW. You must show that you have rigorously attempted all questions.

If you are absent for a significant assessment then a medical certificate is required. Failure to provide a medical certificate may result in a zero score for the relevant assessment piece.

These processes are in place to ensure the validity and fairness of student results for EPW in mathematics. If you have any questions please don't hesitate to ask myself or your mathematics teacher.

Stephen Corcoran
Head of Mathematics