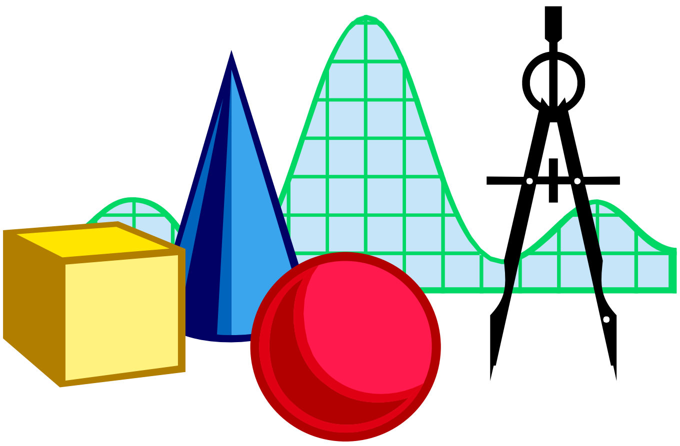
# ST STEPHEN'S SCHOOL
# CARRAMAR

# GEOMETRY & TRIGONOMETRY
# 2008



*COURSE OUTLINE*

# St Stephen's School, Carramar

# 2008 Course outline – Geometry and Trigonometry

| SEMESTER ONE | | | | |
|---|---|---|---|---|
| **Topic** | **Syllabus Objectives** | **Weeks** | | **Assessment Items** |
| Trigonometry Revision<br>Chapter 1 | 1.1 – 1.3, 2.4 | 0 – 3 | Term 1 | EPW 1 Wk 2 (7%) |
| Arc Length<br>Chapter 1 | 1.6, 1.7 | 4 | Term 1 | |
| Radian Measure<br>Chapter 4 | 1.4, 1.5, 1.8 | 5 – 6 | Term 1 | Test 1 Wk 6 (6%)<br>Ch 1 & 4 |
| Introduction to Vectors I<br>Chapters 2 & 3 | 3.1 – 3.7 | 7 – 9 | Term 1 | Test 2 Wk 9 (6%)<br>Ch 2 & 3 |
| Vectors II<br>Chapter 5 | 3.10 – 3.13 | 10 | Term 1 | |
| Vectors II<br>Chapter 5 continued | 3.10 – 3.13 | 1 – 2 | Term 2 | EPW 2 Wk 1 (7%) |
| Trigonometric Graphs<br>Chapter 6 | 2.5 | 3 – 4 | Term 2 | Assignment 1 Wk 4 (2%) |
| Revision and Exams | | 5 – 7 | Term 2 | Semester 1 Exam (15%) |
| Trigonometric equations<br>Chapter 6 | 2.6 | 8 – 9 | Term 2 | Test 3 Wk 9 (6%)<br>Ch 5 & 6 |
| Trigonometric Identities I<br>Chapter 8 (first half) | 2.1 – 2.3 | 10 | Term 2 | |

| SEMESTER TWO | | | | |
|---|---|---|---|---|
| **Topic** | **Syllabus Objectives** | **Weeks** | | **Assessment Items** |
| Trigonometric Identities II<br>Chapter 8 (second half) | 2.7, 2.8 | 1 – 3 | Term 3 | EPW 3 Wk 2 (7%) |
| Relative Vectors<br>Chapter 7 | 3.13 | 3 – 5 | Term 3 | Test 4 Wk 5 (6%)<br>Ch 7 & 8 |
| Vectors III<br>Chapter 9 | 3.8, 3.9, 3.11 | 6 – 9 | Term 3 | Test 5 Wk 9 (5%)<br>Ch 9 |
| Vector Geometry in Space<br>Chapter 11 | 4.1 – 4.11 | 10 | Term 3 | |
| Vector Geometry in Space<br>Chapter 11 (cont.) | 4.1 – 4.11 | 2<br>(Y11 retreat week 1) | Term 4 | |
| Polar Coordinates<br>Chapter 10 | 1.9 – 1.11 | 3 – 4 | Term 4 | Test 6 Wk 4 (6%)<br>Ch 10 & 11<br>Assignment 2 Wk 4 (2%) |
| Revision and Exams | | 4 – 6 | Term 4 | Semester 2 Exam (25%) |

Student text: **Geometry and Trigonometry, *Sadler, 1993***

# Term 1

## *Trigonometry Revision* — *Ch 1*

| | |
|---|---|
| 1.1 | The Fundamental Ratios (SOH-CAH-TOA). |
| 1.2 | Sine and Cosine Rules. |
| 1.3 | Area of a Triangle. |
| 2.4 | Exact Values, Surds, Rationalising the Denominator. |
| 1.6, 1.7 | Arc Length, Longitude/Latitude. |

## *Radian Measure* — *Ch 4*

| | |
|---|---|
| 1.4 | Radian Measure of an Angle. |
| 1.5 | Degree/Radian Conversion. |
| 1.6 | Determining Arc Length. |
| 1.8 | Areas of Sectors and Segments. |

## *Introduction to Vectors I* — *Ch 2 & 3*

| | |
|---|---|
| 3.2, 3.3 | Concept of Vector, Vector Quantities, Addition and Subtraction. |
| 3.7 | Special Vectors, Scalar Multiplication. |
| 3.6 | Parallelogram Rule |
| 3.4, 3.5 | Component form (**i** and **j** notation). |
| 3.1 | Position Vector |

# Term 2

## *Vectors II* — *Ch 5 & 7*

| | |
|---|---|
| 3.11, 3.12 | Vector, Cartesian and Parametric Equation of a Line. |
| 3.10 | Vector and Cartesian Equation of a Circle. |
| 3.13 | Use of Vectors to Solve 2D Problems,<br>Relative Displacement & Relative Velocity. |

## *Trigonometric Graphs* — *Ch 6*

| | |
|---|---|
| 2.1 | Trigonometric Graphs, Period, Amplitude, Phase Shifts. |

## *Trigonometric Equations* — *Ch 6*

| | |
|---|---|
| 2.2 | Solution of Trigonometric Equations within a Finite Domain. |

## *Trigonometric Identities I* — *Ch 8 (first half)*

| | |
|---|---|
| 2.3 | Pythagorean, Parity, Complementary, Periodicity, Phase. |
| 2.4 | Definition of Tan Function. |
| 2.5 | Proving Simple Identities |

## Term 3

### *Trigonometric Identities II* — *Ch 8 (second half)*

| | |
|---|---|
| 2.7 | Addition, Double Angle Formulae |
| 2.8 | Transformation of Trigonometric Equations |

### *Vectors III* — *Ch 9*

| | |
|---|---|
| 3.8 | Dot Product. |
| 3.9 | Angle Between Vector, Parallelism, Perpendicularity |
| 3.11 | Vector Equation of a Line (Revisited) |

### *Vector Geometry in Space* — *Ch 11*

| | |
|---|---|
| 4.1 | Introduction to 3D Cartesian Space. |
| 4.2 | Notation of 3D Vectors. |
| 4.3 | Specific Vectors (zero, opposite, equality). |
| 4.4 | Length Formula. |
| 4.5 | Unit Bases **i**, **j** and **k**. |
| 4.6 | Addition of Vectors (Parallelogram, Components). |

## Term 4

### *Vector Geometry in Space* — *Ch 11*

| | |
|---|---|
| 4.7 | Scalar Multiplication. |
| 4.8 | Dot Product. |
| 4.9 | Angle Between Vectors, Parallel, Perpendicularity. |
| 4.10 | Vector Equation of a Sphere. |
| 4.11 | 3D Geometry (Lines in Vector Space). |

### *Polar Coordinates* — *Ch 10*

| | |
|---|---|
| 1.9 | Polar Coordinates in $(r, \theta)$ notation. |
| 1.10 | Graphing with Polar Coordinates |
| 1.11 | Polar/Cartesian Conversions |

# 2007 Assessment Policy: Geometry and Trigonometry

**Assessment Weightings**

| | |
|---|---|
| EPWs (×3) | 21% |
| Tests (×6) | 35% |
| Exams (×2) | 40% |
| Assignments (×2) | 4% |

At the end of Year 11, each student will receive a mark out of 100 and a grade (A, B, C, D or E) based on the following:

1. Use of grade related descriptors for each level of achievement.
2. The final year score accumulated during the year.
3. Examination of student files by St Stephen's School staff of Curriculum Council moderators as may be necessary.

Every mathematics student is required to keep a "Student Assessment File" that must contain all assignment/project work, tests, extended pieces of work and Semester Exams. That is, the file must contain every piece of work by the student that contributes towards the final mark.

Failure on the part of a student to provide any given piece of assessment from the "Student Assessment File" may result in awarding a grade upon insufficient evidence.

**St Stephen's School Assessment Policy**

Refer to the Secondary School Assessment Policy contained within the Student Work Record.

A Progressive Record of Assessments is contained at the rear of the Student Work Record. Students are encouraged to use this to help monitor their academic progress across all their subjects.

**Mathematics Learning Area**
**St Stephen's School Carramar**
**January 2008**

**Extended Pieces of Work (EPW) Assessment Policy.**
**Year 11 and 12 TEE Mathematics Courses.**

To the student.

An extended piece of work is any assignment, problem solving activity, project or investigation that you are required to complete out of class. After you hand up the completed EPW to your mathematics teacher he/she will give you a validation or test to verify that the EPW is your work. Consequently an EPW consists of two parts. A "take home" component and the "in class" assessment or validation.

The out of class or take home component will be given a weighting of no more than 20% of the total EPW weighting. On the same day that the out of class component is due you will be required to sit the in class validation. When your validation has been marked, your result is added to the mark that you received for the out of class component. The mark allocated to the out of class component is predominantly an effort mark. If you complete this component in a neat, clear and concise format then you can quite easily obtain a perfect mark. While the validation aims to check that the EPW is your own work, it also tests your level of understanding of the content and processes pertained within the EPW. The validation is likely to extend you beyond the mathematical requirements of the out of class component.

For Example: Betty received a mark of 4 out of 5 for the take home part of her investigation. On the day that this was due she sat the validation and she received a mark of 16 out of 20. So her final mark for this investigation is 4 + 16 = 20 out of a possible 25.

Your mathematics teacher may decide to give you an EPW at a designated time that does not contain a take home component. In this case the assessment is 100% weighted towards the in class component and marked similarly to a test.

If you hand up the out of class component late without an acceptable reason marks will be *deducted* at the rate stated in the St Stephen's Assessment Policy contained within your Student Work record. If you do not satisfactorily complete the out of class component you will receive a zero score for the EPW. You must show that you have rigorously attempted all questions.

If you are absent for a significant assessment then a medical certificate is required. Failure to provide a medical certificate may result in a zero score for the relevant assessment piece.

These processes are in place to ensure the validity and fairness of student results for EPW in mathematics. If you have any questions please don't hesitate to ask myself or your mathematics teacher.

Stephen Corcoran
Head of Mathematics

**Mathematics Learning Area**
**St Stephen's School Carramar**
**January 2008**

**Student Notes for Assessments and Examination Length.**

**Year 11 and 12 TEE Mathematics Courses.**

<u>Student Notes</u>

- Curriculum Council Approved Statistical Tables book allowed in all assessments.
- 2 A4 pages double sided, like the TEE for all assessments. At the teachers' discretion this can be changed to be 0 or 1 page but this must be stated in writing on the front page of the assessment and students given sufficient notice.

<u>Exam Length</u>

- 2.5 hours + 10  semester one exam
- 3 hours + 10 semester two exam

Stephen Corcoran
Head of Mathematics

# D503 GEOMETRY AND TRIGONOMETRY

## (YEAR 11) – 2006-2008

### *Rationale*

Geometry and Trigonometry and other subjects that lead to Year 12 TES subjects in Mathematics must fulfil a variety of needs. They should be both a satisfying continuation of the work of earlier years and an adequate preparation for tertiary studies. To do this these subjects must present mathematics as an organised body of useful knowledge and provide students with the skills and confidence necessary to apply this knowledge in practical situations.

These demands are met by offering studies in a range of topics in modern mathematics. The topics have the potential for useful application and are within the capabilities of the more mathematically able students of this age group. Students graduating from secondary school with a knowledge of these areas will appreciate the power of mathematics to provide a systematic way of understanding and interpreting the world around them.

Geometry and Trigonometry provides a consolidation and extension of algebraic, geometric and trigonometric skills, and an introduction to vector methods. It is intended for students desiring a strong mathematical preparation for tertiary studies and will be invaluable for those proceeding to the more mathematically or scientifically orientated courses. Together with the Year 11 subject, Introductory Calculus, it provides a suitable preparation for the Year 12 subject, Calculus.

### *Educational Objectives*

This subject seeks to present mathematics as an organised body of knowledge that will provide students with a sound basis for later work in mathematics and other subjects. In addition the subject attempts to develop the following skills and attitudes.

#### Cognitive Objectives

Students will be expected to:

- recall mathematical facts and traditional terminology
- acquire mathematical concepts
- understand mathematical relationships
- acquire manipulative and computational skills
- use mathematical facts, traditional terminology, concepts, relationships and skills in routine ways
- comprehend information in oral and written forms including graphical, diagrammatic and tabular presentations
- select and use appropriate forms for representing mathematical data and relationships
- recognise and extend patterns and make conjectures, predictions and inferences from information given in oral and written forms
- understand and use deductive reasoning and proof
- apply suitable mathematical techniques and problem-solving strategies to both routine and non-routine situations
- select and use different technologies appropriately
- communicate mathematical ideas and results in both oral and written forms
- compare results with expectations and verify the suitability and reasonableness of a result.

#### Affective Objectives

It is highly desirable that students:

- develop an interest in mathematics and acquire a positive attitude towards its use and power
- show a willingness to participate and persevere in the learning of mathematics
- develop confidence in their ability to use mathematics effectively
- appreciate the benefits of using technology in mathematics
- display responsibility for their organisation, presentation and learning of mathematics
- interact in a constructive and cooperative manner with peers and teachers and respond constructively to advice.

## Recommended Preparation

A desirable preparation is a strong background in algebra and trigonometry.

The recommended preparation for the Year 12 subject Calculus is the successful completion of both Introductory Calculus and Geometry and Trigonometry. However, it is possible to study Geometry and Trigonometry and Calculus concurrently, after the successful completion of Introductory Calculus.

A set of *Counselling Notes* outlining the subject options in the mathematics area for Year 11 and Year 12 students has been distributed to schools. Additional copies can be obtained from the Curriculum Council.

## *Teaching – Learning Program*

The topics, or objectives within topics, can be taught in any order in keeping with the needs of teachers and students.

### 1. Applications of Trigonometry and Radian Measure (25 hours)

This section begins with a review of trigonometry as a way of finding distances and angles in geometrical figures. Examples are not restricted to two dimensions. Simple three-dimensional applications will also serve as preparation for section 4.

Radians are introduced as the natural measure of angle because of their direct association with arc length. Formulas involving angles are usually simpler if radian measure is used. This advantage will become even more apparent when the calculus of trigonometric functions is studied in the TES subject Calculus.

Finally, polar coordinates are introduced as a means of specifying position in the plane by magnitude and direction. This theme will recur in this subject with vectors and with complex numbers in the TES subject Calculus.

1.1 Use trigonometric ratios for finding distances and angles in right triangles.

1.2 Establish and use the sine and cosine rules to find distances and angles in triangles in two and three dimensional situations.

1.3 Establish and use the formula for the area of a triangle:

$$\text{Area} = \frac{ab\sin C}{2}.$$

1.4 Develop the concept of radian measure.

1.5 Establish the relationship between degree and radian measure of angles, and convert from one measure to the other.

1.6 Determine arc lengths in circles.

1.7 Determine distances along circles of constant latitude or constant longitude on the surface of the earth.

1.8 Determine the areas of sectors and segments of circles.

1.9 Develop the concept of polar coordinates $(r,\theta)$ in the plane, where $r \geq 0$.

1.10 Draw and read graphs of $r$ = constant, $\theta$ = constant, and $r = \theta$.

1.11 Establish the relationship between Cartesian and polar coordinates in the plane, and convert from one system to the other.

**Notes:**

- Applications of the sine and cosine rules should include examples in two and three dimensions, and from a variety of sources such as surveying, navigation and construction.
- The cosine rule deserves special attention as it is the basis of subsequent work in this subject. It is used to establish the addition theorems in 2.7, and the dot product formulas in 3.9 and 4.9.
- It can also be used to establish the basic inequality for the lengths of the sides of a triangle: $|a| \leq |b| + |c|$.
- This inequality appears in various guises at later stages in this subject, and at higher levels of mathematics.
- The formulas for arc length and sector area may be derived as proportions of total circumference and total area of circle, and for this the choice of angle measure is unimportant. While students may be more familiar with degree measure, they need to be encouraged to use radian measure and to appreciate its advantages.
- Applications of radian measure calculations could include rotating objects, in physics and in sport.
- Distances along parallels of latitude should not be confused with shortest distances. However the calculation of great circle distances between points with equal latitude is not beyond the capabilities of the better students in this subject, and teachers may wish, as an extension, to explain how this can be done.

### 2. Trigonometric Functions (25 hours)

A thorough understanding of the trigonometric functions is an important foundation for the successful study of mathematics at higher levels. However, there are also enough worthwhile applications which are easily understood and appreciated by students at this level to justify their inclusion here.

The behaviour of sine and cosine as functions can be easily motivated and illustrated by a study of the co-ordinates (cos $\theta$, sin $\theta$) of a point on the unit circle, and their dependence on the angle $\theta$.

The properties of the trigonometric functions are deduced from their definitions as ratios associated with right triangles. Special emphasis is given to periodicity, amplitude and phase. They are illustrated by graphs wherever possible.

2.1 Develop the concept of sine and cosine as functions, and establish and use the following properties:

Pythagorean –

$\sin^2 x + \cos^2 x = 1,\ |\cos x| \leq 1,\ |\sin x| \leq 1$

parity –

$\cos(-x) = \cos x,\ \sin(-x) = -\sin x$

complementarity –

$\sin x = \cos(\pi/2 - x),\ \cos x = \sin(\pi/2 - x)$

periodicity –

$\sin(x + 2\pi) = \sin x,\ \cos(x + 2\pi) = \cos x$

phase –

$\sin x = \cos(x - \pi/2),\ \cos x = \sin(x + \pi/2)$

2.2 Define the tangent function as the ratio of sine to cosine, and establish and use the following properties:

parity –

$\tan(-x) = -\tan x$

periodicity –

$\tan(x + \pi) = \tan x$

2.3 Use the properties listed in 2.1 and 2.2 to prove simple identities.

2.4 Evaluate sin *x*, cos *x* and tan *x* using calculators, and establish the exact values at multiples of $\pi/6$ and $\pi/4$.

2.5 Investigate and compare graphs of

$y = \sin x$, $y = \cos x$, $y = \tan x$ with

$a \sin b(x + c)+d$, $a \cos b(x + c) +d$

$a \tan b(x + c) +d$, and identify the effects of the constants *a*, *b*, *c* and *d* on amplitude, period, phase, and the locations of zeros and turning points.

2.6 Find all solutions of sin *x* = k, cos *x* = k and tan *x* = k on a given finite domain.

2.7 Use the addition and double angle formulas for sine, cosine and tangent:

$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 2\cos^2\theta - 1$

$= 1 - 2\sin^2\theta$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

2.8 Use the conversions

$a\cos\theta + b\sin\theta = r\sin(\theta \pm \delta)$ or $r\cos(\theta \pm \delta)$

**Notes:**

- The reciprocal trigonometric ratios secant, cosecant and cotangent are not required in this subject. However, it may be considered desirable to briefly discuss these, since they are mentioned in most calculus and trigonometry texts.
- As in most branches of mathematics, much of trigonometry can be deduced from a few fundamental results. Students should gain some experience of this deductive aspect of mathematics, and for this reason manipulation of trigonometric identities is important. However, it should not be allowed to dominate the study of trigonometry. Identities encountered in this subject should be straightforward and not require artificial tricks.
- The use of suitable computer software and graphics calculators is highly recommended for investigating the roles of a, b, c and d in the linear scale changes studied in 2.5.
- Formulas for the general solutions of trigonometric equations over the entire real line are not required.

## 3. Vector Geometry in the Plane (40 hours)

This section is intended as an introduction to vector terminology and methods, with applications based on coordinate geometry and trigonometry. For this reason most of the vectors studied here are displacement vectors. However, other types of vectors are introduced for illustrative purposes.

Vectors will be represented in both their algebraic form as ordered pairs of real numbers and their geometric form as directed line segments. Each form has its advantages. Algebraic manipulation is usually easier with ordered pairs, whereas our understanding of vector concepts is enhanced by regarding vectors as directed line segments.

3.1 Represent plane vectors as ordered pairs (*a*, *b*).

3.2 Develop the concept of displacement vectors in the plane as line segments with magnitude and direction, including:

equality of vectors

opposite vectors

the zero vector.

3.3 Establish and use the formula $|(a,b)| = \sqrt{a^2 + b^2}$

for the length of a vector in the plane.

3.4 Represent the free vector (*a*,*b*) in the form

($a\boldsymbol{i} + b\boldsymbol{j}$), where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the standard unit vectors.

3.5 Develop the concept of the position vector of a point in the Cartesian plane.

3.6 Add plane vectors using:

the parallelogram rule

addition of components.

3.7 Multiply vectors by scalars and extend this to subdividing line segments internally and externally in given ratios.

3.8 Develop the concept of the dot product of plane vectors, and the formula $\boldsymbol{a} \bullet \boldsymbol{b} = a_1b_1 + a_2b_2$, where $\boldsymbol{a} = (a_1, a_2)$ and $\boldsymbol{b} = (b_1, b_2)$

3.9 Establish the formula $\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$ and use it to:

calculate the angle between vectors

identify parallelism and perpendicularity.

3.10 Establish and use the vector form of the equation of a circle in the plane:

$|\boldsymbol{r} - \boldsymbol{d}| = \rho$

together with its Cartesian equivalent.

3.11 Establish and use the vector equation of a line in the plane in its various forms:

one point and slope : $\boldsymbol{r} = \boldsymbol{r}_1 + \lambda\boldsymbol{l}$

two points : $\boldsymbol{r} = \boldsymbol{r}_1 + \lambda(\boldsymbol{r}_2 - \boldsymbol{r}_1)$

normal : $\boldsymbol{r} \bullet \boldsymbol{n} = c$

together with their parametric and Cartesian equivalents.

3.12 Interpret the role of the parameter λ in the equations in 3.11, including when λ is time and $\boldsymbol{l}$ is velocity.

3.13 Solve problems in two-dimensional geometry, including real-life applications, using the concepts and formulas established above, and graphical methods where appropriate.

**Notes:**

- Free vectors are equal if they have the same magnitude and direction. The concept of free vectors, and distinguishing between vector and scalar quantities, should be approached using geometric representations and physical examples such as velocity, acceleration and force.
- Relative displacement and relative velocity should be developed as an application of vector subtraction. Navigation problems involving apparent and true velocities of aircraft and ships operating in cross-currents should be considered.
- The dot product should be used to establish the triangle inequality $|\boldsymbol{a}+\boldsymbol{b}| \le |\boldsymbol{a}| + |\boldsymbol{b}|$
- The parametric equations for the lines in 3.11 are one point and slope:

  $$x = x_1 + \lambda l_1 \text{ and } y = y_1 + \lambda l_2$$

  two points:

  $$x = x_1 + \lambda(x_2 - x_1) \text{ and}$$
  $$y = y_1 + \lambda(y_2 - y_1)$$

  The cartesian equations for the lines in 3.11 are

  one point and slope: $y - y_1 = m(x - x_1)$

  two points: $\dfrac{y - y_1}{y_2 - y_1} = \dfrac{x - x_1}{x_2 - x_1}$

  normal: $ax + by = c$
- A flexible approach to problem solving should be adopted which could include using function graphing on a graphics calculator to solve closest position and collision problems. The parametric graphing facility on graphics calculators can be used to advantage for 3.11, 3.12 and 3.13.
- Vector equations of lines and circles should be used to solve problems concerning intersections and tangencies of lines and circles.
- Students should be exposed to a wide variety of applications. In planar geometry applications might include finding perpendicular bisectors of sides of a triangle, proving the concurrency of the medians of a triangle and proving perpendicularity and bisection of the diagonals of a rhombus. Applications such as Global Positioning System (GPS), radar, and ultrasound scanning are appropriate.
- Algebraic properties of the dot product such as $\boldsymbol{a} \bullet (\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{a} \bullet \boldsymbol{b} + \boldsymbol{a} \bullet \boldsymbol{c}$ are needed for certain proofs of the type mentioned above. These should be used implicitly, without direct reference.

## 4. Vector Geometry in Space (20 hours)

The geometry of three-dimensional space is conceptually more difficult than the geometry of the plane.

However, the transition from two to three dimensions is facilitated by the vector approach. In many cases all that is required is the inclusion of an extra component. Here we concentrate on those aspects of three-dimensional geometry which are direct generalisations from planar geometry.

4.1 Represent free vectors in space in Cartesian form as ordered triples $(a,b,c)$.

4.2 Develop the concept of displacement vectors in space as directed line segments with magnitude and direction, including

  equality of vectors

  opposite vectors

  the zero vector.

4.3 Establish and use the formula

$$|(a,b,c)| = \sqrt{a^2 + b^2 + c^2}$$

for the length of a free vector in space.

4.4 Represent the free vector $(a,b,c)$ in the form $a\boldsymbol{i} + b\boldsymbol{j} + c\boldsymbol{k}$, where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are the standard unit vectors.

4.5 Develop the concept of the position vector of a point in space.

4.6 Add vectors in space using:

  the parallelogram rule

  addition of components.

4.7 Multiply vectors by scalars and extend this to subdividing line segments internally and externally in given ratios.

4.8 Develop the concept of the dot product of plane vectors and the formula:

$$\boldsymbol{a} \bullet \boldsymbol{b} = a_1 b_1 + a_2 b_2 + a_3 b_3, \text{ where}$$
$$\boldsymbol{a} = (a_1, a_2, a_3) \text{ and } \boldsymbol{b} = (b_1, b_2, b_3)$$

4.9 Establish the formula $\boldsymbol{a} \bullet \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, and use it to:

  calculate the angle between vectors

  identify parallelism and perpendicularity.

4.10 Establish and use the vector form of the equation of a sphere:

$$|\boldsymbol{r} - \boldsymbol{d}| = \rho$$

together with its Cartesian equivalent.

4.11 Establish and use the vector equation of a line in space in the form $\boldsymbol{r} = \boldsymbol{r}_1 + \lambda \boldsymbol{l}$ together with its parametric equivalent.

4.12 Solve problems in three-dimensional geometry, including real-life applications, using the concepts and formulas established above, and graphical methods where appropriate.

**Notes:**

- This section should be treated as a reinforcement of the material developed in section 3. At this level the similarities, rather than the differences, between two- and three-dimensional geometry should be emphasised.
- The applications studied here should be similar to those in section 3, with an emphasis on real life situations.

### *Time Allocation*

The subject has been designed to be completed through a structured education program of approximately 120 hours in any suitable contexts and series of learning experiences. Typically the subject will be studied over the period of one school year. For administrative reasons schools wishing to vary this delivery pattern (e.g. over a shorter period or over a longer period up to two school years) are required to notify the Chief Executive Officer of the Curriculum Council.

## Subject Completion

Students must complete the school's structured educational and assessment program for a subject in order to be eligible to receive a grade unless there are exceptional and justifiable circumstances. In situations where the school considers that insufficient information has been gathered to justify the award of a grade for the subject, a result of U (for unfinished) should be allocated. The Curriculum Council offers the flexibility for the U to be converted to a grade after the final grades have been submitted. Further details on assessment and grading are provided in Volume I of the Syllabus Manuals.

# *Assessment Structure*

Assessment structures are an integral part of all Accredited Subjects.

The structure specifies:

1. the components and learning outcomes to be included in assessment
2. weightings to be applied to these components
3. the types of assessment considered appropriate for the subject.

**Table 1**

| **Syllabus Content** | **Weighting percentage** |
|---|---|
| Applications of trigonometry and radian measure | 20-25 |
| Trigonometric functions | 20-25 |
| Vector geometry in the plane | 35-40 |
| Vector geometry in space | 15-20 |

**Table 2**

| **Learning Outcomes** | **Weighting percentage** |
|---|---|
| Lower order cognitive objectives * | 45-55 |
| Higher order cognitive objectives ** | 45-55 |

* which include recall of skills, acquisition of concepts and routine use of mathematical knowledge.

** which describe processes such as generalisation, justification, deduction and the application of mathematical techniques and problem-solving strategies in non-routine ways.

**Table 3**

| **Types of Assessment** | **Weighting percentage** |
|---|---|
| Extended pieces of work | 15-30 |
| Other forms of assessment | 0-20 |
| Tests and Examinations | 50-75 |

The assessment program must provide students with the opportunity to demonstrate achievement of the requirements of the subject.

AND

Students must complete the requirements of the subject.

**Supporting Notes**

1. In the assessment program developed by schools, due attention must be given to the full range of educational objectives. It is recommended that, in order to assess the achievement of the higher order cognitive objectives, **at least three (3)** extended pieces of work be included in the assessment program.
2. In many assessment tasks it is appropriate and desirable for students to work cooperatively and obtain help from various sources. However, assessment in mathematics should focus on the level of knowledge and understanding a student has attained and not the manner in which the learning has taken place.

   In all cases where students can collaborate or obtain outside help for assessable work, schools must ensure that marks awarded are measures of the student's own knowledge and understanding. Accordingly, validation is regarded as an integral part of the assessment procedure.
3. Student performance in the subjects will be graded on a scale of A, B, C, D and E according to the degree to which students have achieved the stated objectives of the subject.

   A set of grade-related descriptors, which indicate the standard of student achievement required for each grade, is available for these subjects and should be used when grading student performance.

**Notes on Table 3**

**Extended Pieces of Work:** These are assessment tasks which allow students the opportunity to demonstrate higher order cognitive skills such as verification, justification, generalisation, proof, interpretation, deduction and application. Only tasks which meet these requirements can be included as EPWs. They may involve an in-class and/or out of class component and may be completed over an extended period of time. The intent is to set tasks that can be completed free from the pressures of time. Careful consideration of the time allocated to complete such tasks is therefore essential. Projects which involve higher order skills may be considered as EPWs.

**Other Forms of Assessment:** There are other forms of assessment that may be used to determine the extent to which students have achieved the objectives of the course. Assignments, projects, checklists, homework, teacher observation, and oral presentations should be included in this category.

**Notes:**

- While out of class assessment tasks at times allow (in fact may require) students to utilise resources beyond those typically available while being assessed under supervised conditions, it is expected that work finally submitted for assessment should be both known to and understood by the students concerned.
- Schools are responsible for determining procedures which should be applied to eliminate the likelihood of cheating, plagiarism, collusion and the like.
- School-based initiatives to exercise some form of ‘control’ over out-of-class assessment tasks are considered desirable as a means of both protecting students and retaining the integrity of using school assessments for external purposes.
- The following are examples of ‘controls’ that may be legitimately used:
  - professional judgement
  - in-class supervised tasks with open access to notes and other resources
  - achievement on ‘at home’ tasks measured by a brief in-class test on key concepts
  - research done out of class with reports completed in class under supervised conditions
  - teacher/student interview
  - a combination of in-class and out-of-class components.

## *Grade-Related Descriptors*

Grade-Related Descriptors describe the student performance standards that are used to award grades in this subject. Schools delivering this subject have been provided with a copy of the document. Additional copies may be purchased from the Curriculum Council.