SOLUTIONS

Exponential Notation:

1. **D** $\frac{m^4 n}{3}$

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2. **C** $2\sqrt{14}$

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3. $120\ 000 \times 0.96^{10} = 79\ 780$

$\therefore$ **E**

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4. **a** x^7
b $8x^8y^3$

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5. **a** $x^{3/5}$
b $x^{2/3}$

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6. **a** $\left(\sqrt{16}\right)^3 = 4^3 = 64$

b $(27^{1/3})^5 = 3^5 = 243$

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7. $a^{[2(3/2) + 6(5/2)]}\ b^{[-4(3/2) + 8(5/2)]} = a^{18}b^{14}$

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8. **a** 3.02
b 0.46

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9. **a** 5.74×10^6
b 7.8×10^{-5}

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10. **a** 729
b 7

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11. **a** $8\sqrt{15}$
b $13\sqrt{8} = 26\sqrt{2}$

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12. **a** $\sqrt{63}$
b $\sqrt{216}$

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13. **a** $15\sqrt{5} + 4\sqrt{5} = 19\sqrt{5}$

b $\dfrac{10\sqrt{5}}{6\sqrt{5}} = \dfrac{5}{3}$

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14. **a** $\dfrac{6\sqrt{5}}{5}$

b $\dfrac{2\sqrt{18}}{6} = \dfrac{6\sqrt{2}}{6} = \sqrt{2}$

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15. $\dfrac{5\left(\sqrt{5}+\sqrt{3}\right)}{5-3} = \dfrac{5\left(\sqrt{5}+\sqrt{3}\right)}{2}$

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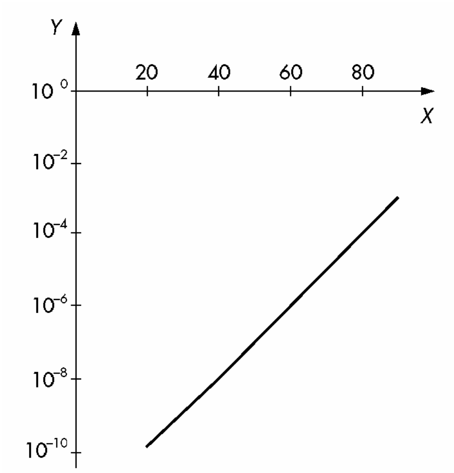
16. **a**

x	20	40	60
y	10^{-10}	10^{-8}	10^{-6}

b



c This is an exponential growth graph, plotted with a logarithmic scale.
d $y = 10^{-12}$
e Ten times
f Trial and error leads to $x = 63$. Check: $10^{(63/10\,-\,12)} = 2 \times 10^{-6}$. Hence an increase of 3 decibels doubles the intensity.

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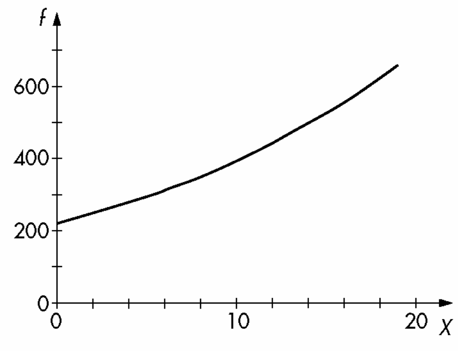
17. a

n	0	5	7	12	17	19
f	220	293.7	329.6	440	587.3	659.3

b



c The relationship is exponential growth. It starts at 220 and increases, doubling each 12 frets.
d It seems to double the frequency. In the formula, increasing *n* by 12 increases the exponent by 1, and thus multiplies the frequency by 2. Thus it will always happen.
e For frets further up the guitar, the length is progressively shorter. Consequently, halving these lengths will make the frets closer together.

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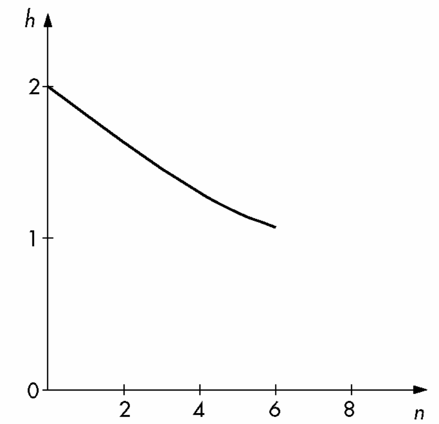
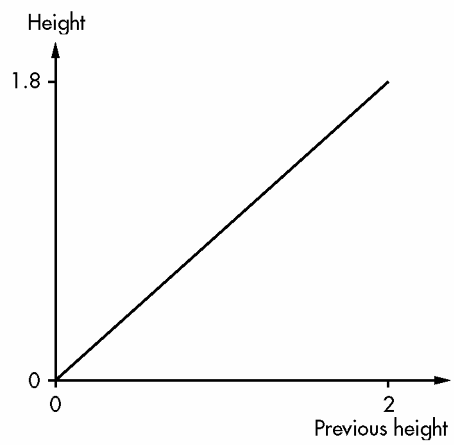
18. a

n	0	1	2	3	4	5	6
h	2.00	1.80	1.62	1.46	1.31	1.18	1.06

b



c This is exponential decay, starting at 2.00, and decreasing by 10% each bounce. $h = 2 \times 0.9^n$.
d



e This is direct linear variation, or direct proportion. $h_{n+1} = 0.9h_n$
f 7
g According to the formula it never quite reaches zero height on a bounce, so continues forever. However reality suggests that it will stop after about 50 bounces. The mathematical model is not as accurate as it could be at that stage!

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19. **a** $1.104^5 = \$1.64$
b 7
c 14
d 21
e $c = 1.104^t$
f 7 years
g $1.104^{(t+7)} = 1.104^7 \times 1.104^t = 2 \times 1.104^t$ so adding 7 years will always double the cost.

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20. **a** 2.4×10^6
b 6.4×10^{-6}

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21. **a** 650 000
b 0.000 000 071

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22. **a** $6a^7b^3$
b $\frac{5a^{10}}{b^2}$

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23. **a** $5ab^3$
b $\frac{p^3}{q^2}$

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24. **a** 7
b 0.5

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25. **a** 5 900 000 000; 384 000
b $C = 2 \times \pi \times 3.84 \times 10^5 = 24.13 \times 10^5 = 2.413 \times 10^6$; answer 2.413×10^6 km.
c We have to divide 5.9×10^9 by $3.84 \times 10^5 = 15\,364 = 1.54 \times 10^4$

Trigonometry:

1. $0.2 \times 60 = 12$
$\therefore$ **C**

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2. $X = 15 \sin 25°$
$= 6.3$ m
$\therefore$ **B**

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3. $\cos \theta = \dfrac{4}{6}$
$\theta = 48° \ 11'$
$\therefore$ **E**

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4. **A** S21°E

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5. **a** 0.6820
b 0.9497

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6. **a** 16.68°
b 69.38°

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7. $\cos 34° = a \div 15$, so $a = 15 \cos 34° = 12.44$

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8. $\cos 29° = 18 \div a$, so $a = 18 \div \cos 29° = 20.58$

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9. $\tan \theta = 2.6 \div 5.4$, so $\theta = 25.71° = 25° \ 42' \ 36''$

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10. $\sin \theta = 1.2 \div 8.5$, so $\theta = 8.12°$ or $8° \ 07'$

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11. $\cos 13° = 3.1 \div a$, so $a = 3.1 \div \cos 13° = 3.18$; answer 3.18 m

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12. $\tan 78° = (h - 1.5) \div 12$, so $h = 12 \tan 78° + 1.5 = 58$ m

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13. $\cos 18° = d \div 35$, so $d = 35 \cos 18° = 33.3$ km

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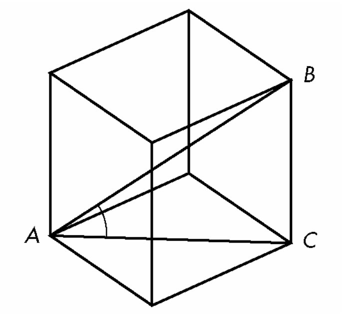
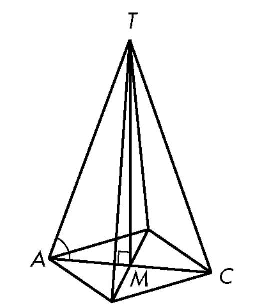
14. $\tan \theta = 12 \div 7$, so $\theta = 59.74°$. Answer: N59°45'W or 300°15'T

15.



Using the theorem of Pythagoras, the base diagonal AC is 14.14 cm so in ΔABC, tan θ = 10 ÷ 14.14, and θ = 35.26° = 35° 16'

16.



Using the theorem of Pythagoras, the base diagonal AC is 16.970 cm so AM is 8.485 cm. In ΔATM, cos θ = 8.485 ÷ 25, and θ = 70.16° = 70° 06'

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17. **a** tan 30° = h ÷ (b + 20), so b + 20 = h ÷ tan 30°, or b = h ÷ tan 30° – 20
b tan 40° = h ÷ b, so b = h ÷ tan 40°
c h ÷ tan 30° – 20 = h ÷ tan 40°
1.732h – 20 = 1.192h, so 0.54h = 20 or h = 37.04. The tower is 37 m high.
d b = 37 ÷ tan 40° = 44.10, hence b + 20 = 64.10. The positions are 44 m and 64 m.

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18. **a** tan 35° = h ÷ 100, so h = 100 tan 35° = 70.02. Height is 70 m.
b tan 25° = 70 ÷ d, so d = 70 ÷ tan 25° = 150.1. Distance is 150 m.
c $150^2 + 70^2$ = 27 400, so distance is 166 m.
d tan θ = 100 ÷ 150, so θ = 33.69° or 33°41'. Bearing is 236.32°T or S56°19'W.

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19. **a** The length TC is the hypotenuse of a triangle with base 3 m and height 2.5 m. Using the theorem of Pythagoras, $3^2 + 2.5^2$ = 15.25, so TC is 3.9 m.
tan $\angle BTC$ = 3 ÷ 3.9, so $\angle BTC$ = 37.57°, so the required angle is 75.14° or 75°08'.
b The required angle STB is 90° + $\angle BTA$.
In ΔFAT, $TA^2 = FA^2 + TF^2$, so TA = 3.9 m.
tan $\angle BTA$ = 3 ÷ 3.9, so $\angle BTA$ = 37.57°, so the required angle is 127.57° or 127°34'.
c The required angle is $\angle ATE$. In ΔATF, tan $\angle ATF$ = 3 ÷ 2.5, so $\angle ATF$ = 50.19°.
Hence $\angle ATE$ = 100.38° = 100°23'
d Base = 6. Height = TC = 3.9. Area = bh ÷ 2 = 11.7 m^2.

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20. **a** 12°30'
b 15°06'

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21. **a** 10.33°
b 43.9°

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22. **a** 0.6157
b 0.9406

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23. The angle south of east is 22°. Thus cos 22° = $d \div 15$, so d = 15 cos 22° = 13.9 km

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24. tan $B = 3 \div 4$, so B = 36.87° = 36°52'. Answer: N36°52'W or 323.13°T.

Algebra:

1. Since each side is the same length, the area is equal to
A $(x - 3)(x - 3)$

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2. This is a difference of perfect squares question.
E $4x^2 - 9$

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3. **a** $3a^2 + 2ab$
b $x^2 + 8x + 12$

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4. **a** $5(x^2 + x - 6) = 5x^2 + 5x - 30$
b $(x - 1)(x^2 - x - 12) = x^3 - 2x^2 - 11x + 12$

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5. **a** $4x(x + 2)$
b $5x(x + 2y + 3xy)$

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6. **a** $(x + 5)(x - 5)$
b $(2y - 7x)(2y + 7x)$

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7. **a** $(x + 4)(x + 8)$
b $(x - 7)(x + 4)$

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8. **a** $5(9x^2 - 4) = 5(3x - 2)(3x + 2)$
b $[(x + 5) - (x - 4)][(x + 5) + (x - 4)] = 9(2x + 1)$

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9. $2(3x^2 + x - 2) = 2(3x - 2)(x + 1)$

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10. $(x + 5)^2 – (7y)^2 = (x + 5 – 7y)(x + 5 + 7y)$

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11. Let $X = x + 1$, then the expression is
$6X^2 – X – 2 = (3X –2)(2X + 1)$
$= [3(x + 1) – 2][2(x + 1) + 1]$
$= (3x + 1)(2x + 3)$

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12. $(x^2 + 10x + 25) – 4 – 25 = (x + 5)^2 – 29 =$
$(x + 5 – \sqrt{29})(x + 5 + \sqrt{29})$

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13. $\dfrac{(x+2)^2}{(x+3)(x-3)} \times \dfrac{(x+3)^2}{(x+2)(x-2)} =$
$\dfrac{(x+2)(x+3)}{(x-3)(x-2)}$

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14. $\dfrac{(x+2)^2}{(x+3)(x-3)} \times \dfrac{(x-3)^2}{(x+2)(x-2)} =$
$\dfrac{(x+2)(x-3)}{(x+3)(x-2)}$

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15. $\dfrac{(2x+3)(x-8)}{(x-8)(x+6)} = \dfrac{2x+3}{x+6}$

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16. $\dfrac{(x-4)(x+4)(x+5)^2}{(x-5)(x+5)(x+4)^2} = \dfrac{(x-4)(x+5)}{(x-5)(x+4)}$

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17. $\dfrac{5(x+5)+2(2x-4)}{10} = \dfrac{5x+25+4x-8}{10} =$
$\dfrac{9x+17}{10}$

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18. **a** $x(a – x) + x(b) = ax – x^2 + bx$ OR $x(b – x) + ax = bx – x^2 + ax$
b Large rectangle area is ab. Smaller rectangle area is $(a – x)(b – x)$; $ab – (a –x)(b – x)$
c When $a = 5$, $b = 7$, $x = 2$, then $bx – x^2 + ax = 14 – 4 + 10 = 20$.
When $a = 5$, $b = 7$, $x = 2$, then $ab – (a – x)(b – x) = 35 – (3)(5) = 35 – 15 = 20$.
d $ab – (a – x)(b – x) = ab – [ab – ax – bx + x^2] = ab – ab + ax + bx – x^2 = bx + ax – x^2$

19.
a $x + 3$

b $\dfrac{4}{a}$

20. **a** $\frac{5x+12}{6}$

b $\frac{2x^2+7x+2}{2x}$

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21. **a** $\frac{xy}{4}$

b 4

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22. $\frac{y-4}{y+2}$

Relationships and Variation:

1. **A** The highest power of x is 2.

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2. The constant of variation is the coefficient of x

C $\frac{2}{5}$

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3. **B** $C = 2T + 2$

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4. **a** $(x-9)(x+9)$

b $(x-8)(x+10)$

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5. **a** 4, 5, 6

b 10, –6, –2

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6. $y = x^2 + 10x + 25 + 8 - 25 = (x+5)^2 - 17$

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7. $y = x^2 + 3x + 2.25 + 1 - 2.25 = (x+1.5)^2 - 1.25$

Turning point is (–1.5, –1.25)

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8. **a** Y-intercept is (0, –10), $y = (x-5)(x+2)$, so the X-intercepts are (–2, 0) and (5, 0).

b Y-intercept is (0, –64), $y = (x-8)(x+8)$, so the X-intercepts are (–8, 0) and (8, 0).

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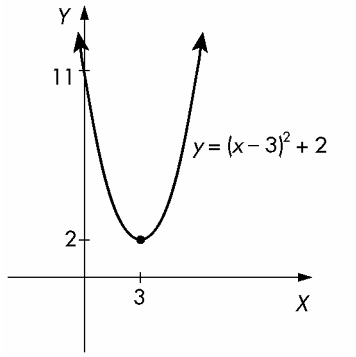
9. $y = x^2 - 6x + 11$, so Y-intercept is (0, 11). The graph does not cross the X-axis, because neither part of the expression for y can ever be negative.



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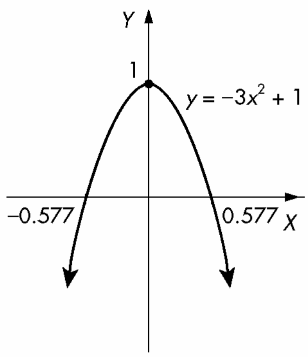
10. Y-intercept is (0, 1). This is the turning point.

$y = 1 - 3x^2 = (1 - \sqrt{3}\,x)(1 + \sqrt{3}\,x)$. $y = 0$ when $x = \frac{1}{\sqrt{3}}$ or $-\frac{1}{\sqrt{3}}$, that is $\pm$ 0.577.

The graph crosses the X-axis at 0.577 and –0.577.



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11. The highest level is the h-value at the turning point.
In turning point form, $h = -(x^2 - 5x + 6.25) + 6.25$. So the height reached is 6.25 metres.

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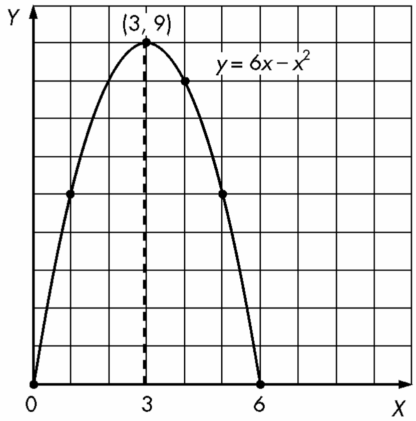
12. **a**

x	0	1	2	3	4	5	6	7
y	0	5	8	9	8	5	0	–7

b



c (3, 9)
d X-intercepts are (0, 0) and (6, 0); Y-intercept is (0, 0).
e $6x - x^2 = 6$ has solutions when $x^2 - 6x + 6 = 0$. This solution uses the formula, but completing the square or an iterative solution is OK.

$$x = \frac{6 \pm \sqrt{36 - 24}}{2} = 3 \pm 1.73 = 1.27 \text{ or } 4.73$$

Check: $6 \times 1.27 - 1.27^2 = 6.0$, and $6 \times 4.73 - 4.73^2 = 6.0$
f The ball was 6 m high on the way up, at 1.27 seconds, and on the way down, at 4.73 seconds.
g The ball only reaches 9 m. Since it does not reach the 12 m level there are no times when this happens.

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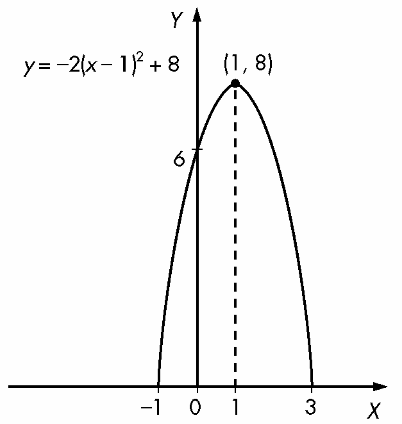
13. **a** Quadratic; the graph is not straight, and has a parabolic shape.
b Turning point is (1, 8).
$y = -2x^2 + 4x - 2 + 8 = -2x^2 + 4x + 6$ so the Y-intercept is (0, 6).
When $y = 0$, $-2x^2 + 4x + 6 = 0$, $x^2 - 2x - 3 = 0$, $(x - 3)(x + 1) = 0$.
So the X-intercepts are (–1, 0) and (3, 0).



c Solved above. $y = 6$, showing that the path started 6 metres above ground level.
d $y = 6$ means that $-2x^2 + 4x + 6 = 6$, hence $-2x(x - 2) = 0$, so $x = 0$ and 2. This means that the water was 6 m high at the start on the way up ($x = 0$) and also when it was 2 metres from the hose on the way down ($x = 2$).
e The highest point was at the turning point (1, 8); height 8 metres.
f The graph does not reach as high as $y = 10$, so the water did not reach that point. There is thus no solution to the equation, since there was no value of x for which it occurred.

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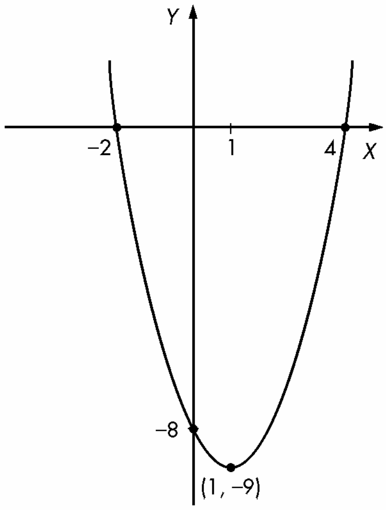
14. **a**



b True when p or q, or both, are zero.
Either or both X- and Y-intercepts will include the point (0, 0), so the graph must pass through it.

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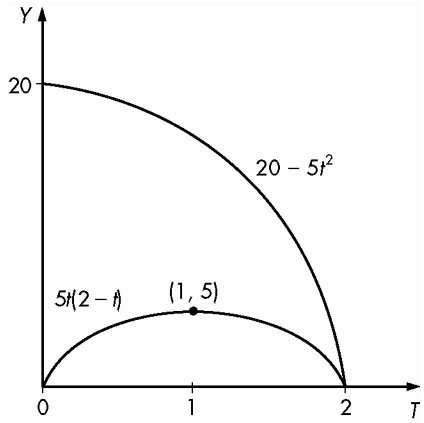
15. **a** 20 metres.
b When $y = 0$, $20 = 5t^2$, so $t = 2$. It hits the ground after 2 seconds.
c Y-intercept is (0, 20). This is the turning point.
$y = 5(4 – t^2) = 5(2 – t)(2 + t)$. T-intercepts are (2, 0) and (–2, 0).



d When t is 2, the ball hits the ground. Its turning point is at $t = 1$, $y = 5$. See graph.
e Yes. If it started at 45° then it travels for 2 seconds horizontally. This means it travelled 20 m, so it hit the brick just as they both landed.

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16. **a** $y = kx$, so $10 = 5k$, $k = 2$
b $y = \dfrac{k}{x}$, so $10 = \dfrac{k}{5}$, $k = 50$

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17. $y = 4x$

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18. $y = kx$, $10 = 4k$, so $k = 2.5$. Hence $y = 2.5x = 2.5 \times 15 = 37.5$

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19. $c = ka$, $2700 = 120k$, so $k = 22.5$. Hence $c = 22.5a = 22.5 \times 80 = 1800$. Answer \$1800.00

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20. $V = kr^3$, $800 = 5.75^3k$, so $k = 4.2$.
Hence $V = 4.2r^3$. So $1000 = 4.2 \times r^3$. Hence $r = 6.2$. Answer 6.2 cm

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21. $xy = 10$ or $y = \dfrac{10}{x}$

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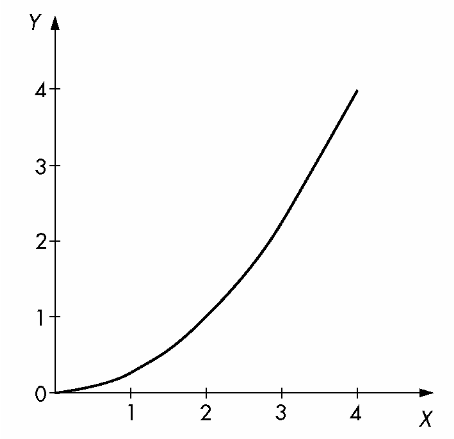
22. **a**

x	0	1	2	3	4
y	0	0.25	1	2.25	4

b



c $1.5^2 \div 4 = 0.56$

d $10 = \dfrac{x^2}{4}$. So $40 = x^2$, and $x = 6.3$

e If $x = 1$, $y = 0.25$. If $x = 1.1$, $y = 0.3025$.
The ratio is $0.3025 \div 0.25 = 1.21$, so the increase in x (from 1 to 1.1) increases y by 21%.

f This is direct square variation; doubling x multiplies y by 4.

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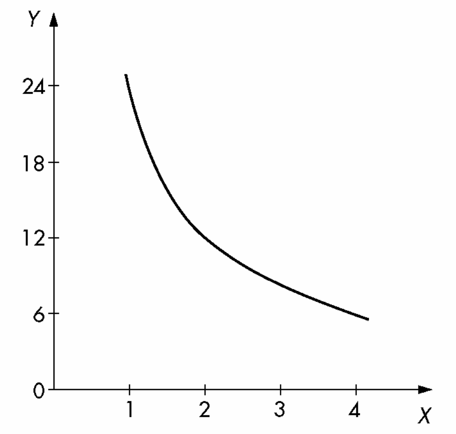
23. **a**

x	1	2	3	4
y	24	12	8	6

b



c 24 ÷ 1.5 = 16

d $2.5 = \frac{24}{x}$. So $x = \frac{24}{2.5} = 9.6$

e If $x = 1$, $y = 24$. If $x = 1.1$, $y = 21.82$.
The ratio is 21.82 ÷ 24 = 0.909, so the increase in x (from 1 to 1.1) decreases y by 9.09%.
f This is inverse variation; doubling x multiplies y by 0.5.
g Dividing by 0 has no meaning.

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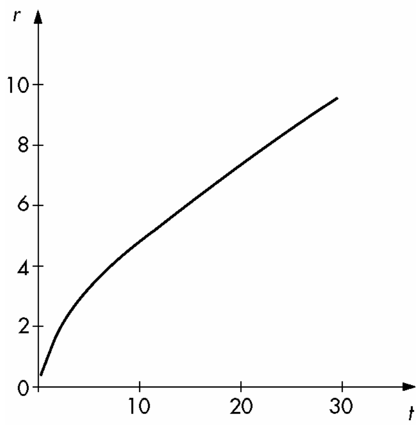
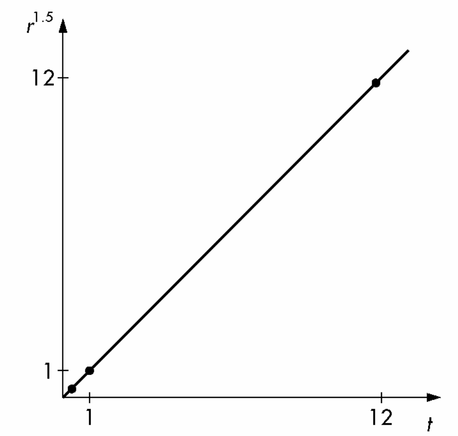
24. **a**



b It is possible, because the curve looks as if it may pass through the origin. More tests are needed.
c 0.2417, 1, 11.88
d



The values of $r^{1.5}$ and t are equal, so the graph will be straight, and will go through the origin when extended. Therefore it shows direct linear variation.
e $t = r^{1.5}$
f $9.545^{1.5} = 29.49$, so the relationship holds.

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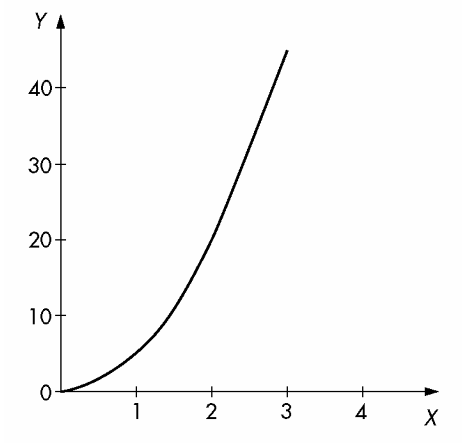
25. **a**



b $n = 2$
c All values are 5.
d $y = 5x^2$